Informational Cycles in Search Markets*

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Abstract

I study a sequential search model where buyers face an unknown distribution of offers and learn about the distribution from other buyers' actions. Each buyer observes whether a randomly chosen buyer traded in the previous period. I show that a cyclical equilibrium exists where the informational content of observing a trade fluctuates: a trade is good news about the distribution in every other period and bad news in the remaining periods. This leads to fluctuations in the volume and probability of trading. They fluctuate more if the unknown distribution is bad rather than good. A steady-state equilibrium where buyers are more likely to continue searching than in the cyclical equilibrium is less efficient than the cyclical equilibrium. A market that starts at date one converges to the cyclical equilibrium for some parameter values.

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1 Introduction

Search markets where searchers are uncertain about some market characteristics are prevalent. Different characteristics of a market may be unknown to searchers. For example, a job-seeker may be uncertain about the distribution of pay packages across employers, how many other job-seekers are applying for jobs, and how impatient the others are. Searchers learn about these unknown characteristics from the offers of the sellers they contact and from additional sources of information. A job-seeker learns from the pay packages of the interviewed employers, but also from the unemployment rate, from advertised wages, and by hearing if a friend has found a job.

In this paper, I show that learning about unknown market characteristics from other searchers' actions can lead to cycles in the volume and probability of trading. In reality, trade volume and probability fluctuate on labour markets, but also on real estate and other search markets. I focus on one unknown market characteristic, the distribution of sellers' offers, and one additional source of information, other searchers' actions. In reality, a house-hunter may not know the distribution of house values net of prices in the 8th district of Vienna and learns by hearing if a colleague has bought a house. In the paper, a searcher learns by observing if a randomly drawn searcher traded yesterday. Observing a trade is good news about the distribution if yesterday many searchers traded high-value offers. Observing a trade is, conversely, bad news if many searchers traded low-value offers. I show that a cyclical equilibrium exists where observing a trade is good news in every other period and bad news in the remaining periods. The fluctuations in the informational content of a trade translate into fluctuations in the volume of trading through the searchers' optimal behaviour. I thus show that information generated on a search market can lead to endogenous cycles.

This informational explanation of fluctuations is appealing for three reasons. First, endogenous cycles are delivered within an equilibrium of a parsimonious model. The environment is stationary (especially, lacks aggregate shocks), there are neither spillovers from other sectors nor adverse selection, and buyers are both rational and ex-ante homogeneous. Second, the type of

Table 1: *Average volatility in the job-finding rate and in the number of private houses sold in the US.*

	Job-finding rate	Number of houses sold
Recession	0.01825	0.00363
Boom	0.00899	0.00301

Notes: The table reports average volatility in a synthetic recession (boom), constructed from all recessions (booms) from 1951 to 2004 (job-finding rate) and from 1968 to 2016 (houses sold). See the Appendix for details.

information that I consider is realistic and present in many markets. In the model, a buyer observes if another buyer traded; in reality, a person hears if a colleague has bought a house and if a friend has found a job. Third, the model's predictions are in line with the US labour-market and housing data. In the model, fluctuations in the volume and probability of trading are larger if the unknown distribution of offers is bad rather than good, where good means better offers on average. Table 1 shows that in the US, fluctuations in the job-finding rate and in the number of private houses sold are larger in recessions than in booms.

In my model sellers are not strategic and have infinite capacity. A fixed amount of buyers enters the market in each period. Each buyer meets a randomly drawn seller and decides whether to accept the seller's offer or to continue to search. Continuing is costly because buyers discount future payoffs. Buyers are uncertain about the distribution of sellers' offers: if the distribution is good, more sellers sell a high-value item and fewer a low-value item than if the distribution is bad. The distribution is fixed throughout the operation of the market. In the full model, a buyer learns about the unknown distribution from her own experience and from a private "trade signal". Her own experience is the offers of the sellers that she meets. The trade signal reveals if yesterday a randomly drawn buyer traded ("a trade" is observed) or did not trade ("no trade" is observed). The informational content of the signal is determined in equilibrium. A trade is good news about the unknown distribution if yesterday many buyers traded with high-value sellers and bad news if many buyers traded with low-value sellers.

I study stationary symmetric equilibria of the model, where stationarity means that the endogenous variables have the same value in every $K \geq 1$ periods. In any equilibrium, all buyers trade high-value items. The only real decision is, thus, made by buyers who meet low-value sellers.

The three main results of the paper are as follows. First, I show that a cyclical equilibrium exists for an open set of parameter values and characterise it. To argue that the sole driver of cycles is learning from others' actions, I show that in a benchmark where buyers learn only from their own experience, only steady-state equilibria exist. In the cyclical equilibrium the endogenous variables fluctuate between two values across time. The equilibrium strategy says that in every other period some buyers condition their behaviour on the trade signal outcome (i.e., trade a low-value item after observing bad news and continue after observing good news) and in the remaining periods no buyer conditions her behaviour on the signal outcome.

Buyers' behaviour generates cycles in the volume of trading as follows. Suppose that yesterday very few buyers traded low-value items (while everyone trades high-value items) regardless of the distribution of offers. In this case, yesterday the volume of trading was low under the bad distribution and high under the good distribution. Thus, a trade that took place yesterday is good news about the distribution. Also, this trade event is very informative because the volume of trading differed a lot under the two distributions. This means that today all buyers who observe no trade become very pessimistic about the distribution and trade low-value items. Thus, the volume of trading is higher today than yesterday under both distributions. However, today the volume is higher under the bad rather than the good distribution because more buyers observe no trades. Thus, a trade that takes place today is bad news about the distribution. But this trade event is not very informative because today the volume of trading is similar under the two distributions. This is because today more buyers trade low-value items under both distributions as compared to yesterday. The low informational content of a trade today means that very few buyers tomorrow, just like yesterday, become pessimistic enough to trade low-value items even after observing bad news.

The fluctuations in the volume of trading are greater if the distribution of offers is bad rather than good. The reason is that there are more low-value sellers around if the distribution is bad. Their amount matters because information plays a role only for a buyer's decision about a low-value offer. The trade signal coordinates the actions of buyers more effectively within a period if the distribution is bad, which creates larger fluctuations.

The second main result is that the cyclical equilibrium is more efficient than one steady state. The cyclical equilibrium coexists with two steady-state equilibria for different parameter values. The steady-state equilibrium where buyers do not condition their behaviour on the trade signal is less efficient than the cyclical equilibrium. The other steady-state equilibrium, where in all periods some buyers condition their behaviour on the trade signal (i.e., trade a low-value item after observing bad news), is more efficient than the cyclical equilibrium. In both pairwise comparisons, more buyers condition their behaviour on the trade signal in the more efficient equilibrium, which moves the market closer to the efficient complete-information benchmark.

Since the cyclical equilibrium is more efficient than a steady state for some parameter values and less efficient for others, the model suggests that from efficiency viewpoint fluctuations on some real-life submarkets should be more worrisome than those on others. The cyclical equilibrium is more efficient than a steady state if buyers are more patient and if the value dispersion is larger. In reality, searchers with higher savings may be more patient. The value dispersion may be larger in a submarket for white-collar as opposed to blue-collar jobs and for commercial as opposed to residential real estate.

As the third result, I show that a market that has a concrete starting date converges to the cyclical equilibrium for an open set of parameter values. The cyclical equilibrium is, thus, a natural limit that some markets reach rather than a curiosity that can be sustained only in the long run.

Literature. My paper integrates two branches of literature: on explanations to fluctuations within an equilibrium and on learning about an unknown state in search markets. The paper's main contribution is to propose a novel mechanism, information on others' trades, as a driver of fluctuations.

Many different drivers of fluctuations have been suggested earlier.[1] The

[1]I focus on models with stationary environments where fluctuations occur within an

real business cycle theory proposes exogenous shocks (see Frisch, 1933, and Slutzky, 1937, for the seminal contributions).[2] Other suggestions are spillovers (see, for example, Caplin and Leahy, 1993), boundedly rational agents (see, for example, De Bondt and Thaler, 1985, Abreu and Brunnermeier, 2003, and Scheinkman and Xiong, 2003), and ex ante heterogeneous agents (see, for example, Conlisk et al., 1984, Sobel, 1984, and Woodford, 1992). The suggestion most related in spirit to mine is informational: adverse selection.[3] The driver of cycles in the adverse selection models is the lack of information about the value of an individual seller's offer, but in my model the driver is information about the unknown distribution of offers.

Learning about an unknown state in search models has been studied earlier.[4] These models, if they are dynamic and have more than two time periods, focus on steady-state equilibria. Asriyan et al. (2017) and Mauring (2017) study learning from a trade signal and Kaya and Kim (2015) and Kim (2017) from delay.[5] In them, a trade or delay is always either bad or good news about the market conditions because these papers focus on steady states. Conversely, in my model's cyclical equilibrium a trade is bad news in some periods and good news in others.

In Section 2 I introduce the model and the equilibrium concept. Section 3 shows that only steady-state equilibria exists in two benchmark models. Section 4 shows that both steady-state and cyclical equilibria exist in the full model. I compare efficiency of the full model's equilibria in Section 5. Section 6 analyses a market with a concrete starting date. I conclude by discussing alternatives to the model's assumptions in Section 7.

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equilibrium, as in my model. In many models with multiple equilibria, agents' different expectations about future payoffs sustain cycles. The most related among these are search models by Diamond and Fudenberg (1989) and Fershtman and Fishman (1992).

[2]The most related paper with exogenous shocks is Zeira (1994), where agents' learning about randomly changing demand generates cycles.

[3]See, for example, Janssen and Karamychev (2002), Janssen and Roy (2004), Daley and Green (2012), Kultti et al. (2015), Fuchs et al. (2016), and Maurin (2017).

[4]See, for example, Benabou and Gertner (1993), Dana (1994), Fishman (1996), Janssen et al. (2011), Lauermann (2012), Janssen and Shelegia (2015), Kaya and Kim (2015), Asriyan et al. (2017), Janssen et al. (2016), Kim (2017), Lauermann et al. (2017), and Mauring (2017).

[5]Learning from others' exits in strategic experimentation literature has been studied by Murto and Välimäki (2011), Cripps and Thomas (2016), and others.

2 Model

I first describe the setup of the model and then the equilibrium concept.

2.1 Setup

Time t is discrete and runs till $+\infty$. The market starts at $t_1 = -\infty$ (except in Section 6 where it starts at $t_1 = 1$). The market is characterised by state $\theta \in \{L, H\}$ that is fixed for all periods.

Buyers. In each period t a mass one of buyers enter the market. Each buyer has a unit demand and discounts the future at rate $\delta \in (0, 1)$. A buyer searches sequentially for a good offer. A buyer dies at the end of her second life period.[6] I call the buyers who entered today "young" buyers and who entered yesterday and did not exit "old" buyers. Buyers do not know the state of the market and their prior belief is $\pi := P(\theta = H)$.

Sellers. The sellers are infinitely lived and have infinite capacity. The indirect utility that a seller offers to a buyer is determined by the state of the market, θ. If the state is bad ($\theta = L$), all sellers have a low-value offer $v_L > 0$. If the state is good ($\theta = H$), half of the sellers have a low-value offer v_L and half have a high-value offer v_H with $v_H > v_L$.

Timing. First, new buyers enter and each buyer is randomly matched to a seller. A buyer sees the offer of the seller, $v \in \{v_L, v_H\}$, and updates her beliefs about the state. Then she decides whether to accept the offer. The buyers who trade exit the market. At the end of the period old buyers die and young buyers who did not trade are carried over to the next period.

Information. Buyers update their beliefs about the state using Bayes' rule. In the main part of the paper, a buyer updates her beliefs based on her own experience and by observing something about others' actions. A buyer's own experience is the offer of the seller that she meets, $v \in \{v_L, v_H\}$. Offer v_H reveals that the state is good. Offer v_L is bad news: a buyer's posterior belief after v_L is lower than her prior.

A buyer b learns from others' actions from a private "trade signal": by

[6] I discuss alternatives to this and other assumptions in Section 7.

observing at t whether a randomly drawn buyer b' traded at $t-1$, without observing the offer that b' saw. If b observes that b' traded, I say that b observes a "trade" (outcome T_{t-1}) and if she observes that b' did not trade, I say that b observes "no trade" (outcome N_{t-1}). Conditional on the state and date, the realisations of the signal are i.i.d. The signal's precision is determined in equilibrium: $P(T_t|\theta) =: \tau_t^\theta$ is the equilibrium probability that a randomly drawn buyer trades in state θ at date t. A trade at t is informative if the equilibrium probability of a trade at t differs across states.

Strategies. A young buyer's strategy specifies for each possible private history whether to accept the offer of the seller she meets or to continue to search. A young buyer optimally accepts v_H: life does not get better in this model. An old buyer's strategy is whether to accept or reject the offer she receives and she optimally accepts both v_L and v_H. Thus, a relevant strategy only specifies whether a buyer who is born at t and meets a v_L-seller accepts v_L or continues. Formally, a (relevant) strategy σ_t is a mapping from the space of a young buyer's private histories (conditional on meeting a v_L-seller) to the space of all probability distributions over her actions "accept" and "continue", $\sigma_t : v_L \times \{T_{t-1}, N_{t-1}\} \to \Omega(\{A, C\})$, where Ω is the set of all probability distributions over accepting v_L (A) and continuing (C).

2.2 Equilibrium

I study the model's symmetric stationary equilibria. A strategy profile σ_t^* is an equilibrium if for all (v, i), where $v \in \{v_L, v_H\}$ is the offer and $i \in \{T_{t-1}, N_{t-1}\}$ the signal outcome that the buyer observes, σ_t^* is

(a) optimal: σ_t^* is a best response of a buyer to all other buyers using σ_t^*;

(b) uses Bayes' updating: the posterior odds are $\frac{P(H|v,i)}{P(L|v,i)} = \frac{\pi}{1-\pi} \frac{P(v|H)}{P(v|L)} \frac{P(i|H)}{P(i|L)}$, where $P(x|\theta)$ is the equilibrium probability of event x in state θ;

(c) consistent: the buyers' beliefs are consistent with the strategy σ_t^*;

(d) stationary: for all endogenous variables x and periods t, $K \in \mathbb{N}$ exists such that $x_t = x_{t+K}$. If $K = 1$, the equilibrium is called a steady state.

Intuitively, in all equilibria a young buyer who meets a v_L-seller accepts v_L if she is pessimistic enough about the state and continues if she is optimistic enough. A critical belief $\bar{\pi}$ plays a role throughout the equilibrium analysis so I define it here:

$$\bar{\pi} := \frac{2v_L(1-\delta)}{\delta(v_H - v_L)}. \tag{1}$$

The critical belief decreases in the potential benefit of continuing, $v_H - v_L$, and in the discount factor, δ.

3 Benchmarks

I show that only steady-state equilibria exist in two benchmark cases. In the first benchmark, buyers know the state. In the second, buyers do not know the state and learn about it only from their own experience. In both benchmarks, any optimal strategy for a single buyer is an equilibrium because there is no interaction between buyers.

3.1 Buyers know the state

Only steady-state equilibria exist if buyers know the state.

Proposition 1. *Suppose that buyers know the state θ. A strategy whereby a young buyer who meets a v_L-seller*

(i) accepts v_L is the unique equilibrium if $\theta = L$.

(ii) accepts v_L is the unique equilibrium if $\theta = H$ and $1 < \bar{\pi}$.

(iii) continues is the unique equilibrium if $\theta = H$ and $\bar{\pi} < 1$.

Proof. In the Appendix. □

The result is intuitive. If the state is bad, only low-value offers are around. A young buyer optimally accepts one rather than delays accepting a low-value offer because of discounting. If the state is good, a young buyer is better off continuing after meeting a v_L-seller if the cutoff belief $\bar{\pi}$ is low enough (defined in (1)): if the potential benefit of continuing, $v_H - v_L$, and the discount factor, δ, are large enough.

For the rest of the paper, I assume that a young buyer who meets a v_L-seller continues in the unique equilibrium of the good state, i.e., that $\bar{\pi} < 1$. If the condition did not hold, all equilibria in the rest of the paper would be trivial: young buyers accept any first offer.

3.2 Buyers do not know the state and learn only from their own experience

I show that only steady-state equilibria exist if buyers do not know the state and learn only from their own experience.

Proposition 2. *Suppose that buyers do not know the state θ and learn only from their own experience. A strategy whereby a young buyer who meets a v_L-seller*

(i) accepts v_L (steady state 0) is the unique equilibrium if $\pi < \frac{2\bar{\pi}}{\bar{\pi}+1}$.

(ii) continues is the unique equilibrium if $\frac{2\bar{\pi}}{\bar{\pi}+1} < \pi$.

Proof. In the Appendix. □

Figure 1a (on page 13) illustrates the regions of the parameter space where the two steady states exist. The result is intuitive. A young buyer optimally continues after receiving a low-value offer if she is optimistic enough and accepts the offer if she is pessimistic enough. Her posterior belief is lower than the prior because a low-value offer is bad news: there are more v_L-sellers in the market if the state is bad rather than good. Thus, to sustain the equilibrium where the buyer continues, it is not sufficient that her prior exceeds the critical belief $\bar{\pi}$ ($\pi > \bar{\pi}$), she must be more optimistic ex ante ($\pi > \frac{2\bar{\pi}}{1+\bar{\pi}}$).

4 Buyers do not know the state and learn from their own experience and the others' trades

In this section I derive the steady-state equilibria and a cyclical equilibrium of the full model where buyers learn from their own experience and from the trade signal. The signal reveals whether one randomly drawn buyer traded or did not trade yesterday. The trade signal introduces interaction between buyers' optimal policies: a buyer trades only if it is optimal for her to do so and all buyers' trading decisions together determine the content of the trade signal, thus, the optimal decision of a single buyer tomorrow.

4.1 Steady-state equilibria

I show here that three different steady-state equilibria in pure strategies are supported in partly overlapping regions of the parameter space.[7]

Proposition 3. *A strategy whereby a young buyer who meets a v_L-seller*

(i) accepts v_L (steady state 0) is an equilibrium if $\pi < \frac{2\bar{\pi}}{\bar{\pi}+1}$.

(ii) accepts v_L after observing no trade and continues after observing a trade (steady state 1) is an equilibrium if $\frac{(\sqrt{5}-1)\bar{\pi}}{(\sqrt{5}-1)\bar{\pi}+(\sqrt{3}-1)(1-\bar{\pi})} < \pi < \frac{(3-\sqrt{5})\bar{\pi}}{(3-\sqrt{5})\bar{\pi}+(2-\sqrt{3})(1-\bar{\pi})}$.

(iii) continues (steady state 2) is an equilibrium if $\frac{3\bar{\pi}}{2\bar{\pi}+1} < \pi$.

In the steady-state equilibria, a trade is (weakly) good news.
No steady-state equilibria where a trade is strictly bad news exist.

Proof. In the Appendix. □

[7] I focus on pure-strategy equilibria here because degenerate sets of parameter values support mixed-strategy equilibria, first, in the benchmark models and, second, in the full model if the market starts at $t_1 = 1$.

The steady states are named so that a young buyer is more likely to continue in a steady state with a higher number. In all steady states either a trade provides no news (because all buyers trade in their entry period in both states) or is good news about the state. A trade is (weakly) good news in all steady states because a trade is (weakly) more likely in the good state. A trade is more likely because there are more sellers who trade with a high probability in the good state: v_H-sellers trade with probability one, whereas v_L-sellers trade with a (weakly) lower probability.

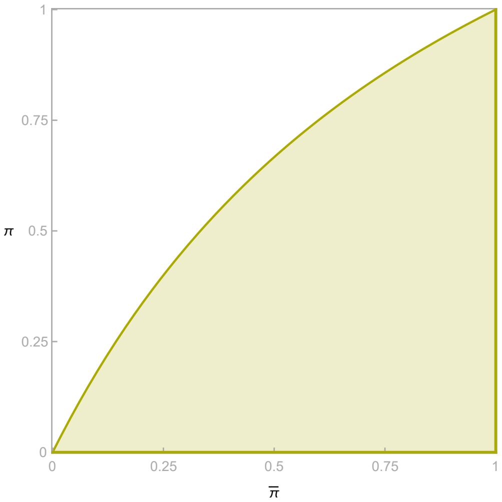
The regions of the parameter space that support the steady states are depicted in Figures 1a, 1b, and 1c (on page 13). The regions that support steady states 0 and 1 overlap. In the gap between the regions that support steady states 1 and 2, I show in Section 4.2, a cyclical equilibrium is supported. The ordering of the regions is intuitive. The more optimistic buyers are ex ante, the more willing they are to continue in a steady state.

4.2 A cyclical equilibrium

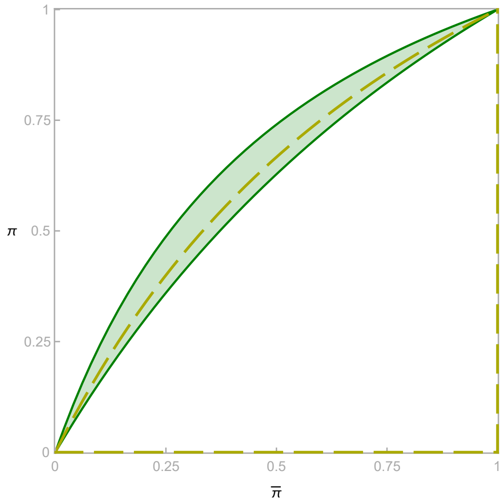
I show that a cyclical equilibrium, where only information on trades sustains the cycles, is supported by an open set of parameter values. In this equilibrium, the volume and probability of trading fluctuate between two values over time. A trade is good news in one period, and bad news in the next period. I call a "trade at t" the event that a buyer trades at t despite this trade being observed only at $t+1$. Let τ_t^θ denote the probability of a randomly drawn buyer trading in period t in state θ. For clarity of exposition, I call periods $t, t+2, t+4, ...$ odd and periods $t+1, t+3, t+5, ...$ even. The names could be swapped because I consider the long run, but not in Section 6 where the market starts at $t_1 = 1$.

Proposition 4. *A strategy whereby a young buyer who meets a v_L-seller*

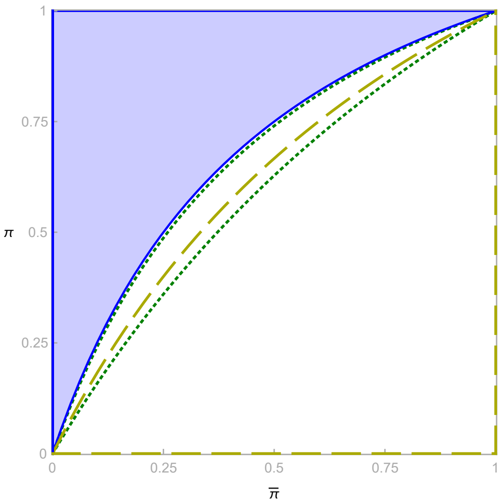
- *(i) in an odd period, continues, and*
- *(ii) in an even period, accepts v_L after observing no trade and continues after observing a trade,*



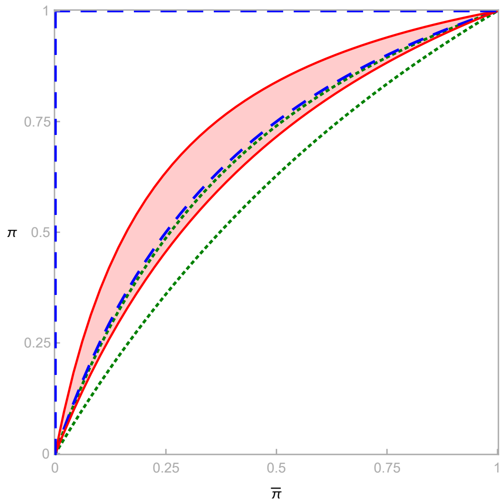
(a) *Steady state 0 (shaded region).*



(b) *Steady state 1 (shaded region; steady state 0 between dashed lines).*



(c) *Steady state 2 (shaded region; steady state 0 between dashed and steady state 1 between dotted lines).*



(d) *Cyclical equilibrium (shaded region; steady state 1 between dotted and steady state 2 between dashed lines).*

Figure 1: *A market that starts at* $t_1 = -\infty$. *A young buyer who meets a* v_L*-seller: in steady state 0, accepts* v_L*; in steady state 1, accepts* v_L *after observing no trade and continues after observing a trade; in steady state 2, continues; in the cyclical equilibrium, in an odd period continues, and in an even period accepts* v_L *after observing no trade and continues after observing a trade.*

is an equilibrium if $\frac{12\bar{\pi}}{12\bar{\pi}+(7-\sqrt{5})(1-\bar{\pi})} < \pi < \frac{4\bar{\pi}}{4\bar{\pi}+(3-\sqrt{5})(1-\bar{\pi})}$.

In the cyclical equilibrium, a trade in an odd period is good news and in an even period is bad news. The probabilities of trading are $\tau^H_{odd} = \frac{\sqrt{5}-1}{2}$, $\tau^H_{even} = \frac{7-\sqrt{5}}{6}$, $\tau^L_{odd} = 0$, *and* $\tau^L_{even} = 1$. *The probability fluctuates more in the bad state than in the good state.*

The volume of trading is low in odd periods and high in even periods. The volumes are $Vol^H_{odd} = \frac{\sqrt{5}+1}{4}$, $Vol^H_{even} = \frac{7-\sqrt{5}}{4}$, $Vol^L_{odd} = 0$, *and* $Vol^L_{even} = 2$. *The volume fluctuates more in the bad state than in the good state.*

Proof. In the Appendix. □

I explain how the equilibrium strategy sustains cycles in the volume of trading (the argument for the probability of trading is analogous). Young buyers' optimal actions differ in odd and even periods, which leads to different amounts of young buyers trading not only across periods, but also across states. This is the main driver of cycles in the aggregate volume of trading. Old buyers' actions are not crucial for the cycles because their optimal behaviour is the same across periods and states, and their amounts are relatively similar across states in any period.

To understand the effect of young buyers' actions, suppose that trade volume is low at t because young buyers are only willing to trade high-value items. This makes a trade good news for young buyers at $t+1$: those who observe a trade are less willing and those who observe no trade are more willing to trade low-value items. If the pessimistic ones trade, then trade volume is high at $t+1$ and a trade can be bad news for buyers at $t+2$. This is because buyers at $t+2$ correctly infer that many of the trades at $t+1$ took place with low-value items, which are abundant if the state is bad. A trade that is bad news due to this, however, is not as bad news as no trade was for buyers at $t+1$. This is because the signal becomes less precise if some (rather than no) young buyers accept low-value items in both states. Thus, at $t+2$ both young buyers who observe a trade and who observe no trade are optimistic enough not to trade the low-value item. But then only high-value items generate trades with young buyers at $t+2$, just like at t.

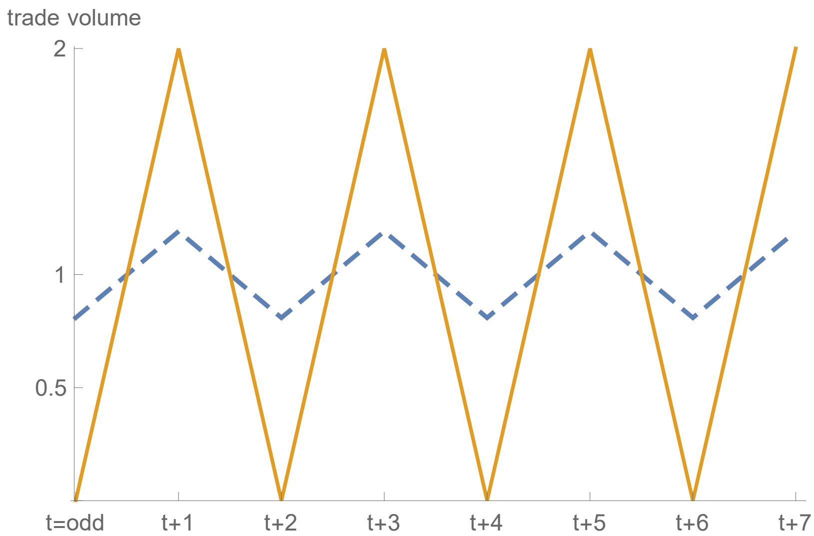


Figure 2: *The volume of trading in the cyclical equilibrium in the good state (blue dashed) and bad state (orange).*

Recall that only steady-state equilibria exist if buyers learn only from their own experience: cycles are sustained by information about others' trades.

Figure 2 depicts the volume and Figure 3 the probability of trading in the cyclical equilibrium. In an odd period, young buyers only trade high-value items, which leads to a lower volume and probability of trading in the bad as compared to the good state. In an even period, young buyers who observe no trade accept low-value offers. More no trade events are observed if the state is bad, which leads to a higher volume and probability of trading in the bad as compared to the good state.

Data supports the model's prediction that the probability of trading fluctuates more in the bad rather than the good state. The top row in Figure 4 depicts average fluctuations in the US job-finding rate and the bottom row fluctuations in the probability of a trade in the model.[8] The volatility is larger in recessions than in booms both in the data and the model.

The red shaded region of the parameter space in Figure 1d (on page 13) supports the cyclical equilibrium. The regions of the parameter space that

[8]Please see the Appendix for a similar result on the volume of trading and the number of residential houses sold in the US. The Appendix also contains details on how the data is compiled and additional evidence from regressions.

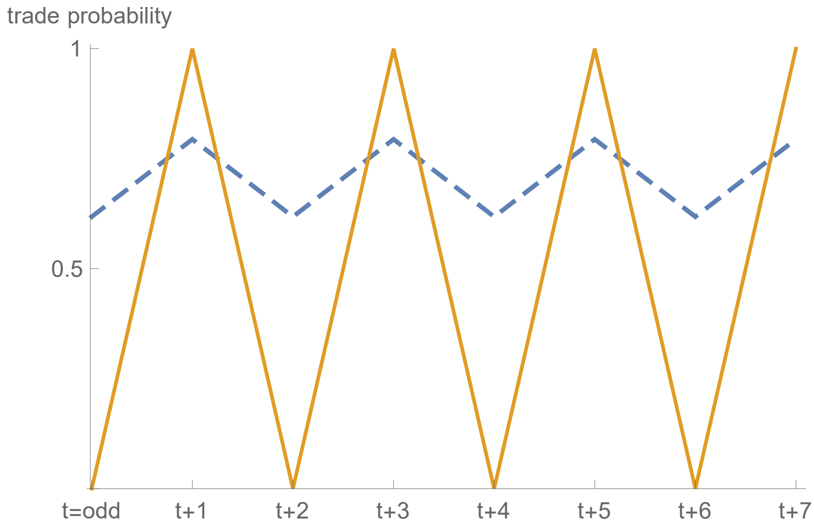


Figure 3: *The probability of trading in the cyclical equilibrium in the good state (blue dashed) and bad state (orange).*

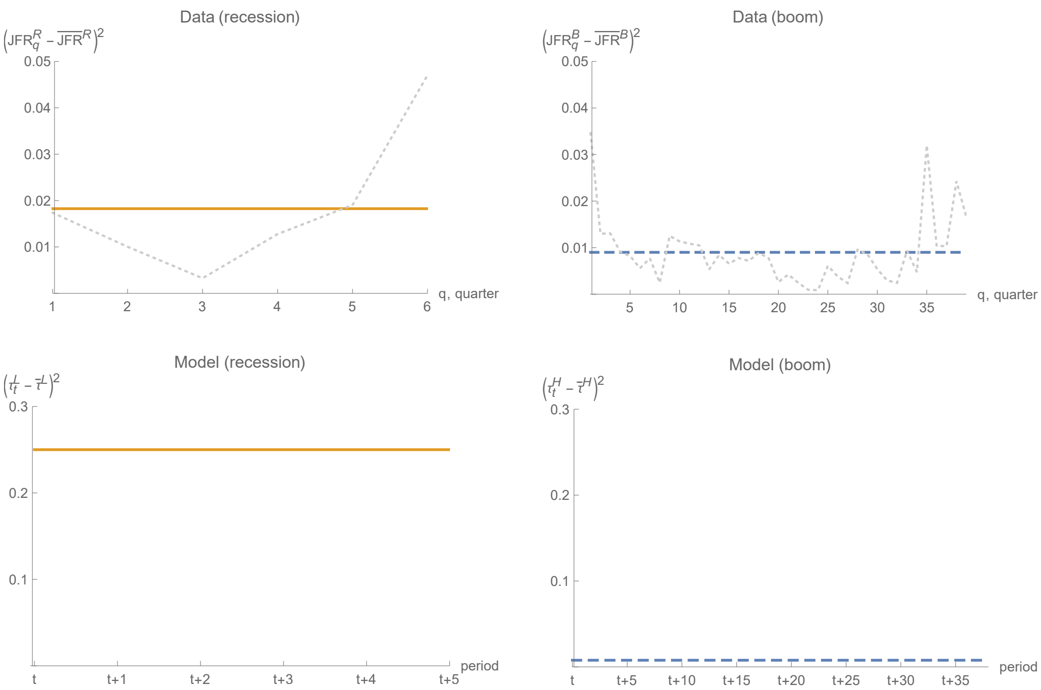


Figure 4: *Average volatility of the US job-finding rate (top panel) and of the model's probability of trading (bottom panel) in an average recession (orange solid) and boom (blue dashed).*

support steady state 0 and the cyclical equilibrium do not overlap. The regions that support steady state 1 and steady state 2 overlap with the region that supports the cyclical equilibrium, but no region nests another. This is intuitive: in even periods, buyers use the same strategy in the cyclical equilibrium as in steady state 1 and in odd periods the same strategy as in steady state 2. I compare efficiency of the two equilibria that are supported by the same parameter values in Section 5.

5 Efficiency

I compare the efficiency of the different equilibria in the regions of the parameter space where multiple equilibria exist. Efficiency is measured by an entering buyer's expected value from participating in the market. I show that the cyclical equilibrium is more efficient than one steady state, which contradicts the common wisdom that cycles reduce efficiency.

Proposition 5. *In the region of the parameter space where the cyclical equilibrium coexists with*

(i) the steady state where a young buyer accepts v_L only after no trade (steady state 1), the steady state is more efficient.

(ii) the steady state where a young buyer continues after v_L (steady state 2), the cyclical equilibrium is more efficient.

Proof. In the Appendix. □

In the two pairwise comparisons, a young buyer is more likely to accept v_L in the more efficient equilibrium. To understand why this increases efficiency, recall that if the state is bad and known, it is efficient for a young buyer to accept v_L. Conversely, if the state is good and known, it is efficient for her to continue after v_L. If the state is unknown, buyers react to the informative trade signal more in the more efficient equilibrium. A higher probability of accepting v_L, thus, increases efficiency.

For a fixed prior belief, the cyclical equilibrium is more efficient than the co-existent steady state if the critical belief $\bar{\pi}$ is lower. The model suggests, thus, that fluctuations are less worrisome in (sub-)markets where $v_H - v_L$ or δ is higher (see (1)). Fluctuations are less worrisome on a labour market where the support of offered wages is larger (for example, white-collar as opposed to blue-collar jobs), on a real-estate market where the support of offered values net of prices is larger (for example, commercial as opposed to residential real estate), and any market where searchers are more patient (for example, have higher as opposed to lower savings).

6 A market that starts at $t_1 = 1$

Consider the full model that starts at $t_1 = 1$ instead of $t_1 = -\infty$. I show that the cyclical equilibrium as described in Proposition 4 is reached from an open set of parameter values. The cyclical equilibrium is, thus, a natural limit that some markets reach.

Proposition 6. *Suppose that the market starts at $t_1 = 1$.*

(i) The steady state where a young buyer accepts v_L (steady state 0) is reached at $t = 1$ if $\pi < \frac{2\bar{\pi}}{\bar{\pi}+1}$.

(ii) The steady state where a young buyer continues after v_L (steady state 2) is reached at $t = 2$ if $\frac{4\bar{\pi}}{3\bar{\pi}+1} < \pi$.

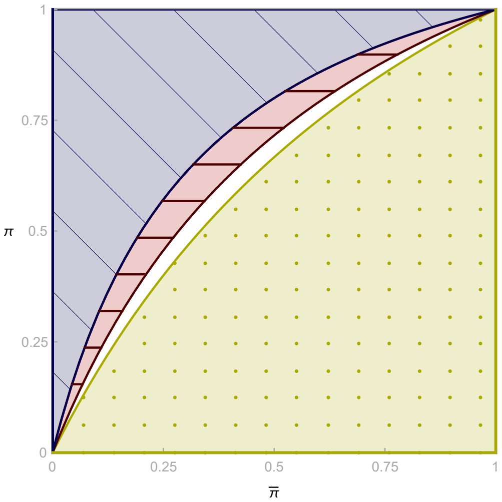
(iii) The cyclical equilibrium is reached as $t \to \infty$ if $\frac{12\bar{\pi}}{12\bar{\pi}+(7+\sqrt{5})(1-\bar{\pi})} < \pi < \frac{4\bar{\pi}}{3\bar{\pi}+1}$.

The three sets of parameter values are disjoint.

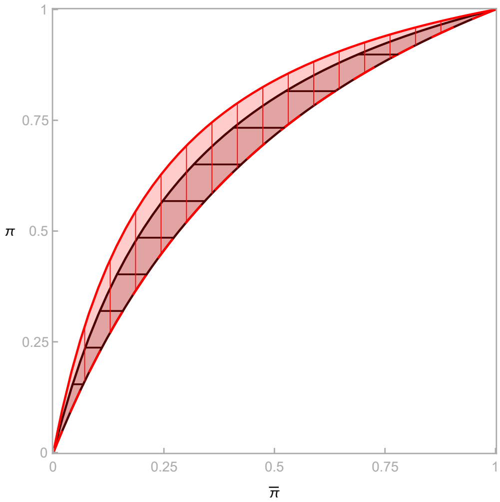
Proof. In the Appendix. □

The result of Proposition 6 is summarised in Figure 5a. The ordering of the three regions is intuitive: if buyers are more optimistic ex ante, they are more willing to continue in equilibrium.[9]

[9]For some values of the parameters in the white region in Figure 5a, longer cyclical equilibria are reached in the limit. In these equilibria, a trade is good news in one period and bad news in the next consecutive $K - 1 > 1$ periods.



(a) *Limit equilibria: steady state 0 (yellow dots), cyclical equilibrium (dark red horizontal lines), and steady state 2 (blue diagonal lines).*



(b) *The cyclical equilibrium is reached in a market that starts at $t_1 = 1$ (dark red horizontal lines) versus $t_1 = -\infty$ (red vertical lines).*

Figure 5: *A market that starts at $t_1 = 1$.*

Figure 5b shows that a smaller region of the parameter space sustains a cycle in a market that starts at $t_1 = 1$ as compared to a market that starts at $t_1 = -\infty$. The reason is that at $t = 1$ no trade information is observed if $t_1 = 1$: the young buyers' incentives to continue at $t = 1$ are different on a market without a past (i.e., if $t_1 = 1$) as opposed to a market with an infinite past (i.e., if $t_1 = -\infty$).

I measure the rate of convergence to the cyclical equilibrium by calculating how far the probability of trading at a certain date is from its long-run value. In the bad state, the probability converges to its long-run value immediately. In the good state, at $t = 2$ the probability of trading is about 5% higher than the long-run value τ^H_{even} and at $t = 3$, about 3% lower than the long-run value τ^H_{odd}.[10] At $t = 6$ the probability is about 0.1% higher and at $t = 7$ about 0.1% lower than the corresponding long-run values. The convergence path is depicted in Figure 6.

[10]Please see the Appendix for the exact calculations.

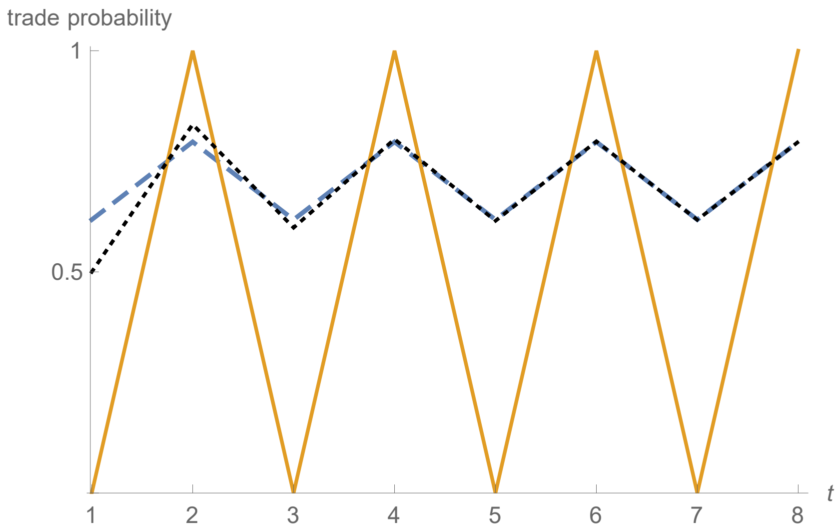


Figure 6: *The convergence of trading probabilities to the cyclical equilibrium: the probability of trading in the good state if* $t_1 = 1$ *(black dotted) and if* $t_1 = -\infty$ *(blue dashed); and in the bad state (orange solid).*

7 Concluding discussion

I show that a cyclical equilibrium exists for open sets of parameter values under several modifications to the model's assumptions.

7.1 Price-setting

Suppose that sellers are strategic, discount future payoffs at some rate $\delta_s < 1$, and have zero marginal costs. The model's results are robust to price-setting in the following forms: take-it-or-leave-it offers by buyers and Nash bargaining with fixed bargaining weights. The intuition is that in both cases, a seller receives a fixed fraction of the value of its product to a buyer, v, if he sells his product. Conversely, if he waits, he gets the same fixed fraction tomorrow, which is worse because of discounting.

7.2 Positive fractions of low-and high-value sellers in both states

Let the fraction of v_H-sellers be μ^θ in state θ with $0 \le \mu^L < \mu^H < 1$. A cyclical equilibrium exists for certain values of μ^L and μ^H.

Proposition 7. *Let the fraction of v_H-sellers be μ^θ in state θ with $0 \le \mu^L < \mu^H < 1$. Consider a strategy whereby a young buyer who meets a v_L-seller*

(i) in an odd period, continues, and

(ii) in an even period, accepts v_L after observing no trade and continues after observing a trade.

The necessary and sufficient conditions for the strategy profile to be an equilibrium are that

1. *a trade in an odd period is good news ($\tau^H_{odd} > \tau^L_{odd}$) and a trade in an even period is bad news ($\tau^H_{even} < \tau^L_{even}$), where the trade probabilities are $\tau^\theta_{odd} = \frac{\mu^\theta+z}{2-\mu^\theta+z}$ and $\tau^\theta_{even} = (2-\mu^\theta)^{-1}\left[1+\frac{2(1-\mu^\theta)^2}{2-\mu^\theta+z}\right]$ for $z := \sqrt{\mu^\theta(4-3\mu^\theta)}$ and $\theta = L, H$;*
2. *no buyer wants to deviate, i.e., that*

$$\frac{\tilde{\pi}}{1-\tilde{\pi}}\frac{1-\mu^L}{1-\mu^H}\cdot\max\left\{\frac{\tau^L_{even}}{\tau^H_{even}},\frac{\tau^L_{odd}}{\tau^H_{odd}}\right\} < \frac{\pi}{1-\pi} < \frac{\tilde{\pi}}{1-\tilde{\pi}}\frac{1-\mu^L}{1-\mu^H}\frac{1-\tau^L_{odd}}{1-\tau^H_{odd}},$$

where $\tilde{\pi} := (\mu^H-\mu^L)^{-1}\left[\frac{(1-\delta)v_L}{\delta(v_H-v_L)}-\mu^L\right]$.

The probability of trading fluctuates more in the bad than in the good state. Two different sufficient conditions for the equilibrium to exist are that $\mu^L = 0$ and that $\mu^H < \frac{2}{7}(3-\sqrt{2})$.

Proof. In the Appendix. □

The second sufficient condition guarantees that the trade probabilities are ordered as necessary: τ^θ_{even} decreases in μ^θ and τ^θ_{odd} increases in μ^θ for all $\mu^\theta < \frac{2}{7}(3-\sqrt{2})$. The necessary and sufficient conditions are satisfied for a range of parameter values. For example, if $\pi = 1/2$, $\mu^L = 1/4$ and $\mu^H = 1/2$, then the trade probabilities are $\tau^H_{odd} = 0.618$, $\tau^H_{even} = 0.794$, $\tau^L_{odd} = 0.434$, and $\tau^L_{even} = 0.814$. The cyclical equilibrium is supported by all $\tilde{\pi} \in (0.31, 0.39)$. This example illustrates the more general point that a trade signal outcome does not have to reveal a state for the cyclical equilibrium to exist.

7.3 Long-lived buyers

Consider a model where buyers can live for ever, but survive till the next period with a fixed probability $\delta \in (0,1)$. The survival probability replaces the discount factor and ensures the existence of a steady-state equilibrium. A buyer observes in each period of life whether a randomly drawn buyer traded in the previous period or did not trade. That is, the buyer can tell trades apart from exits due to the exogenous destruction rate.

Proposition 8. *Let buyers be infinitely-lived and in each period survive to the next period with probability $\delta \in (0,1)$. Sufficient conditions for a strategy whereby a buyer who meets a v_L-seller*

(i) and knows that the state is good, continues,

(ii) and does not know the state,

- *in odd periods, continues and*
- *in even periods, continues after observing a trade, and accepts v_L after observing no trade,*

to be an equilibrium are that $\frac{2\tilde{\pi}}{2\tilde{\pi}+\tau^H_{even}(1-\tilde{\pi})} < \pi < \frac{4\tilde{\pi}}{3\tilde{\pi}+1}$, with τ^H_{even} given below.

In equilibrium, a trade in an odd period is good news ($\tau^H_{odd} > \tau^L_{odd}$) and a trade in an even period is bad news ($\tau^H_{even} < \tau^L_{even}$). The probabilities of trading are $\tau^L_{odd} = 0$, $\tau^L_{even} = 1$, $\tau^H_{odd} = \frac{1}{2}$, and

$$\tau^H_{even} = \frac{1}{2\delta^3}\left(16 + 6\delta - 2\delta^2 + \delta^3 - \sqrt{256 + 192\delta - 28\delta^2 - 40\delta^3 + 4\delta^5 + \delta^6}\right).$$

The probability fluctuates more in the bad than in the good state.

Proof. In the Appendix. □

7.4 Partially observed "prices"

Consider a model where the majority of the buyers (fraction $1 - \varepsilon$) observe a trade signal as before (that is, without observing the value at which the trade/no trade took place) and a minority (fraction ε) observe not only whether another buyer traded, but also the "price": the value v at which this trade/no trade took place. For small enough ε, a cyclical equilibrium is sustained by a strategy that is similar to the one described in Proposition 4.

Proposition 9. *A strategy whereby a young buyer who meets a v_L-seller*

(i) in an odd period, accepts v_L after observing a trade at value v_L and continues otherwise, and

(ii) in an even period, accepts v_L after observing no trade or no trade at value v_L and continues otherwise,

is an equilibrium for an open set of parameter values if $\varepsilon < \bar{\varepsilon}$ for some $\bar{\varepsilon} > 0$.

In the cyclical equilibrium, a trade in an odd period is good news and in an even period is bad news. A trade at value v_L in an odd period is good news and in an even period is bad news.

Proof. In the Appendix. □

The modification in the strategy comes about because a trade (no trade) at value v_L tells a buyer not only about the event of a trade (no trade) at v_L, but also that another buyer met a v_L-seller. It is as bad news as observing another low-value offer. Since a trade at v_L in an even period is bad news, a young buyer who in an odd period meets a v_L-seller and observes a trade at v_L becomes so pessimistic about the state that she accepts v_L in equilibrium.

Intuitively, the informational content of a trade at an unknown value can fluctuate, but it is ex ante plausible that a trade at value v_L is bad news in all periods. The reason why a trade at v_L is good news in an odd period is because of the composition effect: there are many more old buyers if the state is good and they accept low-value offers.

If ε is large, an equilibrium in these strategies does not exist because the probability that a v_L-seller trades is higher in the bad than in the good state in all periods. If $\varepsilon = 1$, for example, in the good state half of the young buyers observe a trade at v_H, learn that the state is good and reject v_L. Conversely, in the bad state, all buyers observe trade information about a v_L-seller, which leads to more than a half of the young buyers accepting v_L. Thus, a trade at v_L is bad news in all periods.

7.5 Markov state

Consider a model where the state of the market may change in each period. Suppose that the state is persistent: in each period, the state is more likely to remain the same than to change. The model without a starting date (i.e., if $t_1 = -\infty$) cannot be solved anymore because the changing state and the information structure make the environment nonstationary. In particular, the signal introduces dependencies across periods $t + 1$, t and $t - 1$, which means that the entire history of states matters for a buyer's optimal decision.

In a model with a starting date (i.e., if $t_1 = 1$), I can show for $t = 1, ..., 6$ that for certain parameter values the expected trade probabilities fluctuate as required for a cyclical equilibrium: a trade in periods $t = 1, 3, 5$ is good news and a trade in periods $t = 2, 4, 6$ is bad news. I do not know if in the limit the market converges to a cycle in expectations.

A Appendix

The Appendix contains details on the empirical evidence and the proofs.

A.1 Empirical evidence

I provide the details about the data that I refer to in the Introduction and in Section 4.2. The data sources are (1) for the job-finding rate (JFR_q): Shimer (2012); (2) for the number of houses sold (S_q): Datastream (where the source for the number of existing houses sold is the US National Association of Realtors and for new houses sold is the US Census Bureau); (3) for the recession dates: NBER business cycle quarterly reference dates. All data is

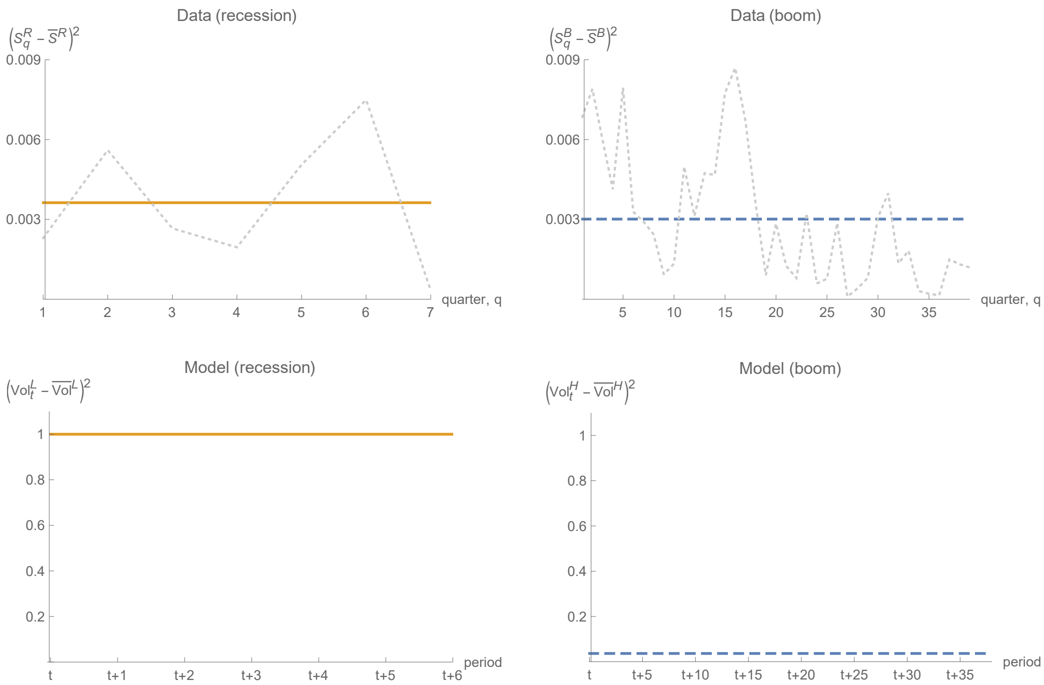


Figure 7: *Average volatility of the number of houses sold in the US (top panel) and of the model's volume of trading (bottom panel) in an average recession (orange solid) and boom (blue dashed).*

quarterly, was deseasonalised by the source, and detrended using the Hodrick-Prescott filter with multiplier 1600 by me.

To assess whether the fluctuations are larger in recessions than in booms, I provide graphical evidence and run a simple regression. Figures 4 and 7 provide the graphical evidence. The top panels of Figure 4 depict the volatility of the US job-finding rate and of Figure 7 the volatility of the US number of houses sold in a synthetic recession and boom.

The volatility measure of the synthetic recession is constructed as follows (top-left panels in Figures 4 and 7). If X is the variable of interest (job-finding rate JFR or number of houses sold S), then the volatility in X in the qth quarter of the synthetic recession R is measured by

$$(X_q^R - \bar{X}^R)^2 := \frac{\sum_r (X_q^r - \bar{X}^r)^2}{\sum_r \mathbb{I}(Q_r \geq q)},$$

where X_q^r is the value of X in the qth quarter of recession r, $\bar{X}^r$ is the mean value of X in recession r: $\bar{X}^r := \frac{\sum_q X_q^r}{Q_r}$, Q_r is the length of recession r in quarters, and $\sum_r \mathbb{I}(Q_r \geq q)$ is the number of recessions that are at least q quarters long. I measure the volatility in quarter q for a recession r as the deviation from recession r's mean (rather than from the mean across all recessions) because the data spans fifty years and there is no good reason to assume that, say, the mean job-finding rates in the early 1980s recession and the Great Recession were similar. The volatility in the synthetic boom B (top-right panels in Figures 4 and 7) is measured analogously. This gives the synthetic data, i.e., the grey dotted lines, in the top panels of Figures 4 and 7. The average volatility in the synthetic recession or boom (the coloured lines in the top panels of Figures 4 and 7 and the numbers reported in Table 1) is a simple average across quarters. The average volatility of the job-finding rate is about 100% and of the number of houses sold about 20% higher in a recession than in a boom.

The volatility in the model (the bottom panels of Figures 4 and 7) is measured as follows. The volatility in a "recession" (the bad state) is measured by the squared difference between the value of the variable of interest (the probability or volume of trading) at t, Z_t, and its mean: $(Z_t^L - \bar{Z}^L)^2$, where $\bar{Z}^L := \frac{Z_{odd}^L + Z_{even}^L}{2}$. The volatility in a "boom" (the good state) is measured analogously.

As further evidence that fluctuations are larger in recessions than in booms in the data, I present the results of a regression where the squared difference between the value of the variable of interest, $Y_q \in \{JFR_q, S_q\}$, and its 5-quarter moving average is the dependent variable and a recession dummy is the independent variable. I use the 5-quarter moving average to capture the effect that the fundamentals have: the average value of the variable is allowed to be different in recessions and booms. The results are robust to using the 3-quarter or 7-quarter moving average instead. I run two regressions: for $Y \in \{JFR, S\}$,

$$(Y_q - \bar{Y}_q)^2 = \alpha_Y + \beta_Y D(recession_q) + \varepsilon_Y, \tag{2}$$

Table 2: *Fluctuations in the US job-finding rate and the US number of private houses sold in recessions and booms.*

Job-finding rate		Number of houses sold	
α_{JFR}	β_{JFR}	α_S	β_S
0.0012***	0.0016***	0.0009***	0.0019**
(0.0002)	(0.0005)	(0.0002)	(0.0006)

Notes: Results of regression (2). *** $p < 0.001$, ** $p < 0.01$, * $p < 0.05$.

where $\bar{Y}_q$ is the 5-quarter moving average and $D(recession_q)$ is the recession dummy. Table 2 reports the results. Both β_{JFR} and β_S are positive at the 99% confidence level, which suggests that fluctuations in the job-finding rate and in the number of houses sold are larger in recessions than in booms.

A.2 Benchmarks

Proof of Proposition 1. Here buyers know the state. For each state, I derive the conditions under which a young buyer prefers continuing to accepting a low-value offer and vice versa. Suppose first that the state is L. A young buyer's value from continuing and accepting any offer when old is $V^L = \delta v_L$, because only v_L-sellers are on the market. A young buyer optimally accepts v_L because the discount rate is less than one.

Suppose now that the state is H. A young buyer optimally either accepts only v_H or accepts either offer. Her value from continuing and accepting any offer when old is

$$V^H = \delta \frac{v_H + v_L}{2},$$

because she is equally likely to meet a v_L-and a v_H-seller. The probability of meeting a v-seller is equal to the fraction of these sellers because of random matching. A young buyer optimally continues after meeting a v_L-seller if v_L is less than the continuation value, $v_L < V^H$, that is, if

$$\bar{\pi} = \frac{2v_L(1-\delta)}{\delta(v_H - v_L)} < 1,$$

where $\bar{\pi}$ is defined in (1), as claimed in the Proposition. □

Proof of Proposition 2. Here, a buyer does not know the state and learns about it only from her own experience, i.e., from the offers of the sellers that she meets. I derive the conditions under which a young buyer prefers continuing to accepting a low-value offer.

Consider any period t. For a young buyer with posterior belief π', the expected value of continuing and accepting either offer when old is

$$V(\pi') = \delta\left[\frac{\pi'}{2}v_H + \left(1 - \frac{\pi'}{2}\right)v_L\right]. \tag{3}$$

She meets a v_H-seller with probability a half only if the state is good. With the rest of the probability, she meets a v_L-seller. The buyer optimally continues if $V(\pi') > v_L$, or if $\pi' > \frac{2v_L(1-\delta)}{\delta(v_H-v_L)} = \bar{\pi}$, where the cutoff belief $\bar{\pi}$ is the same as in the known-state benchmark.

Let $\pi(v_L)$ denote a young buyer's posterior belief that the state is good after meeting a v_L-seller. Her posterior odds are

$$\frac{\pi(v_L)}{1-\pi(v_L)} = \omega\frac{P(v_L|H)}{P(v_L|L)} = \omega\frac{\frac{1}{2}}{1} = \frac{\omega}{2},$$

where $\omega := \frac{\pi}{1-\pi}$ denotes the prior odds. I focus on the posterior odds throughout because the odds contain the same information as the posterior belief but are easier to interpret. The posterior odds are lower than the prior odds because meeting a v_L-seller makes the buyer more pessimistic about the state.

A young buyer optimally continues after meeting a v_L-seller if $V(\pi' = \pi(v_L)) > v_L$ or $\frac{\omega}{\omega+2} > \bar{\pi}$, which can be rearranged to give the exact condition in the Proposition. The argument holds both if the economy starts at $t_1 = 1$ and at $t_1 = -\infty$ because a buyer's optimal decision only depends on the distribution of sellers' offers and that is given. □

A.3 Buyers do not know the state and learn from their own experience and the others' trades

Proof of Proposition 3. (i) Steady state 0: A young buyer who meets a v_L-seller accepts v_L.

If buyers use this strategy, a randomly drawn buyer trades with probability one in both states, which makes the trade signal uninformative. But if the signal is uninformative, then this equilibrium exists for the same parameter values as the same equilibrium in a market where buyers learn only from their own experience (see Proposition 2).

(ii) Steady state 1: A young buyer who meets a v_L-seller accepts v_L after observing no trade and continues after observing a trade.

I first derive the equilibrium objects (amounts of old buyers and trading probabilities) assuming that the strategy profile constitutes an equilibrium and then derive the conditions under which no buyer has an incentive to deviate. Let O_t^θ denote the amount of old buyers and τ_t^θ the probability of a randomly drawn buyer trading at t in state θ. The amounts of buyers are measured at the start of a period, after entry. In order to write down the equilibrium objects for both states using one set of equations, I let μ^θ denote the fraction of v_H-sellers in state θ so that $\mu^H = \frac{1}{2}$ and $\mu^L = 0$.

Given the above strategy, the probability of a trade at t in state θ is

$$\tau_t^\theta = 1 - \frac{(1-\mu^\theta)\tau_{t-1}^\theta}{1+O_t^\theta}, \tag{4}$$

because the only buyers who do not trade are young buyers who met a v_L-seller and observed a trade. The total amount of buyers is the sum of the amounts of young and old buyers.

The young buyers who become old, i.e., buyers who are carried over to $t+1$ are young buyers who meet a v_L-seller and see a trade. So the amount of old buyers at $t+1$ in state θ is

$$O_{t+1}^\theta = (1-\mu^\theta)\tau_{t-1}^\theta. \tag{5}$$

Imposing the steady state condition that $x_t = x_{t+1}$ for all endogenous variables x and solving equations (4) and (5) for $\theta = L, H$ gives $\tau^H = \sqrt{3}-1$ and $\tau^L = \frac{\sqrt{5}-1}{2}$. A trade is good news, as required, because $\tau^H > \tau^L$.

The proposed strategy is optimal if no young buyer wants to deviate. At t, young buyer's posterior odds after meeting a v_L-seller and observing no trade are

$$\frac{\pi(v_L, N_{t-1})}{1-\pi(v_L, N_{t-1})} = \omega \frac{1-\mu^H}{1-\mu^L} \frac{1-\tau^H_{t-1}}{1-\tau^L_{t-1}}, \tag{6}$$

where in a given state, the probability of meeting a v_L-seller is equal to the fraction of these sellers (because of random matching) and the probability of observing no trade is the probability that a randomly drawn buyer did not trade at $t-1$. A young buyer's posterior odds after meeting a v_L-seller and observing a trade are

$$\frac{\pi(v_L, T_{t-1})}{1-\pi(v_L, T_{t-1})} = \omega \frac{1-\mu^H}{1-\mu^L} \frac{\tau^H_{t-1}}{\tau^L_{t-1}}. \tag{7}$$

Plugging in the numbers gives $\frac{\pi(v_L, N_{t-1})}{1-\pi(v_L, N_{t-1})} = \omega \frac{2-\sqrt{3}}{3-\sqrt{5}}$ and $\frac{\pi(v_L, T_{t-1})}{1-\pi(v_L, T_{t-1})} = \omega \frac{\sqrt{3}-1}{\sqrt{5}-1}$.

For a young buyer with belief π', the value of continuing and accepting either offer when old is given by (3). Thus, the cutoff belief is still $\bar{\pi}$ as defined in (1) and the proposed equilibrium strategy is optimal if $\pi(v_L, N) < \bar{\pi} < \pi(v_L, T)$, which, if rearranged, give the exact conditions in the Proposition. A trade being good news ($\tau^H > \tau^L$) guarantees that the conditions are satisfied for an open set of parameter values.

(iii) Steady state 2: A young buyer who meets a v_L-seller continues.

I go through the same two steps as in (ii): I derive first the equilibrium objects given the strategy profile and then the conditions under which no buyer deviates. According to the above strategy, all young buyers who meet a v_L-seller continue at t so the probability of a trade at t in state $\theta = L, H$ is

$$\tau^\theta_t = 1 - \frac{1-\mu^\theta}{1+O^\theta_t}, \tag{8}$$

and the amount of old buyers at $t+1$ is

$$O^\theta_{t+1} = 1 - \mu^\theta. \tag{9}$$

Imposing the steady state condition that $O^\theta_{t+1} = O^\theta_t$, I get that $\tau^H = \frac{2}{3}$ and $\tau^L = \frac{1}{2}$, so a trade is good news.

The strategy is optimal if the most pessimistic young buyer (one who meets a v_L-seller and observes no trade) does not want to deviate and accept v_L. Her posterior odds are given by (6), which becomes $\frac{\pi(v_L,N_{t-1})}{1-\pi(v_L,N_{t-1})} = \frac{\omega}{3}$. Thus, steady state 2 exists if $\bar{\pi} < \frac{\omega}{\omega+3}$, which, once rearranged, gives the exact condition in the Proposition.

Finally, I prove by contradiction that a trade cannot be bad news in a pure-strategy steady-state equilibrium. Since in the above steady states a trade is (weakly) good news, I have to consider only one candidate steady state: where young buyers who meet a v_L-seller continue after observing no trade and accept v_L after observing a trade. The derivation is analogous to that of steady state 1 (where the probability of a trade is replaced by the probability of no trade) so I am brief.

Given the proposed strategy, the probability of a trade in state θ is $\tau^\theta = \frac{\mu^\theta+(1-\mu^\theta)\tau^\theta+O^\theta}{1+O^\theta}$. The buyers who become old are young buyers who meet a v_L-seller and see no trade so the amount of old buyers is $O^\theta = (1-\mu^\theta)(1-\tau^H)$ in state θ. Combining the equations gives $\tau^H = 1$ and $\tau^L = 1$. But a trade is bad news only if $\tau^H < \tau^L$, a contradiction. □

Proof of Proposition 4. I construct a cyclical equilibrium where a trade that takes place in an odd period is good news and a trade that takes place in an even period is bad news. In this equilibrium, in an odd period t (and $t+2$, $t+4$, etc.) buyers behave as in steady state 2: no young buyer accepts v_L. I assume, and verify below, that a trade at t (which is observed at $t+1$) is good news. In an even period $t+1$ (and $t+3$, $t+5$, etc.) buyers behave as in steady state 1: the young buyers who observe bad news about the state, that is, no trade (N_t), accept v_L. I assume, and verify below, that a trade at $t+1$ is bad news, but comparatively "less bad news" than no trade at t (so that all young buyers at $t+2$ optimally reject v_L, whereas the pessimistic young buyers accept v_L at $t+1$). Let O_t^θ denote the amount of old buyers and τ_t^θ the probability of a randomly drawn buyer trading in period t in state θ. I again let μ^θ denote the fraction of v_H-sellers in state θ: $\mu^H = \frac{1}{2}$ and $\mu^L = 0$.

I first derive the equilibrium objects O_t^θ, O_{t+1}^θ, τ_t^θ and τ_{t+1}^θ given the proposed strategy and then show that they satisfy the assumptions made above. I then derive the conditions under which no buyer has an incentive to deviate from the strategy. Finally, I calculate the volumes of trade.

Consider period t. Since at t buyers behave as in steady state 2, the equations for the probability of a trade at t and the amount of old buyers at $t+1$ are given by equations (8) and (9) respectively.

Consider period $t+1$. Since at $t+1$ buyers behave as in steady state 1, the equations for the probability of a trade at $t+1$ and the amount of old buyers at $t+2$ are given by equations (4) and (5) respectively (where t is replaced with $t+1$).

To complete this step, I impose the condition that the cycle is two periods long, i.e., that for $t' = t, t+1$ and all endogenous variables x, $x_{t'+2} = x_{t'}$. The solution is $O_t^H = \frac{\sqrt{5}-1}{4}$, $\tau_t^H = \frac{\sqrt{5}-1}{2}$, $O_{t+1}^H = \frac{1}{2}$, $\tau_{t+1}^H = \frac{7-\sqrt{5}}{6}$, $O_t^L = \tau_t^L = 0$, and $O_{t+1}^L = \tau_{t+1}^L = 1$. The assumptions that I made on the trade probabilities hold: a trade at t is good news because $\tau_t^H > \tau_t^L$ and at $t+1$ is bad news because $\tau_{t+1}^H < \tau_{t+1}^L$.

Next I determine the parameter values for which the proposed strategy is optimal for young buyers. For a young buyer with belief π', the value of continuing and accepting either offer when old is given by equation (3). So the cutoff belief is still $\bar{\pi}$ (defined in (1)). At period $t' = t, t+1$, a young buyer's posterior odds after meeting a v_L-seller and observing signal outcome $N_{t'-1}$ is given by (6) and after signal outcome $T_{t'-1}$ by (7). The solutions are $\frac{\pi(v_L,T_t)}{1-\pi(v_L,T_t)} = \frac{\pi(v_L,N_{t-1})}{1-\pi(v_L,N_{t-1})} = +\infty$ (because a trade at t and no trade at $t-1$ reveal that the state is good), $\frac{\pi(v_L,T_{t-1})}{1-\pi(v_L,T_{t-1})} = \frac{\omega}{2}\frac{7-\sqrt{5}}{6}$, and $\frac{\pi(v_L,N_t)}{1-\pi(v_L,N_t)} = \frac{\omega}{2}\frac{3-\sqrt{5}}{2}$.

The proposed strategy constitutes an equilibrium if a young buyer who meets a v_L-seller at t optimally continues regardless of the signal outcome, $\bar{\pi} < \pi(v_L, N_{t-1}), \pi(v_L, T_{t-1})$, and at $t+1$ optimally continues after a trade, but accepts v_L after no trade, $\pi(v_L, N_t) < \bar{\pi} < \pi(v_L, T_t)$. The binding constraints are $\pi(v_L, N_t) < \bar{\pi} < \pi(v_L, T_{t-1})$, or

$$\frac{\omega(3-\sqrt{5})}{\omega(3-\sqrt{5})+4} < \bar{\pi} < \frac{\omega(7-\sqrt{5})}{\omega(7-\sqrt{5})+12}. \tag{10}$$

The inequalities be rearranged to give the exact conditions in the Proposition. The lower bound in (10) always lies below the higher one so the cyclical equilibrium exists for an open set of parameter values.

Finally, I calculate the volumes of trading. The volume of trades is $Vol^{\theta}_{odd} = \mu^{\theta} + O^{\theta}_t$ in an odd period t and $Vol^{\theta}_{even} = \mu^{\theta} + (1-\mu^{\theta})(1-\tau^{\theta}_t) + O^{\theta}_{t+1}$ in an even period $t+1$, which give the exact numbers in the Proposition. $\square$

A.4 Efficiency

Proof of Proposition 5. In steady state 1, a young buyer who meets a v_L-seller accepts the offer only if she also observes no trade. A buyer's expected utility from participating in the market is

$$W_{ss1} = \frac{\pi}{2} v_H + \frac{\pi}{2}\left[\tau^H \delta \frac{v_H + v_L}{2} + (1-\tau^H) v_L\right] + (1-\pi)[\tau^L \delta v_L + (1-\tau^L) v_L],$$

where $\tau^H = \sqrt{3} - 1$, $\tau^L = \frac{\sqrt{5}-1}{2}$, and π is the prior that the state is good. If the buyer meets a v_H-seller when young (which happens with a probability a half only if the state is good), she accepts the offer. If she instead meets a v_L-seller, she accepts the offer only if she observes no trade and continues otherwise. If she continues, she accepts any offer when old.

In steady state 2, no young buyer who meets a v_L-seller accepts the offer. A buyer's expected value from participating in the market is

$$W_{ss2} = \frac{\pi}{2} v_H + \delta\left[\frac{\pi}{2}\frac{v_H + v_L}{2} + (1-\pi) v_L\right].$$

In the cyclical equilibrium, a buyer's expected utility depends on her entry period. For a buyer born in period t (when no young buyer who meets a v_L-seller accepts v_L), the expected utility from participating in the market, $W_c(t)$, is exactly the same as in steady state 2: $W_c(t) = W_{ss2}$. For a buyer born in period $t+1$ (when a young buyer who meets a v_L-seller accepts the offer only if she also observes no trade), the expected utility from participating in the market is

$$W_c(t{+}1) = \frac{\pi}{2} v_H + \frac{\pi}{2}\left[\tau^H_t \delta \frac{v_H + v_L}{2} + (1-\tau^H_t) v_L\right] + (1-\pi)[\tau^L_t \delta v_L + (1-\tau^L_t) v_L],$$

where $\tau_t^H = \frac{\sqrt{5}-1}{2}$ and $\tau_t^L = 0$.

Comparing efficiency in steady state 1 and in the cyclical equilibrium is the same as comparing W_{ss1} to $\frac{W_c(t)+W_c(t+1)}{2}$. The steady state is more efficient if

$$\frac{\bar{\pi}}{1-\bar{\pi}} > \omega \frac{5+\sqrt{5}-4\sqrt{3}}{4(2-\sqrt{5})},$$

which always holds because the RHS of the inequality is negative.

The comparison between steady state 2 and the cyclical equilibrium boils down to the comparison between W_{ss2} and $W_c(t+1)$: the steady state is less efficient if

$$\frac{(3-\sqrt{5})\omega}{(3-\sqrt{5})\omega+4} < \bar{\pi},$$

which holds because these equilibria coexist if $\frac{(3-\sqrt{5})\omega}{(3-\sqrt{5})\omega+4} < \bar{\pi} < \frac{\omega}{\omega+3}$. □

A.5 A market that starts at $t_1 = 1$

Proof of Proposition 6. The proof for part (i) is separate and for parts (ii) and (iii) is joint. Let μ^θ denote the fraction of v_H-sellers in state θ, $\mu^H = \frac{1}{2}$ and $\mu^L = 0$, and let $\bar{\omega} := \frac{\bar{\pi}}{1-\bar{\pi}}$ where $\bar{\pi}$ is defined in (1).

(i) $\pi < \frac{2\bar{\pi}}{\bar{\pi}+1}$: all buyers trade in their entry period. Steady state 0 is reached at $t = 1$.

Assume that at $t = 1$, a young buyer who meets a v_L-seller optimally accepts v_L (i.e., that $\pi < \frac{2\bar{\pi}}{\bar{\pi}+1}$). But then there are no old buyers at $t = 2$, exactly as at $t = 1$. Hence, all buyers trade at $t = 2$ and in all the following periods. In other words, steady state 0 is reached at $t = 1$.

(ii) $\frac{4\bar{\pi}}{3\bar{\pi}+1} < \pi$: at all t, young buyers who meet a v_L-seller continue. Steady state 2 is reached at $t = 2$.

(iii) $\frac{12\bar{\pi}}{12\bar{\pi}+(7+\sqrt{5})(1-\bar{\pi})} < \pi < \frac{4\bar{\pi}}{3\bar{\pi}+1}$: convergence to cycles where a young buyer who meets a v_L-seller in an odd period continues and in an even period accepts v_L if she observes no trade. The cyclical equilibrium is reached in the limit as $t \to \infty$.

I start the market off at $t_1 = 1$ and show that it converges to the two equilibria in the specified times.

At $t = 1$, I assume that a young buyer who meets a v_L-seller optimally continues (i.e., that $\frac{2\bar{\pi}}{\bar{\pi}+1} < \pi$ or, in terms of odds ratios, $\bar{\omega} < \frac{\omega}{2}$). Then $\tau_1^H = \frac{1}{2}$ and $\tau_1^L = 0$, and a trade at $t = 1$, T_1, reveals the good state.

Consider $t = 2$. The amount of old buyers is $O_2^H = \frac{1}{2}$ and $O_2^L = 1$. At $t = 2$ a young buyer who meets a v_L-seller and sees good news (a trade), is more optimistic about the state than a young buyer who met a v_L-seller at $t = 1$. Since at $t = 1$ the young buyer optimally continued, the more optimistic young buyer at $t = 2$ optimally continues, too. The same argument holds for all the subsequent periods: a young buyer who meets a v_L-seller and sees good news optimally continues. A young buyer who meets a v_L-seller and sees bad news (no trade) at $t = 2$ optimally either continues or accepts v_L. I consider both cases in turn.

(a) Assume that a young buyer who meets a v_L-seller and sees bad news at $t = 2$ optimally continues. I show that the necessary and sufficient condition for this is $\bar{\omega} < \frac{\omega}{4}$.

If at $t = 2$ the pessimistic young buyers continue, only old buyers accept v_L at $t = 2$ and the trade probability in state θ is given by (8) where $t = 2$. The solution is $\tau_2^H = \frac{2}{3}$ and $\tau_2^L = \frac{1}{2}$. Thus, a trade is good news, but not as good news as at $t = 1$. The pessimistic young buyers do not want to deviate at $t = 2$ if $\pi(v_L, N_1) > \bar{\pi}$. The posterior odds are given by (6) where $t = 2$, explicitly, $\frac{\pi(v_L,N_1)}{1-\pi(v_L,N_1)} = \frac{\omega}{4}$. The inequality thus becomes $\frac{\omega}{4} > \bar{\omega}$. Since all young buyers who meet a v_L-seller continue, the amounts of old buyers at $t = 3$ are $O_3^H = \frac{1}{2}$ and $O_3^L = 1$.

Consider $t = 3$ and recall that a trade at $t = 2$ is good news as $\tau_2^H > \tau_2^L$. At $t = 3$, the pessimistic young buyers' posterior odds are given by (6) where $t = 3$. The odds are explicitly $\frac{\pi(v_L,N_2)}{1-\pi(v_L,N_2)} = \frac{\omega}{3}$, which are higher than for the pessimistic young buyers at $t = 2$. Thus, at $t = 3$ the pessimistic young buyers continue, the trade probabilities are exactly like at $t = 2$, and steady state 2 is reached at $t = 2$. The condition that ensures convergence to steady state 2 is $\frac{\omega}{4} > \bar{\omega}$, which, if rearranged, is the condition in the Proposition.

(b) Assume now that a young buyer who meets a v_L-seller and sees bad news at $t = 2$, no trade, accepts v_L. We know from Part (a) that the necessary and sufficient for this to be optimal is that

$$\frac{\omega}{4} < \bar{\omega}. \tag{11}$$

I show that the other necessary and sufficient condition for the market to converge to the cyclical equilibrium is that $\bar{\omega} < \frac{(7-\sqrt{5})\omega}{12}$.

Since here a young buyer at $t = 2$ behaves as in steady state 1, the trade probability in state θ is given by (4) where $t = 2$, The explicit solution is $\tau_2^H = \frac{5}{6}$ and $\tau_2^L = 1$ so a trade at $t = 2$ is bad news.

Consider $t = 3$. Since at $t = 2$ only young buyers who meet a v_L-seller and see a trade continue, the amount of old buyers at $t = 3$ is $O_3^\theta = (1 - \mu^\theta)\tau_1^\theta$, so that $O_3^H = \frac{1}{4}$ and $O_3^L = 0$. The young buyers' posterior are given by (7) where $t = 3$ after meeting a v_L-seller and observing a trade, so that $\frac{\pi(v_L,T_2)}{1-\pi(v_L,T_2)} = \frac{5\omega}{12}$, and by $\frac{\pi(v_L,N_2)}{1-\pi(v_L,N_2)} = +\infty$ after meeting a v_L-seller and observing no trade because no trade at $t = 2$, N_2, reveals the good state. So young buyers who meet a v_L-seller and see no trade at $t = 3$ definitely continue, while those who see a trade could either continue or accept v_L.

Suppose that the young buyers who meet a v_L-seller and see a trade continue at $t = 3$ (to get a cycle). That is, assume that $\bar{\omega} < \frac{5\omega}{12}$. Then at $t = 3$ buyers behave as in steady state 2 and the probability of trading is given by (8) where $t = 3$. The explicit probabilities are $\tau_3^L = 0$ and $\tau_3^H = \frac{3}{5}$. A trade at $t = 3$, thus, reveals the good state. All young buyers who meet a v_L-seller continue at $t = 3$ so $O_4^H = \frac{1}{2}$ and $O_4^L = 1$.

I now show that if $\bar{\omega} < \frac{\omega(7-\sqrt{5})}{12}$, the limit of this process is the cyclical equilibrium. Consider an even period $t + 1$ after an odd period t where a trade was good news with $\tau_t^L = 0$ and $\tau_t^H \in [\frac{3}{5}, \frac{\sqrt{5}-1}{2}]$, and all young buyers who met a v_L-seller continued (we know that such a t exists if $\bar{\omega} < \frac{5\omega}{12}$, for example, $t = 3$). Then the pessimistic young buyers at $t + 1$ are those who meet a v_L-seller and see no trade (N_t). Their posterior odds are given by equation (6), explicitly, $\frac{\pi(v_L,N_t)}{1-\pi(v_L,N_t)} \in \left[\frac{(3-\sqrt{5})\omega}{4}, \frac{\omega}{5}\right]$. But then these buyers definitely accept v_L because condition (11) holds. Buyers at $t + 1$, thus, behave as in steady state 1 so that the probability of trading at $t+1$ in state

θ is given by equation (4), which gives $\tau^H_{t+1} \in \left[\frac{7-\sqrt{5}}{6}, \frac{3}{5}\right]$ and $\tau^L_{t+1} = 1$. Thus, a trade at $t+1$ is bad news. The amounts of buyers carried to $t+2$ are $O^H_{t+2} = \frac{1}{2}\tau^H_{t-1} \in \left[\frac{3}{10}, \frac{\sqrt{5}-1}{4}\right]$ and $O^L_{t+2} = 0$ (see (5)).

Consider $t+2$. To make $t+2$ look like t, I derive the conditions under which all young buyers who meet a v_L-seller optimally continue (i.e., behave as in steady state 2). The pessimistic young buyers are those who meet a v_L-seller and see a trade (T_{t+1}), with posterior odds given by equation (7) (where t is replaced with $t+2$). The explicit odds are $\frac{\pi(v_L,T_{t+1})}{1-\pi(v_L,T_{t+1})} \in \left[\frac{(7-\sqrt{5})\omega}{12}, \frac{3\omega}{10}\right]$. These buyers are the most pessimistic if τ^H_{t+1} takes the lowest allowed value (which is the same as τ^H_t taking the highest allowed value), that is, if $\frac{\pi(v_L,T_{t+1})}{1-\pi(v_L,T_{t+1})} = \frac{(7-\sqrt{5})\omega}{12}$. I assume that

$$\bar{\omega} < \frac{(7-\sqrt{5})\omega}{12}, \tag{12}$$

which guarantees for all permissible τ^H_t that at $t+2$ all young buyers who meet a v_L-seller optimally continue.

But then the trade probabilities at $t+2$ are $\tau^L_{t+2} = 0$ and $\tau^H_{t+2} = \frac{\frac{1}{2}+O^H_{t+2}}{1+O^H_{t+2}} \in [\frac{8}{13}, \frac{\sqrt{5}-1}{2}]$, which are almost the same conditions that I assumed for period t: the interval for the odd-period probability of trading in state H (i.e., for τ^H_{t+2} versus τ^H_t) has shrunk from below. But the binding condition for period $t+1$ from period t, (12), came from the highest allowed value for τ^H_t. Thus, the only limit to this process can be $\tau^H_{odd} = \frac{\sqrt{5}-1}{2}$ and $\tau^H_{even} = 1 - \frac{\tau^H_{odd}}{3} = \frac{7-\sqrt{5}}{6}$. □

Proof of convergence rates. Consider state L. The limit trade probabilities are $\tau^L_{odd} = 0$ and $\tau^L_{even} = 1$, which are reached in periods 1 and 2 respectively.

Consider state H. The limit trade probabilities are $\tau^H_{odd} = \frac{\sqrt{5}-1}{2}$ and $\tau^H_{even} = \frac{7-\sqrt{5}}{6}$. At $t=2$, the probability of trading is $\tau^H_2 = \frac{5}{6}$. The difference between it and the even-period limit value is $\frac{\tau^H_2}{\tau^H_{even}} - 1 = \frac{\sqrt{5}-2}{7-\sqrt{5}} \approx 5.0\%$. At $t=3$, the probability of trading is $\tau^H_3 = \frac{3}{5}$, so that the difference between it and the odd-period limit value is $\frac{\tau^H_3}{\tau^H_{odd}} - 1 = \frac{2(2-\sqrt{5})}{5(1+\sqrt{5})} \approx -2.9\%$.

At $t=6$ and $t=7$, the trade probabilities are $\tau^H_6 = \frac{31}{39}$ and $\tau^H_7 = \frac{21}{34}$. The differences with the limit values are $\frac{\tau^H_6}{\tau^H_{even}} - 1 \approx 0.1\%$ and $\frac{\tau^H_7}{\tau^H_{odd}} - 1 \approx -0.1\%$. □

A.6 Concluding discussion

A.6.1 Positive fractions of low-and high-value sellers in both states

Proof of Proposition 7. Let the fraction of v_H-sellers be μ^θ in state θ with $0 \leq \mu^L < \mu^H < 1$. I show that a cyclical equilibrium exists in this version of the model for open sets of parameter values for certain μ^L and μ^H.

First, I derive the conditions under which it is optimal for a young buyer who knows the state to reject v_L if the state is H, and to accept v_L if the state is L. Only if these conditions are satisfied is information about the state useful for a buyer's optimal decision. A young buyer's continuation value in state θ is

$$V^\theta = \delta[\mu^\theta(v_H - v_L) + v_L].$$

The buyer accepts any offer when old. She gets v_L for sure and the extra value $v_H - v_L$ only if he meets a v_H-seller. She prefers to continue after v_L if $v_L < V^\theta$. I assume that $V^L < v_L < V^H$, or,

$$\mu^L < \frac{(1-\delta)v_L}{\delta(v_H - v_L)} < \mu^H. \tag{13}$$

Now assume that buyers do not know the state. I construct a cyclical equilibrium where buyers use the same strategy as in the case $\mu^H = \frac{1}{2}$ and $\mu^L = 0$ (Proposition 4). The proof is almost the same as for Proposition 4.

The equations for the equilibrium objects, the amounts of old buyers and trade probabilities, are as in the proof of Proposition 4: (8) and (9) for odd periods t and (4) and (5) (where t is replaced with $t+1$) for even periods $t+1$. For general μ^H and μ^L, the solution is $O_t^\theta = \frac{z-\mu^\theta}{2}$, $\tau_t^\theta = 1 - \frac{2(1-\mu^\theta)}{2-\mu^\theta+z}$, and

$$\tau_{t+1}^\theta = (2-\mu^\theta)^{-1}\left[1 + \frac{2(1-\mu^\theta)^2}{2-\mu^\theta+z}\right],$$

where $z := \sqrt{\mu^\theta(4-3\mu^\theta)}$. The probability of trading in an odd period, τ_t^θ, strictly increases in μ^θ while the probability of trading in an even period, τ_{t+1}^H, is convex in μ^θ.

I derive the conditions, first, under which the informational content of trades satisfy the assumptions made in the Proposition and, second, under which no buyer has an incentive to deviate. First, I need that

(a) a trade at t is good news, $\tau_t^H > \tau_t^L$. This holds because τ_t^θ strictly increases in μ^θ.

(b) a trade at $t+1$ is bad news, $\tau_{t+1}^H < \tau_{t+1}^L$. This holds for sure if either $\mu^L = 0$ or if $\mu^H \leq \mu_{min}^\theta$, where $\mu_{min}^\theta := \frac{2}{7}(3 - \sqrt{2})$ minimises τ_{t+1}^θ, but not necessarily for other values of μ^L and μ^H. Broadly, the condition is satisfied if μ_L is close to zero and μ^H is not too close to one.

Second, I derive the conditions under which no buyer wants to deviate. For a young buyer with posterior belief π', the value of continuing and accepting either offer when old is $V(\pi') = \pi' V^H + (1 - \pi')V^L$. A young buyer who meets a v_L-seller prefers to continue if $v_L < V(\pi')$ or

$$\tilde{\pi} := (\mu^H - \mu^L)^{-1} \left[\frac{(1-\delta)v_L}{\delta(v_H - v_L)} - \mu^L \right] < \pi'. \tag{14}$$

The assumptions that I made in the case the state is known, in equation (13), guarantee that $0 < \tilde{\pi} < 1$. For the optimality of the strategy $\pi(v_L, N_t) < \tilde{\pi} < \pi(v_L, N_{t-1}), \pi(v_L, T_{t-1}), \pi(v_L, T_t)$ must hold. If the conditions on the informational content of trades are satisfied, the strategy is optimal if $\pi(v_L, N_t) < \tilde{\pi} < \pi(v_L, T_{t-1}), \pi(v_L, T_t)$.

At $t' = t, t+1$, a young buyer's posterior odds after meeting a v_L-seller and observing no trade ($N_{t'-1}$) are given by (6) and after observing a trade ($T_{t'-1}$) are given by (7) where $t = t'$. The conditions on the posteriors are satisfied for a range of parameter values as long as the conditions on the informational content of trades hold. □

A.6.2 Long-lived buyers

Proof of Proposition 8. I construct an equilibrium where

(i) a buyer with posterior belief $\pi' = 1$ only accepts v_H,

(ii) in an odd period t (and $t+2$, $t+4$, etc.), all buyers with a posterior belief $\pi' < 1$ continue after meeting a v_L-seller (regardless of whether they see N_{t-1} or T_{t-1}), and

(iii) in an even period $t+1$ (and $t+3$, $t+5$, etc.), buyers with a posterior belief $\pi' < 1$ continue after meeting a v_L-seller and observing T_t, and accept v_L after meeting a v_L-seller and observing N_t.

I first derive the probability of trading and amounts of buyers given the above strategy. I then derive the conditions under which no buyer has an incentive to deviate. Let the mass of buyers who are uncertain of the state be denoted by M_t^θ and the total mass of buyers by B_t^θ at t and in state θ as measured at the start of t, after entry. Let μ^θ stand for the fraction of v_H-sellers in state θ: $\mu^H = \frac{1}{2}$ and $\mu^L = 0$.

The probability of trading at t is $\tau_t^\theta = \mu^\theta$, because all buyers trade with a v_H-seller and no buyer trades with a v_L-seller. A trade at t is good news because $\tau_t^H = \frac{1}{2}$ and $\tau_t^L = 0$. The probability of trading at $t+1$ is

$$\tau_{t+1}^\theta = \mu^\theta + (1-\mu^\theta)\frac{M_{t+1}^\theta}{B_{t+1}^\theta}(1-\tau_t^\theta),$$

because all buyers trade with a v_H-seller and a buyer trades with a v_L-seller if she is uncertain of the state and observes no trade. The probabilities at $t+1$ (equivalently, at $t-1$) are explicitly $\tau_{t+1}^H = \frac{1}{2}\left(1 + \frac{M_{t+1}^H}{B_{t+1}^H}\frac{1}{2}\right)$ and $\tau_{t+1}^L = \frac{M_{t+1}^L}{B_{t+1}^L} = 1$, where the last equality follows from the fact that no buyer can know that the state is good if the state is in fact bad. A trade at $t+1$ is bad news because $\tau_{t+1}^H < 1 = \tau_{t+1}^L$. Note that at t, observing no trade (N_{t-1}) and at $t+1$, observing a trade (T_t), reveal the good state (so the equilibrium strategy has to specify what buyers do who know that the state is good, but not what buyers do who know that the state is bad).

Now I derive M_{t+1}^H and B_{t+1}^H. What are the flows into these masses of buyers? First, consider period t. How many buyers who start at t as uninformed, M_t^H, reach $t+1$ as uninformed? An uninformed buyer does not learn the state at t if he meets a v_L-seller and observes a trade (T_{t-1}). These buyers continue and they reach $t+1$ with probability δ. In addition to them, the buyers who enter at $t+1$ start off as uninformed. Thus,

$$M^H_{t+1} = M^H_t(1-\mu^H)\tau^H_{t-1}\delta + 1.$$

Consider period $t+1$. How many buyers who start at $t+1$ as uninformed, M^H_{t+1}, reach period $t+2$ as uninformed? An uninformed buyer does not learn the state at $t+1$ if he meets a v_L-seller and observes no trade (N_t). But all of these buyers accept v_L according to the proposed strategy so no uninformed buyers are carried over to $t+2$ from $t+1$. All of the buyers who enter at $t+1$ start off as uninformed. Thus, $M^H_{t+2} = 1$.

How many buyers who start at t as informed, $B^H_t - M^H_t$, reach period $t+1$ as informed? All informed buyers remain informed, but some of them exit: only those reach period $t+1$ who meet a v_L-seller and survive. Buyers who start t off as uninformed, in the amount M^H_t, reach period $t+1$ as informed if they become informed, don't exit at t, and survive. They become informed if they meet a v_H-seller or observe N_{t-1}. They continue if they meet a v_L-seller, regardless of the signal outcome. Thus,

$$B^H_{t+1} - M^H_{t+1} = (B^H_t - M^H_t)(1-\mu^H)\delta + M^H_t(1-\mu^H)(1-\tau^H_{t-1})\delta.$$

Finally, how many buyers who start at $t+1$ as informed, $B^H_{t+1} - M^H_{t+1}$, reach period $t+2$ as informed? All informed buyers remain informed, but only those reach $t+2$ who at $t+1$ see v_L and survive. Buyers who start $t+1$ off as uninformed, M^H_{t+1}, reach period $t+2$ as informed if they meet a v_L-seller, observe T_t, and survive. Thus,

$$B^H_{t+2} - M^H_{t+2} = (B^H_{t+1} - M^H_{t+1})(1-\mu^H)\delta + M^H_{t+1}(1-\mu^H)\tau^H_t\delta.$$

Combining these equations and imposing that $x_{t'+2} = x_{t'}$ for $t' = t, t+1$ and all endogenous variables x, gives a solution

$$\tau^H_{t+1} = \frac{16 + 6\delta - 2\delta^2 + \delta^3 - \sqrt{256 + 192\delta - 28\delta^2 - 40\delta^3 + 4\delta^5 + \delta^6}}{2\delta^3},$$

which decreases in δ and is in the interval $[\frac{21-\sqrt{385}}{2} \approx 0.69, \frac{3}{4}]$ for all $\delta \in (0,1)$.

Finally, I derive the conditions under which the proposed strategy is optimal. For a buyer with belief π', the value of continuing for one more period and then accepting either offer is given by equation (3) so the critical belief is again $\bar{\pi}$ (defined in (1)).

Let the beliefs of a buyer who has seen h of T_t, i of N_t, j of T_{t+1}, and k of N_{t+1}, be $\pi(h, i, j, k)$. Since odd and even periods alternate, it must be that $h + i \in \{j + k - 1, j + k, j + k + 1\}$. A sufficient condition for the proposed strategy to be optimal is that a buyer who is supposed to continue according to the strategy wants to continue for at least one period and that a buyer who is supposed to accept v_L according to the strategy prefers accepting v_L to continuing for one more period. Then the strategy requires that

(i) buyers who know the state to be good prefer to continue after v_L: $\pi(h, i, j, k) > \bar{\pi}$ for all $h, k \geq 1$,

(ii) buyers who do not know the state and have not seen N_t continue after v_L: $\pi(0, 0, j, 0) > \bar{\pi}$ for all j (i.e., for $j = 0, 1$), and

(iii) buyers who do not know the state and have seen at least one N_t accept v_L: $\pi(0, i, j, 0) < \bar{\pi}$ for all $i \geq 1$ and all j.

Conditions (i) are satisfied as $\pi(h, i, j, k) = 1$ for all $h, k \geq 1$. Of conditions (ii), the stricter is for the more pessimistic buyer, i.e., for $j = 1$ since T_{t+1} is bad news. The stricter condition, $\pi(0, 0, 1, 0) > \bar{\pi}$, can be written as

$$\frac{\pi(0,0,1,0)}{1-\pi(0,0,1,0)} = \omega\frac{1-\mu^H}{1-\mu^L}\frac{\tau^H_{t+1}}{\tau^L_{t+1}} = \frac{\omega\tau^H_{t+1}}{2} > \frac{\bar{\pi}}{1-\bar{\pi}}.$$

Of conditions (iii), the strictest is for the most optimistic buyer, which is for $i = 1$ and $j = 0$ because both N_t and T_{t+1} are bad news. The strictest condition, $\pi(0, 1, 0, 0) < \bar{\pi}$, can be written as

$$\frac{\pi(0,1,0,0)}{1-\pi(0,1,0,0)} = \omega\frac{1-\mu^H}{1-\mu^L}\frac{1-\tau^H_t}{1-\tau^L_t} = \frac{\omega}{4} < \frac{\bar{\pi}}{1-\bar{\pi}}.$$

The conditions can be satisfied simultaneously because $\frac{1}{4} < \frac{\tau^H_{t+1}}{2}$. I rearrange the two inequalities to get the exact conditions in the Proposition. □

A.6.3 Partially observed "prices"

Proof of Proposition 9. Let μ^θ stand for the fraction of v_H-sellers in state θ: $\mu^H = \frac{1}{2}$ and $\mu^L = 0$. Let λ_t^θ denote the conditional probability that a randomly drawn buyer who meets a v_L-seller trades with the seller at t in state θ (I denote the event by TL_t). I denote the event that the buyer does not trade by NL_t.

I first derive the probabilities of trading and amounts of buyers given the proposed strategy and then the conditions under which no buyer has an incentive to deviate. Finally, I derive conditions under which a trade (with a v_L-seller) in an odd period is good news and in an even period is bad news.

Consider an odd period t and state θ. The only young buyers who at t meet a v_L-seller also accept v_L (and don't become old) are those who observe a trade with a v_L-seller (TL_{t-1}). The amount of old buyers at $t+1$ is thus

$$O_{t+1}^\theta = (1-\mu^\theta)\left[1-\varepsilon(1-\mu^\theta)\lambda_{t-1}^\theta\right],$$

where the second $(1-\mu^\theta)$ accounts for the fact that the buyer whose trade information is observed at t met a v_L-seller at $t-1$. The unconditional trading probability at t is

$$\tau_t^\theta = \frac{O_t^\theta + \mu^\theta + (1-\mu^\theta)\varepsilon(1-\mu^\theta)\lambda_{t-1}^\theta}{1+O_t^\theta}.$$

because all old buyer trade, young buyers trade with a v_H-seller always and with a v_L-seller if they observe a trade with a v_L-seller. The probability of a trade with a v_L-seller is

$$\lambda_t^\theta = \frac{O_t^\theta + \varepsilon(1-\mu^\theta)\lambda_{t-1}^\theta}{1+O_t^\theta},$$

because, conditional on meeting a v_L-seller, a buyer trades with a v_L-seller if she is old or if she is young and observes a trade with a v_L-seller.

At an even period $t+1$, young buyers continue if they meet a v_L-seller and see a trade (T_t), a trade with a v_L-seller (TL_t), or a trade/no trade with a v_H-seller. The amount of old buyers at $t+2$ is thus

$$O^\theta_{t+2} = (1-\mu^\theta)\left[(1-\varepsilon)\tau^\theta_t + \varepsilon(1-\mu^\theta)\lambda^\theta_t + \varepsilon\mu^\theta\right].$$

The unconditional trading probability at $t+1$ is

$$\tau^\theta_{t+1} = \frac{O^\theta_{t+1} + \mu^\theta + (1-\mu^\theta)\left[(1-\varepsilon)(1-\tau^\theta_t) + \varepsilon(1-\mu^\theta)(1-\lambda^\theta_t)\right]}{1+O^\theta_{t+1}},$$

because all old buyer trade, young buyers trade with a v_H-seller always and with a v_L-seller if they observe no trade or not trade with a v_L-seller. The conditional trading probability at $t+1$ is

$$\lambda^\theta_{t+1} = \frac{O^\theta_{t+1} + (1-\varepsilon)(1-\tau^\theta_t) + \varepsilon(1-\mu^\theta)(1-\lambda^\theta_t)}{1+O^\theta_{t+1}},$$

because, conditional on meeting a v_L-seller, a buyer accepts v_L if she is old or if she is young and observes either no trade or no trade with a v_L-seller. Note that $\tau^L_{t'} = \lambda^L_{t'}$ and $\tau^H_{t'} > \lambda^H_{t'}$ for $t' = t, t+1$ because $\mu^L = 0$ and $\mu^H > 0$.

Imposing the condition that $x_{t'+2} = x_{t'}$ for all endogenous x and $t' = t, t+1$ and solving the system of equations for the bad state gives trade probabilities $\lambda^L_t = \tau^L_t = \sqrt{\varepsilon\tau^L_{t+1}}$, $\lambda^L_{t+1} = \tau^L_{t+1}$, and τ^L_{t+1} solves

$$\sqrt{\varepsilon\tau^L_{t+1}} = (2-\varepsilon\tau^L_{t+1})(1-\tau^L_{t+1}).$$

The probability τ^L_{t+1} decreases in ε. Letting $y := 3 - \sqrt{5+2\varepsilon\lambda^H_{t+1}}$, the system of equations for the good state collapses to, $\lambda^H_t = 1-y$, $\tau^H_t = \frac{1}{2} + \frac{\lambda^H_t}{2}$, $\tau^H_{t+1} = \frac{1}{2} + \frac{\lambda^H_{t+1}}{2}$, and λ^H_{t+1} solves

$$4(2-\varepsilon\lambda^H_{t+1}) = y[14 - \lambda^H_{t+1}(6-\varepsilon\lambda^H_{t+1}+\varepsilon)].$$

The equilibrium exists for parameter values such that no buyer wants to deviate. For a young buyer with belief π', the value of continuing and accepting either offer when old is given by equation (3) so the critical belief is again $\bar{\pi}$ (defined in (1)). A young buyer who meets a v_L-seller and sees a trade or no trade updates according to equation (7) or (6) respectively. A young buyer who at $t' = t, t+1$ meets a v_L-seller and observes that a buyer b' traded with a v_L-seller updates her beliefs as

$$\frac{\pi(v_L, TL_{t'})}{1 - \pi(v_L, TL_{t'})} = \omega \frac{1-\mu^H}{1-\mu^L} \frac{\lambda^H_{t'}}{\lambda^L_{t'}} \frac{1-\mu^H}{1-\mu^L} = \frac{\omega}{4} \frac{\lambda^H_{t'}}{\lambda^L_{t'}},$$

where the second fraction $\frac{1-\mu^H}{1-\mu^L}$ accounts for the fact that b' must have met a v_L-seller. A young buyer who at $t' = t, t+1$ meets a v_L-seller and observes no trade with a v_L-seller updates her beliefs as

$$\frac{\pi(v_L, NL_{t'})}{1 - \pi(v_L, TL_{t'})} = \omega \frac{1-\mu^H}{1-\mu^L} \frac{1-\lambda^H_{t'}}{1-\lambda^L_{t'}} \frac{1-\mu^H}{1-\mu^L} = \frac{\omega}{4} \frac{1-\lambda^H_{t'}}{1-\lambda^L_{t'}}.$$

No buyer wants to deviate from the proposed equilibrium strategy if

$$\min\{\pi(v_L, T_t), \pi(v_L, TL_t), \pi(v_L, N_{t+1}), \pi(v_L, T_{t+1}), \pi(v_L, NL_{t+1})\} \geq \bar{\pi}, \tag{15}$$

and

$$\max\{\pi(v_L, N_t), \pi(v_L, NL_t), \pi(v_L, TL_{t+1})\} \leq \bar{\pi}. \tag{16}$$

I now explain how we can reduce the set of these constraints considerably. Note that $\pi(v_L, T_t) > \pi(v_L, TL_t)$ because $\tau^H_t > \lambda^H_t$ and $\tau^L_t = \lambda^L_t$. Since no trades take place only with v_L-sellers, the information contained in $N_{t'}$ is exactly the same as that contained in $NL_{t'}$ for any t'. If a trade in an even period $(t+1)$ is bad news, then $\pi(v_L, N_{t+1}) > \pi(v_L, T_{t+1})$. The sets of constraints (15) and (16) thus reduce to $\min\{\pi(v_L, TL_t), \pi(v_L, T_{t+1}))\} \geq \bar{\pi}$, and $\max\{\pi(v_L, N_t), \pi(v_L, TL_{t+1})\} \leq \bar{\pi}$.

I show that the equilibrium exists for ε small enough. The limits of the trading probabilities as $\varepsilon \to 0$ are $\tau^L_t = \lambda^L_t \to 0$, and $\tau^L_{t+1} = \lambda^L_{t+1} \to 1$ in state L and $\tau^H_t \to \frac{\sqrt{5}-1}{2}$, $\lambda^H_t \to \sqrt{5}-2$, $\tau^H_{t+1} \to \frac{7-\sqrt{5}}{6}$ and $\lambda^H_{t+1} \to \frac{17-7\sqrt{5}}{3(3-\sqrt{5})}$ in state H. Thus, in the limit the inequalities on the posteriors that are required for the proposed strategy to constitute an equilibrium can be satisfied for an open set of parameter values v_H, v_L, and δ.

Now I show that for small ε, the trading probabilities are true probabilities so that a solution exists for ε close to zero. In state L, the condition $(2 - \varepsilon\tau_{t+1}^L)(1 - \tau_{t+1}^L) = \sqrt{\varepsilon\tau_{t+1}^L}$ holds. As τ_{t+1}^L decreases in ε, the only way this equality can hold is if $\varepsilon\tau_{t+1}^L$ increases in ε. But $\tau_t^L = \sqrt{\varepsilon\tau_{t+1}^L}$ is thus positive for $\varepsilon > 0$. As τ_{t+1}^L decreases in ε, both τ_t^L and τ_{t+1}^L are less than one for $\varepsilon > 0$.

Similarly, I can show that as ε increases, $\varepsilon\lambda_{t+1}^H$ must increase (and thus y decrease) so that λ_t^H and τ_t^H both increase in ε. Close to $\varepsilon = 0$, both are thus positive (and far from zero). Both λ_{t+1}^H and τ_{t+1}^H are below one for any positive ε, τ_t^H, and λ_t^H. Because everything is continuous, there exists a $\bar{\varepsilon} > 0$ such that the proposed strategy is an equilibrium for an open set of parameter values for all $\varepsilon < \bar{\varepsilon}$. □

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