Selling to Consumers with Intransitive Indifference*

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Abstract

We are indifferent between two cups of coffee when one differs from the other in having only one more grain of sugar. But such an indifference is not transitive, because eventually, after many enough grains of sugar are added, we will become able to tell one cup is sweeter than the other. When consumers feature intransitive indifference, putting a bad deal alongside a good deal can boost the sale of the latter by helping consumers to better appreciate it. When sellers compete for these consumers, they tend not to undercut each other, because undercutting often go un-appreciated. Instead, sellers segregate into providers of good deals and bad deals, with the formers free-riding on the latters in helping consumers better appreciate their good deals, and the latters free-riding on the formers in making consumers less hesitant to buy.

KEYWORDS: intransitive indifference, monopoly, duopoly, compete in utility space

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1 Introduction

We are indifferent between two cups of coffee when one differs from the other in having only one more grain of sugar. But such an indifference is not transitive, because eventually, after many enough grains of sugar are added, we will become able to tell one cup is sweeter than the other. Ever since Luce (1956), economists have developed various utility representations of a decision maker featuring intransitive indifference,[1] but never explored how they may affect classical economic analyses. This paper makes a first step in filling this gap. Specifically, it studies sellers' equilibrium behavior when consumers feature intransitive indifference.

In exploring the implications of intransitive indifference, we are forced to make an explicit choice over two competing interpretations. When a coffee drinker expresses indifference between two cups of coffee, A and B, is he merely having too dull an instrument to distinguish two alternatives that he otherwise would have strict preferences over, or is he genuinely indifferent between the two? To paraphrase the question, suppose he is told that A actually contains less sugar than B does, would he be so health-conscious that he then strictly prefers A to B, or would he remain indifferent on the basis that difference too small to detect is also immaterial to health? Let's call the corresponding interpretations the *dull-instrument* and the *genuine-indifference* interpretations, respectively.

Some authors apparently favor the dull-instrument interpretation. For example, Luce (1956) envisions that a decision maker's expressed preferences may change when he has access to better instruments, and Fishburn (1970a) speaks of the possibility that a coffee drinker breaking his earlier indifference after further taste tests a few moments later.

Other authors, however, seem to favor the genuine-indifference interpretation. For example, when Jamison and Lau (1973) and Fishburn (1975) characterize possible choice functions that may arise from intransitive indifference, they implicitly have in mind a health-conscious coffee drinker who, when choosing among three cups of coffee—with B indistinguishably sweeter than A, C indistinguishably sweeter than B, but C distinguishably sweeter than A—may nevertheless choose B on the basis that it is dominated by none. He does so notwithstanding the fact that he should have inferred from these pairwise comparisons that B contains more sugar than A does, presumably because he is

[1] See, among others, Fishburn (1970a, 1970b) and Beja and Gilboa (1992).

genuinely indifferent.[2]

In this paper, we explicitly espouses the dull-instrument interpretation. This is not to say that we consider the genuine-indifference interpretation implausible. But we conjecture that little change in classical economic analyses arises from the latter kind of intransitive indifference. For example, suppose a coffee drinker buying a cup of Arabica coffee is genuinely indifferent if up to 5% of the Arabica coffee bean is replaced by the cheaper Robusta coffee bean, then the optimal strategy of a monopolist is to simply offer him at the same price a cup of coffee that is 95% Arabica and 5% Robusta.

We hence have in mind a consumer shopping for a mobile phone plan. If given an opportunity to read through all the fine print and to time his phone calls carefully to make the most of the phone plan, the maximum utility he can extract will be u, which we can think of as the phone plan's offered utility. He cares about this offered utility when he makes his purchase, and hence is not genuinely indifferent. However, at the store, due to an imperfect night of sleep, he only has a rough sense of this offered utility, and has to make his purchase decision based on the limited sense data he has, limited by his inability to discern small differences. Traditional game-theoretic tools continue to be applicable in analyzing the strategic interaction between sellers and such a consumer, but ample new results arise from the specific information structure of this game.

Behind many of these new results is the phenomenon that the presence of inferior phone plans, or more generally *deals*, can help the consumer appreciate a better deal. To see this, let's return to the example two paragraphs above, where there are three deals, with A indistinguishably better than B, B indistinguishably better than C, but A distinguishably better than C. Suppose B is the consumer's outside option, and the seller is trying to sell A to him. In the absence of C, the consumer is not able to appreciate the superiority of A. By letting him examine the inferior C, the seller can convince him that A indeed is the best. This phenomenon is the driving force behind, for example, one of our results that sellers of good deals coexist with sellers of bad deals, with the former free-riding the presence of the latter (see below). We shall review evidences of this phenomenon from the psychology and marketing literature in the next subsection.

[2] Kamada (2016), to our best knowledge, is the first to point out this implicit interpretation underlying the kind of choice functions studied by Jamison and Lau (1973) and Fishburn (1975). He then proceeds to characterize an alternative kind of choice functions (called *sophisticated* choice functions) that are more in line with the dull-instrument interpretation.

In modelling a consumer featuring intransitive indifference, we follow the decision-theory literature and introduce the notion of *just noticeable difference* (jnd). For example, suppose a coffee drinker cannot distinguish the sweetness of any two cups of coffee that differ by fewer than 100 grains of sugar, then 100 grains of sugar is his jnd. Much of the decision-theory literature postulates a *deterministic* jnd, which is admittedly unrealistic. In this paper, we follow the discussion in Gilboa (2009) and postulate a probabilistic jnd instead—as we add more and more sugar to cup B, the probability that the coffee drinker can tell that B is sweeter than A increases gradually instead of jumping from 0 to 1. Our way to model such a probabilistic jnd is to assume that how sharp the coffee drinker's taste buds are depends on the quality of his previous-night's sleep, which in turn contains much randomness beyond the grasp of the coffee drinker himself as well as the others.

Section 2 provides further details of our model. Specifically, we study a game with a consumer who features intransitive indifference and up to two sellers. Sellers market one or more deals, with each represented by its offered utility. The consumer demands up to only one deal, but has to base his purchase decisions on his limited sense data, in the form of ordinal rankings of different deals, limited by his inability to discern small differences.

In Section 3, we first study the case of a monopolist marketing a single deal to this consumer. Under some regularity condition on the distribution of the consumer's jnd (which in particular guarantees that he has a non-trivial jnd often enough), there exists a unique equilibrium, which is necessarily in mixed strategies—the monopolist randomizes between offering a good and a bad deals, and the consumer randomizes between buying and not when he cannot compare the offered utility with his reservation utility. Non-degenerate distribution of offered utility from a single seller thus can arise for a purpose different from screening. When the consumer's jnd increases (in FOSD sense), the monopolist's equilibrium profit decreases. Intuitively, the consumer is aware of his inability to compare, and hence is justifiably suspicious. When the consumer is suspicious more often, it is more difficult for the monopolist to sell, which explains its lower profit.

In Section 4, we study the case when the monopolist can at negligible costs market a second deal. The common knowledge that it can do so changes the strategic interaction between it and the suspicious consumer, and as a result its equilibrium profit may be even lower.

However, we show in Section 5 that, if the monopolist can at negligible costs market a sufficiently large number of deals, almost all of which are not meant to make any sales, its equilibrium profit must be close to the full surplus. Intransitive indifference plays a crucial role in this result. Intuitively, when the consumer has difficulty appreciating a genuinely good deal due to his dull instruments, putting some bad deals alongside the good deal can help him make the necessary comparison. The reason why many different bad deals are needed to approximate full surplus extraction is that the consumer may walk into the store with many possible jnd's (depending on the quality of his previous-night's sleep). Different bad deals are needed to cater to these different possible jnds. This result can potentially address a puzzle posted in the marketing literature, namely that sellers are sometimes observed to market inexplicably many different deals, notwithstanding the well-known effect that too many choices can potentially bring consumers headaches.[3]

The most interesting case arises when there are two *ex ante* identical sellers, each marketing only one deal, which we study in Section 6. Compared with the earlier case with a monopolist marketing two different deals, in this case the same number of deals are marketed, but by different sellers, and hence arguably the market is more competitive. However, we show that competition does not always increase the consumer's expected surplus (Proposition 9). The intuition is that, when the consumer features intransitive indifference, competing sellers tend not to undercut each other, because undercutting often goes un-appreciated by the consumer due to his dull instruments. Instead, in equilibrium, one seller specializes in offering a good deal, while another in offering a bad deal. The former free-rides on the latter because the latter helps the consumer to appreciate the good deal the former offers. The latter also free-rides on the former because the existence of a good deal make the consumer less hesitant to buy. In short, *free-riding*, instead of undercutting, is the keyword in understanding sellers' behavior when the consumer features intransitive indifference. With the opportunity to free-ride on the bad-deal seller, the good-deal seller does not need to sweeten its deal too much to make sales, which limits the benefit of competition for the consumer.

Section 7 concludes.

[3]Mochon (2013) articulates the puzzle as follows: "Indeed, one Best Buy store surveyed displayed 114 different TVs in their store. While preference heterogeneity likely accounts in part for this vast array of options, it is unlikely that consumers' preferences are so refined that they require 15 different models of 32-inch televisions, suggesting that something else may be at play here."

1.1 Related Literature

Evidences that a bad deal can help consumers better appreciate a good deal can be found both in the mass media and in the psychology and marketing literature. In a case study reported on the *Wall Street Journal*,[4] when Willimans-Sonoma brought the very first bread maker to the market at US$275 in the 1990s, sales were very bad. After the introduction of a slightly better model at twice the price, sales of the original model skyrocketed. In the psychology literature, Kim, Novemsky, and Dhar (2012) find that the probability of sales increases when two types of gums are priced at 620 and 640 Korean wons, respectively, than when both are priced at 630 Korean wons. In the marketing literature, Mochon (2013) finds that a brand is more likely to be purchased when it is presented with a competitor's brand than when it is presented alone.

Our results that a monopolist would in equilibrium permit a non-degenerate distribution of offered utility—either by randomizing in the one-deal case, or by offering different utilities for different deals in the two-deal case—resemble Salop's (1977) classical result that a monopolist may permit a non-degenerate distribution of price for the same product. However, the mechanisms are different. In Salop (1977), consumers differ in their price elasticities, and the monopolist would like to charge those who have lower price elasticities a higher price. If consumers who have lower price elasticities also have higher search costs, then the monopolist can achieve this goal by permitting a non-degenerate price distribution, because consumers who search less will on average pay a higher price, effecting the sorting pattern the monopolist wants. In our model, the consumer's (in)ability to discern small differences is not correlated with any other characteristic. As such, a non-degenerate distribution of offered utility arises not as a screening device of the monopolist, but rather as an equilibrium phenomenon: had the monopolist always offered a good deal, the consumer will be too willing to purchase even when he cannot tell for sure it is a good deal, generating incentives for the monopolist to replace the good deal with a bad one; but had the monopolist always offered a bad deal, the consumer will be too unwilling to purchase, generating incentives for the monopolist to sweeten its deal.

Our model is also similar to Rubinstein (1993) in that different types of the consumer differ neither in their preferences nor in the information they possess, but rather in their

[4] "Paul Lee: The Williams-Sonoma Bread Maker: A Case Study", *Wall Street Journal*, 10 April 2013, https://blogs.wsj.com/accelerators/2013/04/10/paul-lee-the-williams-sonoma-bread-maker-a-case-study/

ability to process information. Like Salop (1977), Rubinstein's (1993) monopolist also permits a non-degenerate price distribution. However, in terms of the underlying mechanism, Rubinstein (1993) is closer to Salop (1977) than to us. In particular, in Rubinstein (1993), consumers differ in their ability to process information, with less able consumers also being more costly to serve, and hence a non-degenerate price distribution serves as the monopolist's screening device to preclude these consumers. In our model, no correlation of this kind is assumed, and hence the equilibrium non-degenerate distribution of offered utility has nothing to do with screening.[5]

Our result that a non-degenerate distribution of offered utility arises in equilibrium in the multi-seller case resembles that of Salop and Stiglitz (1977), Varian (1980), and Rob (1985). However, our findings that competition (between two identical sellers) does not help bid up the offered utilities (compared to the case with a monopolist marketing up two different deals) and consequently does not necessarily benefit the consumer (Proposition 9) does not have a natural counterpart in these previous studies.

Our paper also contributes to an emerging literature on consumers who have difficulties in making comparisons. One popular way to model such difficulties is called categorization, where the consumer partitions deals into a few categories, and reacts to different deals within the same category in the same way (see, e.g., Chen, Iyer, and Pazgal (2010) and Gul, Pesendorfer, and Strzalecki (2017)). Since a partition structure dictates that the consumer's indifference must be transitive, some of the phenomena that arise only in an environment with intransitive indifference (such as the good-deal seller free-riding on the bad-deal seller) hence cannot arise in this literature.

Another approach to model consumers' difficulties in making comparisons is to postulate that they may simply ignore a competing seller, either because of inattention (de Clippel, Eliaz, and Rozen (2014)), or because the competing seller frames its deal in a format that these consumers find too unfamiliar (Piccione and Spiegler (2012)). A recurring theme of this literature is that, when multiple sellers compete, consumers' surplus is decreasing in their ability to make comparisons. This common finding resonates with some of our results (see Proposition 10). However, since the settings in these studies are

[5]See also Piccione and Rubinstein (2003), where consumers differ *both* in their preferences and in their ability to process information. Again, when these two characteristics are correlated in certain way, the monopolist can use a non-degenerate price distribution (or more precisely a deterministic price sequence that looks random to some consumers) as a screening device to separate consumers with different willingness to pay.

so different from each other, it is not easy to tell whether some common force is at work. We leave this question for future research.

Finally, our paper is also related to Natenzon (forthcoming), who studies an environment where the choice between two deals may be affected by the presence of a third one. We share the premise that the consumer only obtains imperfect information about the corresponding offered utilities. We differ however in both our focus and the driving force. Our paper focuses on sellers' strategic choices of their offered utilities, while Natenzon (forthcoming) assumes an exogenously fixed distribution of these offered utilities. In Natenzon (forthcoming), some pairs of deals are inherently easier to compare, in the sense that the consumer obtains more precise information about the ranking of these pairs than of other pairs. Such asymmetry is absent in our paper, where every pair is *a priori* similar, and contextual inference comes solely from utility gap.

2 The Model

There is a single consumer and up to two sellers. Sellers "compete in utility space" in the fashion of Armstrong and Vickers (2001). That is, they compete by offering various "deals", each involving possibly myriads of options (or bundles of options) with intricate tariffs. We follow Armstrong and Vickers (2001) and assume that the consumer purchases a deal instead of picking and choosing from various deals to make up his own bundle. A deal hence is indivisible, and the consumer demands at most one. Each deal is identified with "the (scalar) level of utility, or 'value for money', offered to" the consumer. We can think of this offered utility as an indirect utility, obtained only after the consumer purchases the deal, painstakingly reads all the fine print at home, and makes the most of the purchased deal. When a seller provides utility u to the consumer, the maximum profit it can extract from the consumer is $\pi(u)$, which decreases in u. We normalize the consumer's reservation utility to 0, and assume that $\pi(u)$ takes the simple linear form of $\pi(u) = \overline{\pi} - u$.[6] We assume that $\overline{\pi} > 0$, and hence there is gain of trade.

We shall carefully distinguish two different concepts: the consumer's ability to *discern* two utilities, and his ability to *compare* them. The consumer's ability to *discern* two utilities depends on how sharp his receptors are. An analogy is that, when we are given two cups

[6]For an example see Armstrong and Vickers (2001, pp.587-8).

of coffee, whether we can tell which one is sweeter depends on how sharp our taste buds are. Specifically, the consumer wakes up in the morning with a random type $d \in [0, \infty)$, depending on the quality of his previous-night's sleep. When the consumer is of type-d, given two utilities, u_1 and u_2, he is able to discern them if and only if $|u_1 - u_2| > d$; i.e., if and only if the two utilities are far enough apart. The type d is hence an inverse measure of the consumer's ability to discern two utilities. If the consumer can discern the two utilities, we write $u_1 \widehat{\succ} u_2$ (respectively, $u_2 \widehat{\succ} u_1$) if he feels that u_1 is higher (respectively, lower) than u_2. If he cannot discern the two utilities, we write $u_1 \widehat{\sim} u_2$ (i.e., $u_1 \widehat{\sim} u_2$ if and only if $u_1 \widehat{\nsucc} u_2$ and $u_2 \widehat{\nsucc} u_1$).

When the consumer cannot discern two utilities (possibly because of poor receptors), he may nevertheless still be able to *compare* them, especially if he receives certain aid. An especially interesting aid he may receive is the existence of a third utility, u_3. For example, if he cannot discern u_1 and u_2 (i.e., $u_1 \widehat{\sim} u_2$) and cannot discern u_2 and u_3 (i.e., $u_2 \widehat{\sim} u_3$), but nevertheless feels that u_1 is higher than u_3 (i.e., $u_1 \widehat{\succ} u_3$), then by some simple logical deduction he should be able to infer that u_1 is actually higher than u_2. We shall assume that consumers are always able to make such kind of inference. In other words, while a consumer may have poor receptors, his rationality is undamaged.

Formally, we follow Kamada (2016) and construct *inferred ordering*, $\succ$, from the more primitive $\widehat{\succ}$ as follows: $u_1 \succ u_2$ if and only if at least one of the following holds:[7]

1. $u_1 \widehat{\succ} u_2$;
2. $\exists u_3$ such that $u_1 \widehat{\succ} u_3$ but $u_2 \widehat{\nsucc} u_3$;
3. $\exists u_3$ such that $u_3 \widehat{\succ} u_2$ but $u_3 \widehat{\nsucc} u_1$.

We write $u_1 \sim u_2$ if and only if $u_1 \nsucc u_2$ and $u_2 \nsucc u_1$.

An implicit assumption here is that u_3, which helps the consumer compare u_1 and u_2, has to come with an actual (as in contrast to fictitious) deal. Without such an implicit assumption, our model would readily collapse into a traditional one. This is because, for example, a consumer with $d = 5$, though not being able to discern $u_1 = 10$ and $u_2 = 7$, would (had the implicit assumption relaxed) be able to compare them if he manages to

[7] Kamada (2016) shows that further iterations of this logic would not result in new inferences. More precisely, if we define $\succ^*$ using the same method (with $\succ$ in place of $\widehat{\succ}$), the new binary relation $\succ^*$ will remain the same as $\succ$.

imagine a fictitious deal with utility $u_3 = 4$.[8] One can think of the consumer's ability to discern two utilities as already including his limited ability to imagine fictitious deals.

While the information the consumer possesses at the time of purchase is ordinal, his payoff remains cardinal. For example, he may find two different deals in the market, with utilities u_1 and u_2, which he cannot discern (i.e., $u_1 \widehat{\sim} u_2$), while he feels that both are higher than his reservation utility 0 (i.e., $u_1 \widehat{\succ} 0$ and $u_2 \widehat{\succ} 0$). These are the only primitive sense data he possesses at the time of purchase. However, his payoff of purchasing the first deal will still be u_1, as in any traditional model. In other words, the consumer's receptors are poor only at the time of purchase, but not when he actually consumes the deal.[9]

We assume that both the consumer and the sellers are risk neutral. Therefore, the consumer makes his purchase decision based on expected utilities, where expectation is taken conditional on the primitive sense data he possesses, and his knowledge of the sellers' strategies. Similarly, each seller maximizes expected profit, where the profit from offering utility u is $\overline{\pi} - u$.

We assume that *the consumer does not know his own type*, possibly because the quality of his previous-night's sleep contains much randomness beyond his grasp. As a result, the consumer's purchasing strategy is independent of his type. It is commonly known that the distribution, F, of his type has support $\mathbb{R}_+$,[10] and admits a density function, f, which in turn satisfy the following assumption:

Assumption 1 *The density function f is weakly decreasing and satisfies the monotone hazard rate property; i.e., $f/(1-F)$ is weakly increasing.*

Examples of a density function satisfying Assumption 1 include that of an exponential distribution.[11]

If F is a point mass at 0, then our model collapses to a traditional one, where in equilibrium a monopolist would offer utility $u = 0$, and the consumer would purchase for

[8] With $d = 5$, the consumer's primitive sense data are $u_1 \widehat{\not\succ} u_2$ and $u_2 \widehat{\not\succ} u_3$ but $u_1 \widehat{\succ} u_3$. From these primitive sense data the consumer can derive $u_1 \succ u_2$, inferring that u_1 is higher than u_2.

[9] The genuine-indifference interpretation mentioned in the Introduction corresponds to the case where the consumer features intransitive indifference both at the time of purchase and at the time of actual consumption.

[10] Throughout this paper, we assume that F has full support on $\mathbb{R}_+$. This assumption is not necessary for any of our results. We can easily handle distributions with finite supports of the form $[0, D]$, where $D < \infty$, at the expenses of slightly messier proofs.

[11] As explained in Footnote 10, we can easily handle distributions with finite support at the expenses of messier proofs. Examples of a finite-support density function satisfying Assumption 1 include that of a uniform distribution.

sure. Gain of trade will be realized for sure, and the monopolist captures all the surplus. We shall refer to this outcome as the *first best* for short.

It turns out that the first best remains an equilibrium outcome when the probability mass nearby 0 is sufficiently close to 1. The intuition is that types with d close to 0 impose a discipline on the monopolist, discouraging it from lowering the offered utility below 0 and exploiting types with larger d. In order for our model to generate results that are qualitatively different from a traditional one, we need F to be sufficiently different from a point mass at 0. The dividing line turns out to be the following condition.[12]

Assumption 2 *The density at $d = 0$ is sufficiently small; specifically, $f(0)\overline{\pi} < 1$.*

Throughout this paper, our solution concept is the standard perfect Bayesian equilibrium, which we shall simply refer to as the *equilibrium.*

3 A Monopolist Marketing A Single Deal

In this section, we start with the simplest possible case where there is a monopolist who markets only a single deal. We can think of the costs of marketing a second deal as being prohibitively high, an assumption that we shall relax in the next two sections. The monopolist's strategy is a distribution of the offered utility u. The consumer's strategy is his probability of purchasing the deal conditional on his primitive sense data (recall that he does not know his own type and hence cannot contingent his purchase probability on it). Since he is faced with only two utilities (u, and his reservation utility, 0) at the time of purchase, $\succ$ is the same as $\widehat{\succ}$. Utility maximization dictates that the consumer purchases with probability 1 (respectively, with probability 0) when his primitive sense data is $u\widehat{\succ}0$ (respectively, $0\widehat{\succ}u$). Therefore, the consumer's strategy can be reduced to his purchase probability when his his primitive sense data is $u\widehat{\sim}0$, which we shall denote by q.

[12]While we shall provide more intuition for the meaning of Assumption 2 in Section 3, let's promptly point out one particular implication of this assumption. For any finite-support F (see Footnote 10) that satisfies Assumptions 1 and 2, the upper limit of its support, D, is larger than $\overline{\pi}$. This is because, since f is weakly decreasing, we have $F(\overline{\pi}) \leq f(0)\overline{\pi} < 1$, and hence $D > \overline{\pi}$. An implication of this observation is that, even when the offered utility is as high as $\overline{\pi}$, thus erasing any profit of the seller, some types of the consumer will still not be able to discern this utility and the reservation utility 0.

Given the consumer's strategy q, the monopolist's profit as a function of its (pure-strategy) utility is

$$\Pi(u;q) = \begin{cases} (\overline{\pi} - u)[1 - F(-u)]q & \text{if } u \le 0 \\ (\overline{\pi} - u)\left(F(u) + [1 - F(u)]q\right) & \text{if } u \ge 0 \end{cases},$$

where, in the case of $u \ge 0$ for example, $F(u)$ is the probability that the consumer can discern u and 0 and hence would purchase with probability 1, and $1 - F(u)$ is the complementary probability that he cannot and hence would purchase with probability q.

Since the profit function has a kink at $u = 0$, we maximize it over the lower sub-range $u \in (-\infty, 0]$ and the upper sub-range $u \in [0, \overline{\pi}]$ separately, and then compare the maximized profit over each sub-range. In the proof of Proposition 1 below, we show that a unique maximizer exists in each sub-range, which we denote by $\underline{u}$ and $\overline{u}(q)$, respectively. Note that the maximizer in the lower sub-range does not depend on q, which can be readily verified by inspection of the profit function in that sub-range. The first best will be an equilibrium outcome iff $\underline{u} = 0 = \overline{u}(1)$. In the Appendix, we show that this is indeed the case if Assumption 2 is violated. In this sense a distribution F that violates Assumption 2 is not different enough from a point mass at 0.

To understand why Assumption 2 guarantees that the first best cannot be an equilibrium outcome, it suffices to understand why $\underline{u} < 0$ under this assumption. Suppose the consumer purchases for sure even when he cannot discern u and 0; i.e., suppose $q = 1$. Suppose the monopolist decreases the offered utlity from $u = 0$ to $u = -\epsilon < 0$. For ϵ small, almost no type of the consumer can detect the decrease, and hence the monopolist earns ϵ more from almost every type. There are approximately $\epsilon f(0)$ types whose receptors are very sharp (i.e., with $d < \epsilon$), who will be able to detect the decrease and hence refuse to purchase. The lost profit from this small subset of types amounts to $\epsilon f(0)\overline{\pi}$. Under Assumption 2, the lost profit is smaller than the grain from exploiting the rest of the types (i.e., $\epsilon f(0)\overline{\pi} < \epsilon$), and hence the monopolist cannot resist the temptation of secretly decreasing the offered utility below 0.

Indeed, for q sufficiently close to 1, types with poor receptors are so trusting and so willing to purchase that it is better for the monopolist to offer the low utility $\underline{u}$ to exploit these types. On the contrary, when q is sufficiently close to 0, types with poor receptors are so untrusting and so unwilling to purchase that the only way to do business with them is to offer the high utility $\overline{u}(q)$ in the hope of convincing them that the deal is good. Neither case can be an equilibrium, because the consumer's best response against $\underline{u}$ is $q = 0$ and that against $\overline{u}(q)$ is $q = 1$. In equilibrium, q must take some intermediate value q^* so that the monopolist is willing to randomize between $\underline{u}$ and $\overline{u}(q^*)$, and the monopolist must randomize in a way that makes the consumer willing to randomize between purchasing or not.

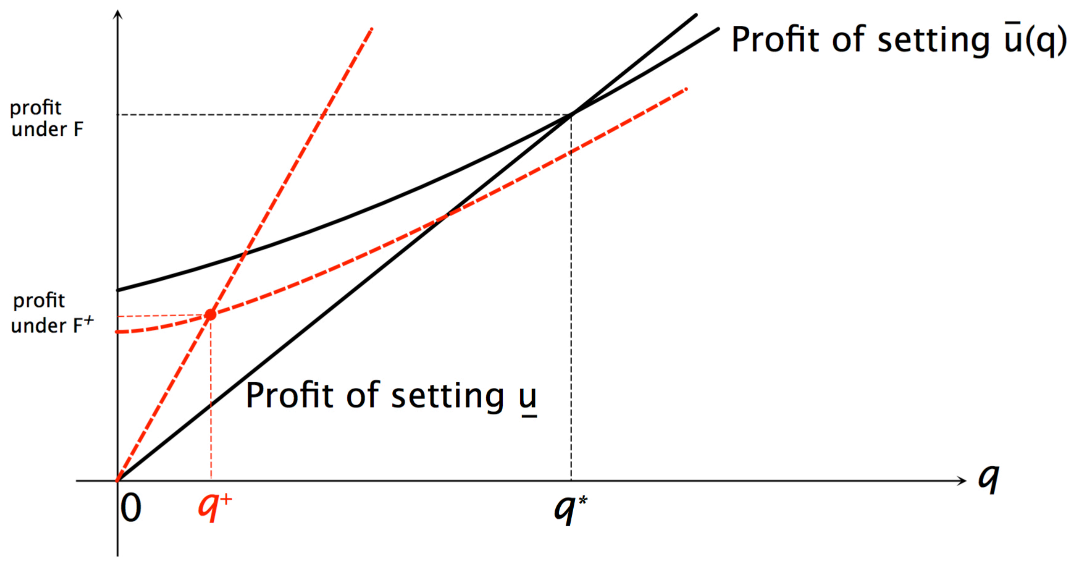

Figure 1: the monopolist's profits in the one-deal case

Figure 1 illustrates how the equilibrium is uniquely determined. In Figure 1, the solid line passing through the origin represents the monopolist's profit if it offers the low utility $\underline{u}$, which in turn increases linearly in q. The solid curve represents the monopolist's profit if it offers the high utility $\overline{u}(q)$, which is convex in q because the monopolist re-optimizes the offered utility when it faces a different q. The convex curve is strictly above the linear line at $q = 0$, and is strictly below at $q = 1$. The shapes of the two profit functions dictate that they cross once and only once at q^*, at which point the monopolist is willing to randomize between the high and the low utilities.

Proposition 1 *In the case of a monopolist marketing up to only one deal, the unique equilibrium is a mixed-strategy equilibrium, where*

- *the consumer purchases with a probability q^* that is strictly between 0 and 1/2 when he* *cannot discern the offered utility and his reservation utility u, and*

- *the monopolist randomizes between a low utility $\underline{u}^*$ that is strictly negative, and a high utility $\overline{u}^*$ that is strictly between 0 and $\overline{\pi}$.*

In equilibrium, gain of trade is not always realised. Some types of the consumer (those with $d \geq$ underlineu*) sometimes (when the monopolist offers the low utility $\underline{u}^*$) obtain strictly negative utility. But on average the consumer obtains strictly positive utility, meaning that the monopolist does not extract the full surplus even conditional on trade.

From Proposition 1, we can also obtain a very rough estimate of how much gain of trade is lost in equilibrium due to intransitive indifference. Notice that every time the monopolist offers the low utility, the consumer either can tell that it is a bad deal (in which case there will be no sale), or cannot tell (in which case he purchases with probability $q^* < 1/2$). Therefore, conditional on a low offered utility, gain of trade is realised with probability at most 1/2. Suppose the monopolist randomized between the high and low utilities with roughly equal probabilities. Then at least about 1/4 of gain of trade is lost in equilibrium due to intransitive indifference.

It will be interesting to see how the monopolist's equilibrium profit changes when the consumer becomes less able to discern two utilities. To answer that question, we compare the monopolist's equilibrium profit under two different distributions of the consumer's types, F and $F^{\dagger}$, where both satisfy Assumptions 1 and 2, but $F^{\dagger}$ first-order stochastically dominates (FOSD) F. In a market featuring $F^{\dagger}$, the consumer has higher type (probabilistically) and hence is less able to discern two utility. Our next proposition says that the consumer's inability to discern utilities actually hurts the monopolist.

Proposition 2 *In the case of a monopolist marketing up to only one deal, the monopolist's equilibrium profit decreases with an FOSD shift in F.*

Figure 1 provides a pictorial proof of Proposition 2. When the distribution of the consumer's types undergoes an FOSD shift from F to $F^{\dagger}$, the consumer is less able to discern utilities. This raises the monopolist's profit from offering a low utility, because the consumer is less able to tell a bad deal. This results in an anti-clockwise tilt of the linear line. On the contrary, the monopolist's profit from offering a high utility is now lower, because the consumer is also less able to tell a good deal. This results in a downward shift of the convex curve. In the new equilibrium, the consumer is less trusting ($q^{\dagger} < q^{*}$), and the monopolist's profit is lower.

This may seems a bit surprising. After all, a consumer who is less able to discern two utilities seem vulnerable to exploitation, and hence should be welcomed by the monopolist. Such an intuition is incomplete, however. Recall that a consumer with difficulty in discerning two utilities is a person equipped with some poor receptors. Although his instruments are poor, he is by no means irrational. He is aware that his instruments are poor, and is rationally untrusting when he finds the offered utility indiscernable from his reservation utility. A monopolist fares worse when the consumer is untrusting, because he cannot be easily convinced even when it is indeed offering him a good deal.

While the consumer's inability to discern utilities hurts the monopolist, it does not always benefit the consumer. Indeed, it is easy to see that the effect of an FOSD shift in F on the consumer's surplus is necessarily non-monotonic. Consider again the distributions F and $F^{\dagger}$, where the latter dominates the former in FOSD sense. In particular, this implies $F(\overline{\pi}) \geq F^{\dagger}(\overline{\pi})$. At the limit when $F^{\dagger}(\overline{\pi}) \searrow 0$, consumers almost surely cannot identify a good deal even when one exists (because the monopolist will never offer a utility beyond $\overline{\pi}$, and hence $\overline{u}$ cannot be larger than $\overline{\pi}$), and recall that whenever the consumer cannot identify a good deal he walks home with 0 surplus. Therefore, the consumer's surplus decreases with an FOSD shift from F to $F^{\dagger}$.

On the other hand, consider yet another distribution $F^{\dagger\dagger}$, which also satisfies Assumptions 1 and 2, and is dominated by F in FOSD sense. In particular, this implies $f(0) \leq f^{\dagger\dagger}(0)$. At the limit when $f^{\dagger\dagger}(0) \nearrow 1/\overline{\pi}$, the first best is an equilibrium outcome, where the consumer walks home with 0 surplus.[13] Therefore, the consumer's surplus increases with an FOSD shift from $F^{\dagger\dagger}$ to F.

Proposition 3 *In the case of a monopolist marketing up to only one deal, the consumer's inability to discern utilities does not always benefit him. Indeed, the effect of an FOSD shift in F on the consumer's surplus is necessarily non-monotonic.*

[13]Continuity holds at the limit, meaning that when $f^{\dagger\dagger}(0) \approx 1/\overline{\pi}$ while still satisfying Assumption 2, the consumer still walks home with approximately 0 surplus. See the proof of Proposition 3 for details.

4 A Monopolist Marketing Two Different Deals

In this section, we continue to study the case where there is only one seller, the monopolist. However, we now assume that the costs of marketing a second deal is negligible, while those of marketing more than two deals remain prohibitively high. We shall show that this improvement in the monopolist's marketing ability may paradoxically hurt its profit.

Formally, we assume that the costs of marketing a second deal are commonly known to be 0. The monopolist can market either a single deal or two different deals. In the case the monopolist markets two different deals, it can potentially offer a different utility for each deal. We shall name those two deals "deal 1" and "deal 2", with associated utilities u_1 and u_2, respectively. The consumer continues to demand at most one deal.

By allowing utilities to take the value of $-\infty$, we can wlog proceed as if both deals are actively being marketed. Marketing a single deal would then correspond to the case where $u_1 > -\infty = u_2$, whereas marketing two different deals would correspond to the case where both utilities are finite.

Some care should be taken in describing a game with two deals. It makes more sense to think of the deals as "anonymous", in the sense that the consumer's purchase decision can only depend on the inferred ordering, $\succ$, of 0, u_1, and u_2, but otherwise cannot depend on the names of the deals. This precludes, for example, the strategy of purchasing deal 1 with probability 4/5 when $u_1 \sim 0 \succ u_2$, while purchasing deal 2 with only probability 1/3 when $u_2 \sim 0 \succ u_1$. Similarly, it precludes the strategy of purchasing deal 1 and deal 2 with probabilities 1/10 and 9/10, respectively, when $0 \prec u_1 \sim u_2 \succ 0$. In other words, the names of the deals are artificial constructs that are for the convenience of we analysts only, but are otherwise meaningless to the consumer. All the consumer can learn about a specific deal is already summarized by the inferred ordering $\succ$.

Since the names of the deals are just artificial constructs that are meaningless to the consumer, we shall follow the convention that "deal 2" is the deal with a lower utility; i.e., $u_1 \geq u_2$ by our convention. This convention does not preclude positive sales for deal 2 in equilibrium. This is because, even though the consumer knows that deal 2 is less desirable than deal 1, a type who cannot compare u_1 and u_2 cannot tell which deal is deal 2 (recall the assumption of anonymity above), and hence may end up purchasing deal 2 by chance.

Under the convention of $u_1 \geq u_2$, there can only be 11 different configurations of primitive sense data the consumer may possibly encounter.[14] For 8 out of these 11 configurations, there is an unambiguous highest-utility option in the resulting inferred ordering $\succ$, and utility maximization dictates that the consumer chooses this highest-utility option.[15] Among the remaining 3 configurations, the consumer's best response is also straightforward when the primitive sense data are $0 \widehat{\prec} u_1 \widehat{\sim} u_2 \widehat{\succ} 0$ (in which case the inferred ordering $\succ$ is the same as $\widehat{\succ}$): in this case, anonymity dictates that the best the consumer can do is to purchase each deal with probability $1/2$.

Therefore, the only two non-trivial cases are

1. the *single-contender* case, where the primitive sense data are $u_2 \widehat{\prec} u_1 \widehat{\sim} 0 \widehat{\succ} u_2$, and hence deal 1 is the only possible good deal for the consumer; and

2. the *all-tied* case, where the primitive sense data are $u_2 \widehat{\sim} u_1 \widehat{\sim} 0 \widehat{\sim} u_2$, and hence both deals 1 and 2 are possibly good deals for the consumer.

How the consumer behaves in these two cases will be determined in equilibrium. Let's denote by $q_1 \in [0, 1]$ the probability that the consumer purchases deal 1 (i.e., the single contender) in the single-contender case, and by $q_2/2 \in [0, 1/2]$ the probability that he purchases each contender in the all-tied case.

The reader may wonder why the monopolist may ever be hurt by its ability to market a second deal at negligible costs. Couldn't it guarantee at least its equilibrium profit in the one-deal case by simply marketing a single deal? The answer is no. If the consumer anticipates that the monopolist markets two different deals, then the event that it markets a single deal will be an off-equilibrium event, and the consumer's off-equilibrium belief in such an event can be quite different from his equilibrium belief in the one-deal case. We present an equilibrium with such a flavor below in Proposition 4. We then provide an

[14] There are 3 configurations where the primitive sense data already form a linear order; for example, $u_1 \widehat{\succ} 0 \widehat{\succ} u_2$. For each of these 3 configurations, the inferred ordering $\succ$ is the same as $\widehat{\succ}$. There are 4 configurations where exactly one pair of utilities are indiscernible; for example, $u_2 \widehat{\prec} u_1 \widehat{\sim} 0 \widehat{\succ} u_2$. For each of these 4 configurations, the inferred ordering $\succ$ is still the same as $\widehat{\succ}$. There are 3 configurations where exactly two pairs of utilities are indiscernible; for example, $u_1 \widehat{\succ} u_2 \widehat{\sim} 0 \widehat{\sim} u_1$. For each of these 3 configurations, the inferred ordering $\succ$ becomes a linear order; for example, from $u_1 \widehat{\succ} u_2 \widehat{\sim} 0 \widehat{\sim} u_1$ the consumer obtains $u_1 \succ 0 \succ u_2$. Finally, there is 1 configuration where all three pairs of utilities are indiscernible. For this configuration, the inferred ordering $\succ$ is also the same as $\widehat{\succ}$.

[15] There are 6 out of 11 configurations where the inferred ordering $\succ$ is a linear order (see Footnote 14), and hence an unambiguous highest-utility option exists. The other two configurations where an unambiguous highest-utility option exists are $0 \widehat{\succ} u_1 \widehat{\sim} u_2 \widehat{\prec} 0$ and $u_1 \widehat{\succ} 0 \widehat{\sim} u_2 \widehat{\prec} u_1$.

example of F such that the monopolist's profit in the equilibrium described in Proposition 4 is lower than its equilibrium profit in the one-deal case.

Proposition 4 *In the case of a monopolist marketing up to two different deals, there exists an equilibrium where*

- *the consumer refuses to purchase whenever there is no apparent highest-utility option (i.e., $q_1^* = q_2^* = 0$),*
- *the monopolist markets two different deals, with offered utilities being mirror images of each other around* 0 *(i.e., $u_1^* > 0 > u_2^*$ and $(u_1^* + u_2^*)/2 = 0$), and*
- *given the consumer's behavior (i.e., $q_1^* = q_2^* = 0$), (u_1^*, u_2^*) is the unique maximizer of the monopolist's profit over all pairs of utilities that are mirror images of each other around* 0.

In this equilibrium, the monopolist's offered utilities are deterministic. Yet gain of trade is still not always realized. The consumer purchases only if he can compare u_1^* and 0, and hence he always obtains strictly positive surplus conditional on a purchase.

To further elaborate on the point we made in the paragraph immediately before Proposition 4, let's consider what would happen if the monopolist deviates from its equilibrium behavior by marketing a single deal. Specifically, suppose the monopolist, instead of offering utilities (u_1^*, u_2^*) as described in Proposition 4, deviates and offers $u_2 = -\infty$ and randomizes between $u_1 = \overline{u}^*$ and $u_1 = \underline{u}^*$ as in Proposition 1. Given the consumer's equilibrium strategy $q_1^* = q_2^* = 0$, the monopolist cannot make any sales when the random utility u_1 takes the value of $\underline{u}^*$. Even when the random utility u_1 takes the value of $\overline{u}^*$, the monopolist makes a sale only if the consumer can discern u_1 and 0. Its profit from such a deviation is hence much lower than its equilibrium profit in the one-deal case.

The reason behind this dismal deviation profit is that the consumer is very untrusting in the single-contender case ($q_1^* = 0$). In the equilibrium described in Proposition 4, the single-contender case is an off-equilibrium event, and in such an event the consumer's off-equilibrium belief can be quite pessimistic.

Note that, in the equilibrium described in Proposition 4, the monopolist never makes any sales from deal 2. The only role of deal 2 is to convince the consumer that deal 1 is a good deal (it is). Specifically, types $u_1^* - u_2^* > d \geq u_1^* - 0$, although unable to *discern* u_1^* and

0, are nevertheless able to *compare* the two. The primitive sense data received by these types are $0 \widehat{\sim} u_1^* \widehat{\succ} u_2^* \widehat{\sim} 0$, which induce the inferred ordering of $u_1^* > 0 > u_2^*$, convincing the consumer to purchase deal 1.

Paradoxically, this helping hand from the second deal can backfire. In the one-deal case, it cannot be an equilibrium for the consumer to be totally untrusting. If the consumer were totally untrusting (i.e., if $q = 0$), the monopolist would not be able to make any sales unless it offers a strictly positive utility, but then the consumer should be totally trusting (i.e., $q = 1$) instead. This is no longer the case when there exists a bad deal purely to help the consumer to appreciate a good deal. Now the consumer who still cannot compare utilities despite this helping hand *can* justifiably remain totally untrusting (i.e., $q_1^* = q_2^* = 0$), worrying that he may inadvertently purchase a bad deal.

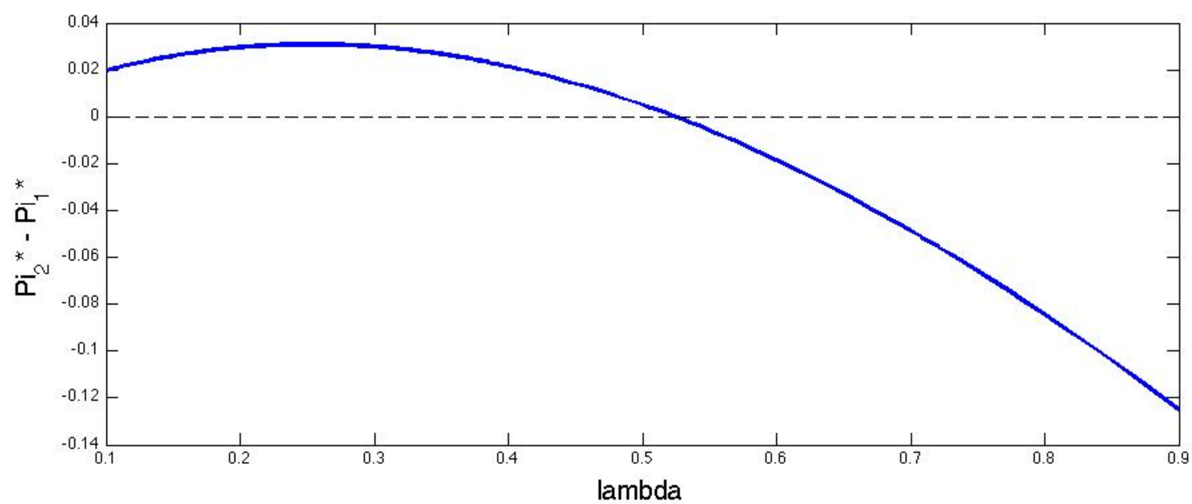

Figure 2: equilibrium profits in the one-deal and two-deal cases

That the consumer is more untrusting in the equilibrium described in Proposition 4 than in the unique equilibrium described in Proposition 1 is the main reason why the monopolist's profit can be lower in the former than in the latter. As an illustration, we compute the monopolist's profit in each of these two equilibria by letting $\overline{\pi} = 1$ and letting F be a member of the exponential class; i.e., $F(d) = 1 - e^{-\lambda d}$. Let Π_1^* denote the monopolist's profit in the equilibrium described in Proposition 1, and Π_2^* that in the equilibrium described in Proposition 4. In Figure 2, we plot $\Pi_2^* - \Pi_1^*$ against λ, the parameter of the exponential distribution. Note that F satisfies Assumption 2 only if $\lambda < 1$.

As λ increases towards 1, the consumer's ability to discern utilities improves.[16] In the one-deal case, this generates more discipline on the monopolist, decreases its incentive to offer a bad deal, and increases its incentive to offer a good deal. Types of the consumer who cannot discern utilities, by free-riding those who can, can hence afford to be more trusting, resulting in a higher q^* (see the proof of Proposition 2). Meanwhile, in the equilibrium described in Proposition 4, the consumer remains totally untrusting. As a result, $\Pi_2^* - \Pi_1^*$ becomes negative as λ increases towards 1.

Proposition 5 *A monopolist's ability to market a second deal at negligible costs may paradoxically hurt its profit. Specifically, there exists an equilibrium (as described in Proposition 4) where the monopolist's expected profit, under certain distributions of types, is lower than its (unique) equilibrium expected profit when marketing a second deal is prohibitively costly.*

While there is a unique equilibrium in the one-deal case, there are multiple equilibria in the two-deal case, with the one described in Proposition 4 being merely one of them. For the sake of completeness, we fully characterize an important sub-class of equilibria, namely the pure-seller-strategy equilibria, in Proposition 6 below. These are equilibria where the monopolist plays a pure strategy, in contrast to the mixed strategy played in the unique equilibrium in the one-deal case. Note that the equilibrium described in Proposition 4 is an example of a pure-seller-strategy equilibrium, where the monopolist offers utilities $(u_1, u_2) = (u_1^*, u_2^*)$ with probability 1.

Proposition 6 *In the case of a monopolist marketing up to two different deals, there is an $q_2^{max} \in (0, 1)$ such that*

- *every pure-seller-strategy equilibrium features an $q_2^* \leq q_2^{max}$;*
- *there exists a strictly decreasing function $u_1(\cdot)$ that maps $[0, q_2^{max}]$ into $(0, \overline{\pi})$ such that, in the pure-seller-strategy equilibrium featuring $q_2^* \in [0, q_2^{max}]$, the monopolist offers deterministic utilities $u_1^* = u_1(q_2^*)$ and $u_2^* = -u_1^*$;*
- *comparing any two pure-seller-strategy equilibria, the monopolist's expected profit is higher and the consumer's expected surplus is lower in the equilibrium with a higher q_2^*; and*
- *if, in addition to Assumption 2, f further satisfies $f(0)\overline{\pi} > 1/2$, then*

[16] Formally, an exponential distribution with a smaller λ dominates one with a larger λ in the FOSD sense.

– *for every* $q_2^* \in [0, q_2^{max}]$*, there exists a pure-seller-strategy equilibrium featuring that specific* q_2^**; and*

– *in the pure-seller-strategy equilibrium featuring* $q_2^* = q_2^{max}$*, the monopolist's expected profit is higher than its (unique) equilibrium expected profit in the one-deal case.*

In other words, every pure-seller-strategy equilibrium resembles the one described in Proposition 4, in the sense that the monopolist markets two different deals, with offered utilities being mirror images of each other (i.e., $u_1^* > 0 > u_2^*$ and $(u_1^* + u_2^*)/2 = 0$). As a result, the single-contender case is always an off-equilibrium event, rendering the exact description of q_1^* payoff-irrelevant. Each pure-seller-strategy equilibrium described in Proposition 6 hence is more precisely an equivalent class of equilibria featuring the same q_2^* but different q_1^*'s.

Comparing different pure-seller-strategy equilibria, the consumer is more trusting in those featuring higher q_2^*. When the consumer is more trusting, the monopolist's expected profit is higher, at the expense of the consumer's expected surplus. The equilibrium described in Proposition 4 is hence the worst for the monopolist and the best for the consumer among all pure-seller-strategy equilibria.

The consumer, however, will never be totally trusting in any pure-seller-strategy equilibrium. This is shown by the fact that q_2^* is capped from above by an upper bound q_2^{max} that is strictly smaller than 1. Therefore, once again, the first best cannot be achieved.

Finally, one may wonder whether there exists any (mixed-seller-strategy) equilibrium where the monopolist does not market deal 2 at all; i.e., $u_2 = -\infty$ with probability 1. Such an equilibrium, if exists, must look exactly like the unique equilibrium in the one-deal case as described in Proposition 1; i.e., the monopolist plays a mixed strategy and randomizes between $u_1 = \overline{u}^*$ and $u_1 = \underline{u}^*$, and the consumer purchases with probability $q_1 = q^*$ when he finds himself in the single-contender case. Let's call such an equilibrium, if exists, the single-deal equilibrium.[17]

When the monopolist can market up to two different deals at negligible costs, the single-deal equilibrium may not exist. The reason is that, whenever the monopolist is to offer $u_1 = \overline{u}^*$—which is a good deal for the consumer because $\overline{u}^* > 0$—it would lament

[17]The single-deal equilibrium is more precisely an equivalent class of equilibria featuring different q_2^*. This is because $u_2 = -\infty$ with pobability 1 implies that the all-tied case is an off-equilibrium event, and hence many different q_2^*'s can be supported by appropriately chosen off-equilibrium beliefs.

the fact that too few types of the consumer can *discern* $\overline{u}^*$ and 0 and hence appreciate this good deal, and would have incentives to bring in the second deal in order to help the consumer to *compare* $\overline{u}^*$ and 0.

Characterizing exactly when the single-deal equilibrium fails to exist turns out to be both tedious and non-illuminating. This is because there are many possible deviations involving "bringing in the second deal", and the single-deal equilibrium will fail to exist as long as one of these deviations is profitable. We shall hence provide only an easy sufficient condition for the non-existence of the single-deal equilibrium, focusing on only one particular deviation, namely the deviation to offering $(u_1, u_2) = (u_1^*, u_2^*)$, where (u_1^*, u_2^*) is as defined in Proposition 4. In the following proposition, Π_1^* and Π_2^* are as defined in the paragraph immediately before Proposition 5.

Proposition 7 *In the case of a monopolist marketing up to two different deals, the single-deal equilibirum does not exist whenever* $\Pi_2^* > \Pi_1^*$.

Proof: This is because the monopolist's equilibrium expected profit in the single-deal equilibrium, if exists, must equal to Π_1^*, while its deviation expected profit is at least Π_2^* if it deviates to setting $(u_1, u_2) = (u_1^*, u_2^*)$, where (u_1^*, u_2^*) is as defined in Proposition 4.[18] ■

For example, we can see from Figure 2 that, when $\overline{\pi} = 1$, and F is an exponential distribution with parameter $\lambda < 1/2$, the single-deal equilibrium does not exist.

5 A Monopolist Marketing Many Deals

While a monopolist's ability to market a second deal at negligible costs may paradoxically hurt its profit (Proposition 5), we can however prove that, its ability to market at negligible costs a sufficiently large number of deals, almost all of which are not meant to make any sales, will necessarily help its profit.

Formally, we assume that the costs of marketing the first n deals are commonly known to be 0, while those of of marketing more than n deals remain prohibitively high. We shall show that, for n sufficiently large, the monopolist's profit in *any* equilibrium is arbitrarily

[18] Recall from Footnote 17 that the single-deal equilibrium can feature many different q_2^*. The deviation profit is exactly Π_2^* if $q_2^* = 0$, and is strictly higher than Π_2^* if $q_2^* > 0$. (See Proposition 6.) Note that q_1^* is irrelevant in calculating the deviation expected profit.

close to its first-best profit $\overline{\pi}$, and hence is higher than its equilibrium profit in the one-deal case. Since the proof is short and constructive, we include it here in the main text and let it help explain the underlying intuition.

Proposition 8 *For any $\varepsilon > 0$, there exists $\overline{n}$ such that, for any $n \geq \overline{n}$, in the case of a monopolist marketing up to n different deals, the monopolist can achieve a profit higher than $\overline{\pi} - \varepsilon$ in any equilibrium.*

Proof: Pick any δ small enough and $\overline{n}$ big enough so that $(\overline{\pi} - \delta)F(\overline{n}\delta) > \overline{\pi} - \varepsilon$. Suppose the monopolist is to market $n \geq \overline{n}$ deals, with offered utilities $u_1 = \delta$, $u_2 = -\delta$, $u_3 = -2\delta$, $\ldots$, and $u_n = -(n-1)\delta$, respectively. When the consumer has type $d < \delta$, he will be able to discern u_1 and 0 and hence tell that u_1 is strictly positive, and will purchase deal 1. When the consumer has type $d \in [\delta, 2\delta)$, he cannot discern u_1 and 0, and cannot discern 0 and u_2, but is able to discern u_1 and u_2, and hence can infer that u_1 is strictly positive, and will also purchase deal 1. More generally, when the consumer has type $d \in [(k-1)\delta, k\delta)$, $k \in \{2, \ldots, n\}$, he cannot discern u_1 and 0, and cannot discern 0 and u_k, but is able to discern u_1 and u_k, and hence can infer that u_1 is strictly positive, and will purchase deal 1. The monopolist's profit is hence at least $(\overline{\pi} - u_1)F(n\delta) \geq (\overline{\pi} - \delta)F(\overline{n}\delta) > \overline{\pi} - \varepsilon$. ■

6 Two Sellers Marketing One Deal Each

In Section 4, we study the case of a single seller marketing up to two different deals. In this section, we study the case where the ability to market the second deal comes from a second seller instead of from the original seller. Specifically, we study the case where there are two identical sellers, each marketing up to only one deal. Arguably the marketing capacity available to the society is the same, in the sense that the costs of marketing two or fewer deals are negligible, while those of marketing more than 2 deals are prohibitively high. The only change from the setting in Section 4 to the current setting is how this marketing capacity is distributed. When this marketing capacity is evenly distributed between two identical sellers instead of being concentrated in the hands of one, the market is more competitive. We shall, however, show that more competition does not always benefit the consumer.

We shall name the two sellers "seller 1" and "seller 2", with associated offered utilities u_1 and u_2 for their respective deals. The consumer continues to demand one and only one deal. Following the second half of Section 4, we focus on pure-seller-strategy equilibria, meaning those equilibria where each seller plays a pure strategy.

As in Section 4, we assume that deals are "anonymous", in the sense that the names of deals are artificial constructs that are for the convenience of we analysts only, but are otherwise meaningless to the consumer. All the consumer can learn about a specific deal is already summarized by the inferred ordering $\succ$. As such, and since sellers play pure strategies, we can follow the convention in Section 4 that "deal 2" is the deal with a lower equilibrium offered utility; i.e., $u_1^* \geq u_2^*$ by our convention. As before, this convention does not preclude an asymmetric equilibrium where the two sellers offer different utilities. This is because, even though the consumer knows that deal 2 is worse than deal 1, if he cannot compare u_1 and u_2, he cannot tell which deal is deal 2, and hence may end up purchasing deal 2 by chance.

As in Section 4, to describe the consumer's strategy, it suffices to describe his behavior in the single-contender case and the all-tied case, which are defined in exactly the same way as in Section 4. Let's continue to denote by $q_1 \in [0, 1]$ the probability that the consumer purchases deal 1 (i.e., the single contender) in the single-contender case, and by $q_2/2 \in [0, 1/2]$ the (necessarily common) probability that he purchases each contender in the all-tied case.

The reader may wonder why more competition does not always benefit the consumer. Wouldn't competing sellers undercut each other and lead to higher offered utilities as in the traditional Bertrand model? The answer is no. For starter, when the consumer features intransitive indifference, undercutting one's opponent does not enable it to capture the whole market, because the consumer often is not able to tell that its offered utility is higher than its opponent's. This reduces one's incentives to undercut its opponent. Indeed, if sellers are anticipated to undercut each other aggressively, the consumer will become fairly trusting, and will be fairly willing to purchase even when he cannot compare utilities. Sellers hence will have incentives to lower their offered utilities in order to exploit this trusting consumer, invalidating the original anticipation.

When the consumer features intransitive indifference, sellers actually free-ride instead of undercut each other. There are two different kinds of free-riding behavior, and are respectively adopted by the two sellers. In a pure-seller-strategy equilibrium, one seller (seller 1) will specialize in offering a good deal, while the other (seller 2) specializes in offering a bad deal. Seller 2 free-rides seller 1's good deal, which keeps the consumer trusting, and offers a bad deal to exploit this trusting consumer.[19] Seller 1, on the other hand, free-rides seller 2's bad deal, which enables the consumer to sometimes recognize the good deal offered by seller 1—when the consumer cannot *discern* u_1 and 0, but can *compare* them with the help of u_2—and avoids the need to offer an even higher utility to win over the consumer.[20] As a result, both sellers manage to alleviate some upward pressure on their offered utilities by free-riding each other, albeit free-riding in very different manners.

A pure-seller-strategy equilibrium in this two-seller case ends up being very similar to one in the two-deal case studied in Section 4, in the sense that the offered utilities of the two deals are mirror images of each other around 0. The consumer purchases either when he can compare utilities—in which case he purchases the better deal and obtains strictly positive surplus—or when he finds himself in the all-tied case—in which case he randomizes between purchasing or not, and randomizes between the two deals when he does purchase, and obtains zero surplus on average. The consumer's surplus hence depends solely on how high the utility offered by deal 1 is, same as in a pure-seller-strategy equilibrium in the two-deal case. It turns out that the free-riding logic mentioned in the

[19]More formally, $u_2 < 0$ can be a best response for seller 2 only when $q_2 > 0$ (because seller 2 can sell a bad deal only when the consumer finds himself in the all-tied case), and $q_2 > 0$ can be part of the consumer's best response only when $u_1 \geq 0$ (otherwise $(u_1 + u_2)/2 \leq u_1 < 0$ and hence the consumer's best response must feature $q_2 = 0$).

[20]More formally, consider the case where $q_1 = 0 < q_2$ (which, as we shall argue soon, can be assumed wlog in our search for pure-seller-strategy equilibria). Suppose seller 2 offers $u_2 = -\infty$, effectively dropping out from the market. Then seller 1's best response is to set $u_1 = \overline{u}(0)$, where $\overline{u}(\cdot)$ is as defined in Section 3. Suppose seller 2 now raises its offered utility, but not too high as to offer the consumer a genuine good deal. Specifically, suppose seller 2 raises its offered utility from $-\infty$ to some finite $u_2 \in (-u_1, 0)$. Such a move of seller 2, instead of imposing competitve pressure on seller 1, actually *raises* seller 1's profit from $(\overline{\pi} - u_1)F(u_1)$ to

$$\Pi_1 := (\overline{\pi} - u_1)\{F(u_1 - u_2) + q_2[1 - F(u_1 - u_2)]\}.$$

More importantly for the consumer, seller 1 would now have incentives to even further *lower* its offered utility. Intuitively, with the help of a finite u_2, the consumer can more often appreciate the good deal that seller 1 is offering him. With a bigger consumer base, seller 1 now has incentives to lower its offered utility. Formally,

$$\begin{aligned}
\left.\frac{\partial \Pi_1}{\partial u_1}\right|_{u_1=\overline{u}(0)} &= -\{F(u_1 - u_2) + q_2[1 - F(u_1 - u_2)]\} + (\overline{\pi} - u_1)(1 - q_2)f(u_1 - u_2) \\
&= -q_2 - (1 - q_2)f(u_1 - u_2)\left[\frac{F(u_1 - u_2)}{f(u_1 - u_2)} - \frac{F(u_1)}{f(u_1)}\right] \\
&< 0,
\end{aligned}$$

where the second equality makes use of the first-order condition $\overline{\pi} - u_1 = F(u_1)/f(u_1)$ that characterizes $u_1 = \overline{u}(0)$, and the inequality makes use of Assumption 1.

previous paragraph and in Footnote 20 will depress u_1 so low that the consumer's surplus in this two-seller case is even lower than that in the pure-seller-strategy equilibrium described in Proposition 4.

Proposition 9 *In the case of two sellers marketing up to only one deal each, in any pure-seller-strategy equilibrium,*

- *the two sellers offer utilities that are mirror images of each other around* 0*; specifically,* $(u_1^*, u_2^*) = (x^*, -x^*)$*, where* $x^* > 0$ *is the unique solution to*

$$\frac{1 - F(2x^*)}{f(2x^*)} = x^* + \overline{\pi};$$

- *the consumer's expected surplus is strictly lower than in the equilibrium described in Proposition 4 for the case of a monopolist marketing up to two different deals.*

In the special case where F belongs to the exponential class, the consumer's expected surplus in any pure-seller-strategy equilibrium is especially easy to calculate. First, using the equation in Proposition 9, we have

$$x^* + \overline{\pi} = \frac{1 - F(2x^*)}{f(2x^*)} = \frac{\exp(-\lambda(2x^*))}{\lambda \exp(-\lambda(2x^*))} = \frac{1}{\lambda},$$

which gives us $x^* = 1/\lambda - \overline{\pi}$. When the consumer has type $d < 2x^*$, he is able to compare u_1^* and 0 (either by directly *discerning* them, or by *comparing* them with the help of u_2^*) and hence will walk home with surplus $u_1^* = x^*$. When the consumer has type $d \geq 2x^*$, on the other hand, he cannot compare u_1^* and 0, and hence will purchase both deals 1 and 2 with the same probability if he ever does any purchase. The consumer hence walks home with zero surplus, given that u_1^* and u_2^* are mirror images of each other around 0. On average, the consumer's surplus is hence

$$x^* F(2x^*) = x^*[1 - \exp(-\lambda(2x^*))] = \left[\frac{1}{\lambda} - \overline{\pi}\right][1 - \exp(-2 + 2\lambda\overline{\pi})],$$

which is strictly decreasing in λ. Recall that, in the special case of the exponential class, a smaller λ represents an FOSD shift in F, meaning that the consumer is less able to discern utilities. We hence have the conclusion that the consumer's expected surplus is increasing in his *in*ability to discern utilities, echoing similar results in Piccione and Spiegler (2012) and de Clippel, Eliaz, and Rozen (2014). Intuitively, when the consumer has difficulty in discerning utilities more often, there are two opposite effects on the consumer's expected surplus. On the one hand, the consumer can recognize the good deal offered by seller 1 less often, which reduces his expected surplus. On the other hand, seller 1 is pressured to further raise its offered utility in order to convince the consumer that its deal is good, which increases his expected surplus. Depending on the distribution of the consumer's types, it is possible that the second effect dominates the first effect.

Proposition 10 *Suppose F belongs to the exponential class. In the case of two sellers marketing up to only one deal each, the consumer's pure-seller-strategy-equilibrium expected surplus increases with an FOSD shift in F.*[21]

A pure-seller-strategy equilibrium, however, may not exist. Indeed, one can prove that it does not exist if $f(0)\overline{\pi} < 1/3$. In order to make sure that Proposition 9 above is not a characterization of the empty set, we numerically demonstrate the existence of a pure-seller-strategy equilibrium given some value of $\overline{\pi}$, and some distribution F that satisfies Assumptions 1 and 2.

Specifically, let $\overline{\pi} = 1$, and F be the exponential distribution with parameter $\lambda = 0.8$. Plugging these into the equation in Proposition 9, one readily computes that $x^* = 0.25$. By Proposition 9, a pure-seller-strategy equilibrium, if exists, must feature $(u_1^*, u_2^*) = (x^*, -x^*) = (0.25, -0.25)$. We can plot seller 1's profit $\Pi_1(u_1, u_2)$ (respectively, seller 2's profit $\Pi_2(u_1, u_2)$) as a function of its offered utility u_1 (respectively, u_2), given the opponent's

[21] As explained in Footnote 10, we can easily handle distributions with finite supports (such as uniform distributions) at the expenses of messier proofs. In the special case where F belongs to the uniform class $U[0, D]$, the equation in Proposition 9 becomes

$$x^* + \overline{\pi} = \frac{1 - F(2x^*)}{f(2x^*)} = D - 2x^*,$$

which gives us $x^* = (D - \overline{\pi})/3$. Therefore, consumers' surplus is

$$x^* F(2x^*) = x^* \frac{2x^*}{D} = \frac{2(D - \overline{\pi})^2}{9D},$$

which is strictly increasing in D if $D > \overline{\pi}$, which in turned is guaranteed by Assumption 2 (see Footnote 12). Recall that, in the special case of the uniform class, a bigger D represents an FOSD shift in F. We hence once again arrive at the same conclusion that the consumer's expected surplus is increasing in his *in*ability to discern utilities.

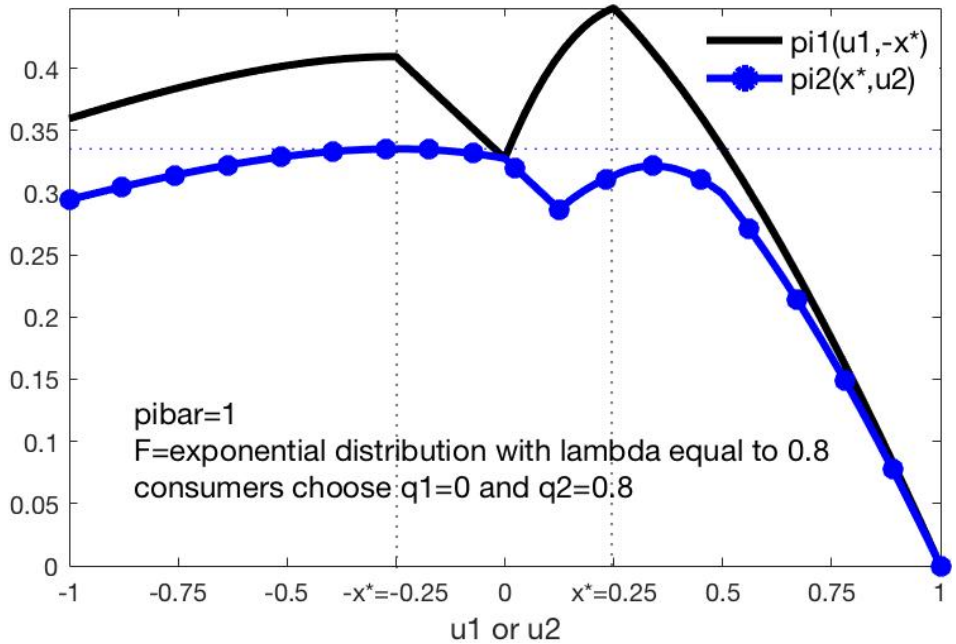

Figure 3: the solid line is seller 1's profit as a function of u_1, given opponent's offered utility $u_2 = -x^*$; the line with little circles is seller 2's profit as a function of u_2, given opponent's offered utility $u_1 = x^*$; both are drawn with $\overline{\pi} = 1$, F being the exponential distribution with parameter $\lambda = 0.8$ (which implies $x^* = 0.25$), and $(q_1, q_2) = (0, 0.8)$

offered utility $u_2 = -0.25$ (respectively, $u_1 = 0.25$), and given any (q_1, q_2). A pure-seller-strategy equilibrium exists iff there exists some (q_1, q_2) such that $u_1 = 0.25$ and $u_2 = -0.25$ are global optima of $\Pi_1(\cdot, -0.25)$ and $\Pi_2(0.25, \cdot)$, respectively. A familiar argument we once used in the proof of Proposition 6 suggests that, in our search for such (q_1, q_2), it is wlog to let $q_1 = 0$, because this does not affect either seller's profit at its candidate-equilibrium offered utility, while weakly lowers its profit at other offered utilities.

This leaves us only one variable to tune with. As we tune q_2, the shapes of $\Pi_1(\cdot, -0.25)$ and $\Pi_2(0.25, \cdot)$ change. In the neighborhood of $q_2 = 0.8$, $u_1 = 0.25$ and $u_2 = -0.25$ indeed become the global optima of $\Pi_1(\cdot, -0.25)$ and $\Pi_2(0.25, \cdot)$, respectively, as shown in Figure 3, verifying the existence of a pure-seller-strategy equilibrium for such $\overline{\pi}$ and F.

Figure 3 is robust to perturbation in q_2, meaning that $(u_1^*, u_2^*, q_1^*, q_2^*) = (0.25, -0.25, 0, q_2^*)$ remains a pure-seller-strategy equilibrium for an open set of q_2^* containing 0.8. Indeed, $\Pi_1(\cdot, -0.25)$ has a kink at $u_1 = 0.25$. As we perturb q_2, the left and right derivatives of $\Pi_1(\cdot, -0.25)$ at $u_1 = 0.25$ will be perturbed, and similarly for the values of $\Pi_1(\cdot, -0.25)$ at the two local optima, but $u_1 = 0.25$ will remain the unique global optimum. The case for

$\Pi_2(0.25, \cdot)$ is slightly different: its slope at $u_2 = -0.25$ remains flat regardless of the value of q_2. Indeed, this property is the geometric meaning of the equation in Proposition 9, which we used earlier to compute x^*. Therefore, as we perturb q_2, $u_2 = -0.25$ remains a local optimum. Since the value of $\Pi_2(0.25, \cdot)$ at $u_2 = -0.25$ is strictly higher than that at the other local optimum, $u_2 = -0.25$ will remain the unique global optimum upon perturbation of q_2.

It is also worth highlighting some interesting features of Figure 3 that are not mentioned in Proposition 9. First, the two sellers earn different profits, with the one offering a bad deal earning strictly less than the one offering a good deal. While seller 2 may envy seller 1, it cannot mimic the latter by also offering a good deal. If it were to do so, the best way to do it is to undercut seller 1, which is shown by the fact that the right local optimum of $\Pi_2(0.25, \cdot)$ is located on the right of 0.25. However, by doing so, seller 2 actually will earn even less than free-riding seller 1 and exploiting the trusting consumer.

Second, sellers' profits as functions of own offered utilities are not quasi-concave. This explains why it is difficult to provide interesting sufficient conditions for the existence of a pure-seller-strategy equilibrium beyond numerical examples such as the one depicted in Figure 3.

7 Conclusion

In this paper, we made a first step in exploring the implications of intransitive indifference in some classical economic analyses. Many of these implications are driven by the phenomenon that the presence of a bad deal can help a consumer appreciate a good deal—a phenomenon that is the signature of intransitive indifference.

To keep our first-pass exercise tractable, we have followed Armstrong and Vickers (2001) and treat as the same any two deals if they offer the same utility. One can imagine many situations in which this assumption would be too restrictive. Consider again our example of a consumer shopping for a mobile phone plan. Consider two different classes of phone plans. The first are all-you-can-eat plans with different monthly fees. The second are complicated plans with highly non-linear tariffs, augmented by myriads of "exceptions" and "conditions". While both classes can be rich enough to span the same range of offered utility, the consumer will find it easier to compare phone plans within the first class than within the second, holding fixed the quality of his previous-night's sleep. This suggests that different manners in which the same utility u is delivered can be associated with different distributions of d, a possibility that our current model assumes away.

Another possibility that our current model assumes away is that how densely the offered utilities from different deals are packed within a given range may also affect the consumer's d. This is related to the *frequency effect* identified in the marketing literature (see, for example, Huber, Payne, and Puto (1982)). The frequency effect, informally, refers to the phenomenon that a consumer will pay more attention when there is a larger number of deals that are densely packed within a given range along a given dimension (i.e., higher frequency begets higher attention). This higher attention in turn makes the consumer more sensitive to small differences along this dimension. All these different possibilities are interesting extensions of our current model, and we leave their exploration for future research.

Appendix A: Omitted Proofs in Section 3

The following lemma will be used in the proof of Proposition 1.

Lemma 1 *Consider the function* $\Pi(u) := (\overline{\pi} - u)[1 - F(-u)]$. *Then* $\Pi(u)$ *is quasi-concave and has a unique maximizer in the sub-range* $u \in (-\infty, 0]$.

Proof: At any $u \in (-\infty, 0)$, we have $\Pi(u) > 0$ and

$$\frac{d\Pi(u)}{du} = f(-u)\left[(\overline{\pi} - u) - \frac{1 - F(-u)}{f(-u)}\right].$$

By Assumption 1, the term in the square parentheses is strictly decreasing in u. This proves quasi-concavity of $\Pi(u)$ and the existence of a unique maximizer in the sub-range $u \in (-\infty, 0]$. ■

Proof of Proposition 1: By Lemma 1, for any $q > 0$, the function $\Pi(u; q)$ is quasi-concave and has a unique maximizer in the sub-range $u \in (-\infty, 0]$. Let $\underline{u} \leq 0$ denote this unique maximizer. Apparently $\underline{u}$ does not depend on q.[22] Moreover, we have

$$\underline{u} < 0 \quad \text{iff} \quad \left.\frac{\partial \Pi(u;q)}{\partial u}\right|_{u=0^-} < 0 \quad \text{iff} \quad f(0)\overline{\pi} < 1,$$

which is guaranteed by Assumption 1.

In the sub-range $u \in (0, \overline{\pi})$,

$$\begin{aligned}\frac{\partial \Pi\,(u;q)}{\partial u} &= -\left(q + F(u)(1-q)\right) + (\overline{\pi} - u) f(u)(1-q),\\ \frac{\partial^2 \Pi}{\partial u^2} &= -2f(u)(1-q) + (\overline{\pi} - u) f'(u)(1-q) < 0,\end{aligned}$$

where the last inequality follows from Assumption 1. Therefore, $\Pi(u; q)$ is strictly concave in u, and has a unique maximizer in the sub-range $u \in [0, \overline{\pi}]$. Let $\overline{u}(q) \geq 0$ denote this unique maximizer. Since

$$\frac{\partial^2 \Pi(u;q)}{\partial u \partial q} = -\left(1 - F(u)\right) - (\overline{\pi} - u) f(u) < 0,$$

[22]If $q = 0$, $\Pi(u; q) \equiv 0$ for any $u \leq 0$, and hence $\underline{u}$ remains a maximizer.

$\overline{u}(q)$ is decreasing in q (strictly so if $\overline{u} \in (0, \overline{\pi})$). It attains its lower bound 0 iff

$$\left.\frac{\partial \Pi(u;q)}{\partial u}\right|_{u=0^+} = -q + \overline{\pi} f(0)(1-q) \leq 0 \quad \text{iff} \quad f(0)\overline{\pi} \leq \frac{q}{1-q}.$$

Let $\overline{q}$ be the unique solution to $f(0)\overline{\pi} = \overline{q}/(1-\overline{q})$. Note that $0 < \overline{q} < 1/2$ by Assumption 2.

At $q = 0$, $\Pi(u;q) = 0$ for any $u \leq 0$, and hence $\Pi(\underline{u};q) = 0 < \Pi\left(\overline{u}\left(q\right);q\right)$. At any $q \in [\overline{q}, 1]$, we have $\overline{u}\left(q\right) = 0$, and hence $\Pi(\underline{u};q) > \Pi(0;q) = \Pi\left(\overline{u}\left(q\right);q\right)$. At any $q \in (0, \overline{q})$, $\Pi(\underline{u};q)$ is linear in q, while $\Pi\left(\overline{u}\left(q\right);q\right)$ is convex in q:

$$\frac{d\Pi\left(\overline{u}\left(q\right);q\right)}{dq} = \frac{\partial \Pi\left(\overline{u}\left(q\right);q\right)}{\partial q} = \left(\overline{\pi} - \overline{u}\left(q\right)\right)\left[1 - F\left(\overline{u}\left(q\right)\right)\right],$$

which is increasing in q. Therefore, $\Pi(\underline{u};\cdot)$ crosses $\Pi\left(\overline{u}(\cdot);\cdot\right)$ once and only once, and crosses from below. Let $q^* \in (0, \overline{q})$ denote the unique solution of $\Pi(\underline{u};q) = \Pi\left(\overline{u}\left(q\right);q\right)$.

Any equilibrium must have $q = q^*$. Indeed, if $q < q^*$ in equilibrium, $\overline{u}(q) > 0$ will be the unique maximizer of $\Pi(u;q)$, leading to $q = 1$ as the consumer's best response, a contradiction. Similarly, if $q > q^*$ in equilibrium, $\underline{u} < 0$ will be the unique maximizer of $\Pi(u;q)$, leading to $q = 0$ as the consumer's best response, a contradiction again. At $q = q^*$, the monopolist is indifferent between offering utility $\underline{u}$ and $\overline{u}(q^*)$. In equilibrium it must randomize between these two in a way that makes the consumer willing to randomize between purchasing and not purchasing when he cannot discern the offered utility and his reservation utility 0.

Let α be the probability that the monopolist offers the bad deal $\underline{u}$ in equilibrium. Conditional on the event that the consumer cannot discern the offered utility and his reservation utility 0, the conditional expected utility of the offered deal is

$$\frac{\alpha\left[1 - F\left(-\underline{u}\right)\right]\underline{u} + (1-\alpha)\left[1 - F\left(\overline{u}(q^*)\right)\right]\overline{u}(q^*)}{\alpha\left[1 - F\left(-\underline{u}\right)\right] + (1-\alpha)\left[1 - F\left(\overline{u}(q^*)\right)\right]}.$$

In order for the consumer to be indifferent between purchasing and not purchasing, this conditional expected utility must be the same as his reservation utility 0, or equivalently,

$$\alpha = \frac{\overline{u}(q^*)\left[1 - F\left(\overline{u}(q^*)\right)\right]}{\left(-\underline{u}\right)\left[1 - F\left(-\underline{u}\right)\right] + \overline{u}(q^*)\left[1 - F\left(\overline{u}(q^*)\right)\right]}.$$

The consumer obtains strictly positive surplus in expectation because he obtains his reservation utility 0 either when he feels that the offered utility is discernibly lower than 0, or when he cannot discern the two, yet with strictly positive probability (more precisely, with probability $(1-\alpha)F(\overline{u}(q^*)) > 0$) he obtains a strictly positive surplus of $\overline{u}(q^*) > 0$. ■

Proof of Proposition 2: Suppose $F^\dagger$ is a distribution that also satisfies Assumptions 1 and 2, and dominates F in the FOSD sense. Let's write the distribution explicitly as an argument of the profit function. Then $\Pi(u; q, F^\dagger) > \Pi(u; q, F)$ for any $q > 0$ and any $u < 0$ (this is because when the monopolist offers a utility lower than the consumer's reservation utility 0, it will fare better if the consumer is less able to discern these two utilities and hence cannot tell for sure that this is a bad deal), and hence $\Pi(\underline{u}^\dagger; q, F^\dagger) > \Pi(\underline{u}; q, F)$ for any $q > 0$, where $\underline{u}^\dagger$ is the unique maximizer of $\Pi(u; q, F^\dagger)$ in the sub-range $u \in (-\infty, 0]$.

On the other hand, $\Pi(u; q, F^\dagger) < \Pi(u; q, F)$ for any q and any $u > 0$ (this is because when the monopolist offers a utility higher than the consumer's reservation utility 0, it will fare worse if the consumer is less able to discern these two utilities and hence cannot appreciate this good deal), and hence $\Pi(\overline{u}^\dagger(q); q, F^\dagger) < \Pi(\overline{u}(q); q, F)$ for any q, where $\overline{u}^\dagger(q)$ is the unique maximizer of $\Pi(u; q, F^\dagger)$ in the sub-range $u \in [0, \overline{\pi}]$.

Recall that $\Pi\left(\underline{u}; \cdot, F\right)$ and $\Pi\left(\overline{u}(\cdot); \cdot, F\right)$ are both increasing functions of q, with the former crossing the latter once and only once and from below at some $q^* \in (0, 1/2)$. Similarly $\Pi\left(\underline{u}; \cdot, F^\dagger\right)$ and $\Pi\left(\overline{u}(\cdot); \cdot, F^\dagger\right)$ are both increasing functions of q, with the former crossing the latter once and only once and from below at some $q^\dagger \in (0, 1/2)$. The facts that $\Pi\left(\underline{u}; \cdot, F\right)$ lies pointwise below $\Pi\left(\underline{u}; \cdot, F^\dagger\right)$ and that $\Pi\left(\overline{u}(\cdot); \cdot, F\right)$ lies pointwise above $\Pi\left(\overline{u}(\cdot); \cdot, F^\dagger\right)$ hence imply $q^* \geq q^\dagger$. The monopolist's equilibrium profit under distribution $F^\dagger$ is hence

$$
\begin{aligned}
\Pi\left(\overline{u}^\dagger(q^\dagger); q^\dagger, F^\dagger\right) &\leq \Pi\left(\overline{u}^\dagger(q^\dagger); q^\dagger, F\right) \\
&\leq \Pi\left(\overline{u}^\dagger(q^\dagger); q^*, F\right) \\
&\leq \Pi\left(\overline{u}(q^*); q^*, F\right),
\end{aligned}
$$

where the first inequality follows from the fact that the monopolist benefits from the higher probability that the consumer can appreciate its good deal, the second inequality from the fact that $\Pi(u; q, F)$ is strictly increasing in q for any $u < \overline{\pi}$, and the third inequality from the optimality of $\overline{u}(q^*)$ given q^* and F. This proves that the monopolist's equilibrium profit decreases with an FOSD shift in F. ■

Proof of Proposition 3: Start with any arbitrary distribution F that satisfies Assumptions 1 and 2. By Proposition 1 the consumer's expected surplus is strictly positive. Construct an *increasing* sequence of distributions $\{F_n\}_{n\geq 0}$ satisfying Assumptions 1 and 2 such that $F_0 = F$ and $f_n(0) \nearrow 1/\overline{\pi}$. Note that, along this sequence, we have F_n dominates F_{n+1} in the FOSD sense for all n. We shall prove that, for n sufficiently large, the consumer's expected surplus in an economy featuring F_n is lower than that in an economy featuring F. To this end, it suffices to prove that the consumer's expected surplus converges to 0 as $n \to \infty$.

Let $\overline{u}_n^*$, $\underline{u}_n^*$, and q_n^* be the corresponding equilibrium variables in an economy featuring F_n. Recall that $\underline{u}_n^*$ maximizes $\Pi_n(u; q_n^*) = (\overline{\pi}-u)[1-F_n(-u)]q_n^*$ over the sub-range $u \in (-\infty, 0]$, and that $q_n^* > 0$. Therefore, $\underline{u}_n^*$ solves the following first-order condition:

$$\overline{\pi} - \underline{u}_n^* = \frac{1 - F_n\left(-\underline{u}_n^*\right)}{f_n\left(-\underline{u}_n^*\right)} \leq \frac{1 - F_n(0)}{f_n(0)} = \frac{1}{f_n(0)} \searrow \overline{\pi},$$

where the inequality follows from Assumption 1. We hence have $\underline{u}_n^* \nearrow 0$, which implies

$$\lim_{n\to\infty} \Pi_n\left(\underline{u}_n^*; q\right) = q\overline{\pi}.$$

We next prove that $q_n^* \nearrow 1/2$. Recall from the proof of Proposition 1 that $\Pi_n\left(\underline{u}_n^*; \cdot\right)$ crosses $\Pi_n(\overline{u}_n(\cdot); \cdot)$ from below at $q_n^* < 1/2$. Therefore, it suffices to prove that, for any $q < 1/2$, $\Pi_n\left(\underline{u}_n^*; q\right) < \Pi_n(\overline{u}_n(q); q)$ for n sufficiently large.

Recall from the proof of Proposition 1 that $\overline{u}_n(q) > 0$ for all $q < \overline{q}_n$, where $\overline{q}_n$ is the unique solution of $f_n(0)\overline{\pi} = \overline{q}_n/\left(1 - \overline{q}_n\right)$. Apparently $\overline{q}_n \nearrow 1/2$. Therefore, for any $q < 1/2$, we have $\overline{u}_n(q) > 0$ for n sufficiently large. Moreover, for such q and n, we have

$$\Pi_n(\overline{u}_n(q); q) > \Pi_n(0; q) = q\overline{\pi} = \lim_{m\to\infty} \Pi_m(\underline{u}_m^*; q),$$

where the inequality follows from the fact that $\overline{u}_n(q)$ is the unique maximizer of $\Pi_n(\cdot; q)$ in the sub-range $u \in [0, \overline{\pi}]$. For any $m > n$, $\Pi_m(\overline{u}_m(q); q) \geq \Pi_n(\overline{u}_n(q); q)$ (recall the proof of

Proposition 2). Therefore, we have

$$\lim_{m\to\infty} \Pi_m(\overline{u}_m(q); q) \geq \Pi_n(\overline{u}_n(q); q) > \lim_{m\to\infty} \Pi_m(\underline{u}_m^*; q),$$

and hence we have $\Pi_m(\overline{u}_m(q); q) > \Pi_m\left(\underline{u}_m^*; q\right)$ for m sufficiently large. Since $q < 1/2$ is arbitrary, we hence have $q_n^* \nearrow 1/2$ as claimed.

Finally, we prove that $\overline{u}_n^* \searrow 0$. Since $\overline{u}_n^* = \overline{u}_n(q_n^*)$, it maximizes $\Pi_n(u; q_n^*) = (\overline{\pi} - u)(F_n(u) + [1 - F_n(u)]q_n^*)$ over the sub-range $u \in [0, \overline{\pi}]$, and hence solves the following first-order condition:

$$\overline{\pi} - \overline{u}_n^* = \frac{F_n(\overline{u}_n^*)}{f_n(\overline{u}_n^*)} + \frac{q_n^*}{(1 - q_n^*) f_n(\overline{u}_n^*)} > \frac{q_n^*}{(1 - q_n^*) f_n(\overline{u}_n^*)} \geq \frac{q_n^*}{(1 - q_n^*) f_n(0)}.$$

Taking limit on both sides, we have

$$\lim_{n\to\infty} \overline{\pi} - \overline{u}_n^* \geq \lim_{n\to\infty} \frac{q_n^*}{(1 - q_n^*) f_n(0)} = \overline{\pi},$$

and hence $\overline{u}_n^* \searrow 0$ as claimed.

That the consumer's expected surplus converges to 0 as $n \to \infty$ now follows from $\overline{u}_n^* \searrow 0$.

Similarly, we can construct a *decreasing* sequence of distributions $\{F_n\}_{n\geq 0}$ satisfying Assumptions 1 and 2 such that $F_0 = F$ and $F_n(\overline{\pi}) \searrow 0$. Note that, along this sequence, we have F_n dominates F_{n-1} in the FOSD sense for all n. As explained in the main text, the consumer's expected surplus converges to 0 as $n \to \infty$, and hence the consumer's expected surplus in an economy featuring F_n is lower than that in an economy featuring F for n large enough. ■

Appendix B: Omitted Proofs in Section 4

We first prove three lemmas that will be used in the proofs of both Proposition 4 and Proposition 6. Define

$$\mathbf{U} := \{(u_1, u_2) \,|\, u_1 \geq 0, u_2 = -u_1\}.$$

Lemma 2 *Consider the case of a monopolist marketing up to two different deals. Suppose the consumer's strategy (q_1, q_2) is such that $q_1 = 0$. Then, for every (u_1, u_2) such that $u_1 > 0$ and $u_1 \geq u_2 > -u_1$, there exists $(u_1', u_2') \in \mathbf{U}$ such that the monopolist makes strictly higher profit (i.e., $\Pi(u_1, u_2) < \Pi(u_1', u_2')$).*

Proof: Fix any $u_1 > 0$. If the monopolist is to offer $u_2 \in (-u_1, 0]$, its profit will be

$$\begin{aligned}
\Pi\,(u_1, u_2) &= (\overline{\pi} - u_1)\,F\,(u_1 - u_2) + q_2\left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)[1 - F\,(u_1 - u_2)] \\
&< \left(\overline{\pi} - \frac{u_1 - u_2}{2}\right)F(u_1 - u_2) + q_2\,(\overline{\pi} - 0)\,[1 - F(u_1 - u_2)] \\
&= \Pi\left(\frac{u_1 - u_2}{2}, -\frac{u_1 - u_2}{2}\right),
\end{aligned}$$

where the strict inequality follows from $u_1 > (u_1 - u_2)/2 > 0$.

If the monopolist is to offer $u_2 \in (0, u_1/2]$, its profit will be

$$\Pi(u_1, u_2) = (\overline{\pi} - u_1)\,F\,(u_1) + q_2\left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)[1 - F\,(u_1)]\,,$$

which is weakly decreasing in u_2 (strictly so if $q_2 > 0$), and hence according to the last paragraph is also strictly worse than $(u_1', u_2') = (u_1/2, -u_1/2) \in \mathbf{U}$.

If the monopolist is to offer $u_2 \in (u_1/2, u_1]$, its profit will be

$$\begin{aligned}
\Pi(u_1, u_2) = {} & (\overline{\pi} - u_1)\,F(u_1 - u_2) \\
& + \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)[F(u_2) - F(u_1 - u_2)] \\
& + (\overline{\pi} - u_1)\,[F(u_1) - F(u_2)] \\
& + q_2\left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)[1 - F(u_1)] \\
\leq {} & \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)(F(u_1) + q_2\,[1 - F(u_1)])\,.
\end{aligned}$$

However, if the monopolist is to offer $\left(u_1', u_2'\right) = (u_2, -u_2) \in \mathbf{U}$, its profit will be

$$
\begin{aligned}
\Pi(u_1', u_2') &= \Pi(u_2, -u_2) \\
&= (\overline{\pi} - u_2)F(2u_2) + q_2\overline{\pi}[1 - F(2u_2)] \\
&> \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)F(2u_2) + q_2\left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)[1 - F(2u_2)] \\
&> \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)(F(u_1) + q_2\,[1 - F(u_1)]) \\
&\geq \Pi(u_1, u_2),
\end{aligned}
$$

where the second inequality follows from $2u_2 > u_1$. ■

Lemma 3 *Consider the case of a monopolist marketing up to two different deals. Suppose the consumer's strategy (q_1, q_2) is such that $q_1 = 0$. Suppose, furthermore, either $q_2 = 0$ or $f(0)\overline{\pi} \geq 1/2$. Then, for every (u_1, u_2) such that $u_1 > 0$ and $u_2 < -u_1$, there exists $(u_1', u_2') \in \mathbf{U}$ such that the monopolist makes weakly higher profit (i.e., $\Pi(u_1, u_2) \leq \Pi(u_1', u_2')$).*

Proof: Fix any $u_1 > 0$. If the monopolist is to offer $u_2 < -u_1$, its profit will be

$$
\begin{aligned}
\Pi(u_1, u_2) = (\overline{\pi} - u_1)F(u_1) \\
&+ (\overline{\pi} - u_1)\,[F(u_1 - u_2) - F(-u_2)] \\
&+ q_2\left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)[1 - F(u_1 - u_2)]\,.
\end{aligned}
$$

Partial-differentiating $\Pi(u_1, u_2)$ wrt u_2, we have

$$
\begin{aligned}
\frac{\partial \Pi(u_1, u_2)}{\partial u_2} &= (\overline{\pi} - u_1)\,[-f(u_1 - u_2) + f(-u_2)] \\
&\quad + q_2\left[-\frac{1 - F(u_1 - u_2)}{2} + \left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)f(u_1 - u_2)\right] \\
&\geq \frac{q_2 f\,(u_1 - u_2)}{2}\left[-\frac{1 - F(u_1 - u_2)}{f\,(u_1 - u_2)} + 2\left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)\right] \\
&\geq \frac{q_2 f\,(u_1 - u_2)}{2}\left[-\frac{1}{f(0)} + 2\overline{\pi}\right] \\
&\geq 0,
\end{aligned}
$$

where the first and the second inequalities follow from Assumption 1, and the third inequality follows from the supposition that either $q_2 = 0$ or $f(0)\overline{\pi} \geq 1/2$. Therefore, (u_1, u_2) is weakly worse than $(u_1', u_2') = (u_1, -u_1) \in \mathbf{U}$. ∎

Lemma 4 *Consider the case of a monopolist marketing up to two different deals. Consider the following constrained maximization problem: the monopolist is to maximize its profit by seting (u_1, u_2), subject to the constraints that $(u_1, u_2) \in \mathbf{U}$, and given the consumer's strategy (q_1, q_2). The monopolist's problem has a unique solution that depends only on q_2 but not on q_1. There exists a strictly decreasing function $u_1(\cdot)$, with $u_1(0) < \overline{\pi}$ and $u_1(1) = 0$, such that, for any $q_2 \in [0, 1]$, the monopolist's unique solution is $u_1 = u_1(q_2)$ and $u_2 = -u_1(q_2)$.*

PROOF: Fix any (q_1, q_2). For any $(u_1, u_2) \in \mathbf{U}$, the monopolist's profit is

$$\Pi(u_1, -u_1) = (\overline{\pi} - u_1)F(2u_1) + q_2\overline{\pi}[1 - F(2u_1)],$$

which apparently depends only on q_2 but not on q_1.

Total-differentiating $\Pi(u_1, -u_1)$ wrt u_1, we have

$$\begin{aligned}\frac{d\Pi(u_1, -u_1)}{du_1} &= -F(2u_1) + 2(\overline{\pi} - u_1)f(2u_1) - 2q_2\overline{\pi}f(2u_1) \\ &= f(2u_1)\left[-\frac{F(2u_1)}{f(2u_1)} + 2(\overline{\pi} - u_1) - 2q_2\overline{\pi}\right]. \end{aligned} \tag{1}$$

By Assumption 1, F/f is weakly increasing, and hence $-F(2u_1)/f(2u_1)$ is weakly decreasing in u_1. Therefore, the term inside the square brackets is strictly decreasing in u_1. This shows that $\Pi(u_1, -u_1)$ is quasi-concave in u_1, and hence admits a unique maximizer, denoted by $u_1(q_2)$.

Since the term inside the square brackets is strictly positive (unless $q_2 = 1$) at $u_1 = 0$ and strictly negative at $u_1 = \overline{\pi}$, we have $u_1(q_2) \in (0, \overline{\pi})$ for all $q_2 \in [0, 1)$. As for $q_2 = 1$, the term inside the square brackets is strictly negative at any $u_1 > 0$ and is 0 at $u_1 = 0$. Therefore, we have $u_1(1) = 0$.

Finally, since an increase in q_2 strictly decreases the term inside the square brackets, $u_1(\cdot)$ is strictly decreasing in q_2. ∎

Proof of Proposition 4: If the monopolist always offers utilities that are mirror images of each other around 0, then $q_2 = 0$ is apparently a best response of the consumer contingent on the all-tied case. Moreover, the single-contender case will never arise, and hence we are free to specify the consumer's (off-equilibrium) belief contingent on such an event. In particular, one possible (off-equilibrium) belief contingent on the single-contender case is that u_1 is strictly negative, while at the same time the consumer's type d is strictly larger than $-u_1$. Against such a belief, $q_1 = 0$ is the consumer's best response contingent on the single-contender case.

On the other hand, if $q_1 = q_2 = 0$, the consumer will never make a purchase unless he finds at least one deal offering strictly positive utility. The monopolist will then make 0 sales if it offers $u_1 \leq 0$ (recall that $u_2 \leq u_1$ by our convention). Therefore, the monopolist's optimal strategy must have $u_1 > 0$. By Lemmas 2 and 3, for every (u_1, u_2) such that $u_1 > 0$ and $u_1 \geq u_2 \neq -u_1$, there exists $(u_1', u_2') \in \mathbf{U}$ such that the monopolist makes weakly higher profit. Therefore, solutions of the constrained maximization problem described in Lemma 4 are also the monopolist's *un*constrained best responses. By Lemma 4, the constrained maximization problem admits a unique solution, namely $u_1^* = u_1(0)$ and $u_2^* = -u_1(0)$, where $u_1(0) \in (0, \overline{\pi})$. ■

We prove Proposition 6 through a series of lemmas.

Lemma 5 *In the case of a monopolist marketing up to two different deals, in any pure-seller-strategy equilibrium, $(u_1^* + u_2^*)/2 \leq 0$.*

Proof: Suppose not. Then the monopolist's equilibrium profit is bounded from above by $\overline{\pi} - (u_1^* + u_2^*)/2 < \overline{\pi}$, because if the consumer purchases deal 2 with positive probability he must also purchase deal 1 with the same probability.

Note that in equilibrium the consumer knows the monopolist's strategy. Therefore, $q_2^* = 1$, as the consumer's unique best response is to purchase either deal randomly when he finds himself in the all-tied case. Given $q_2^* = 1$, however, the monopolist can profit from deviating to $u_1 = u_2 = 0$, because its profit will increase to $\overline{\pi}$, a contradiction. ■

Lemma 6 *In the case of a monopolist marketing up to two different deals, in any pure-seller-strategy equilibrium, if $(u_1^* + u_2^*)/2 < 0$, then $u_1^* = 0$, $u_2^* = -\infty$, and $q_1^* > 0$.*

Proof: Suppose there is a pure-seller-strategy equilibrium where $(u_1^* + u_2^*)/2 < 0$. Then $q_2^* = 0$ (when the consumer finds himself in the all-tied case, he rationally refrains from purchasing any deal). Apparently $u_1^* \geq 0$, otherwise we would have had $q_1^* = 0$ as well, and the monopolist's equilibrium profit would have been 0, and would have profited strictly from deviating to, say, $u_1 = u_2 = \overline{\pi}/2$.

We claim that, if $q_1^* = 1$, then $u_2^* = -\infty$. The presumption of $(u_1^* + u_2^*)/2 < 0$ implies $u_2^* < -u_1^*$. For any $u_2 < -u_1^*$, given $q_1^* = 1$ and $q_2^* = 0$, the monopolist's profit is

$$\Pi(u_1^*, u_2) = (\overline{\pi} - u_1^*)F(u_1^* - u_2),$$

which is maximized by setting $u_2 = -\infty$.

Suppose $u_1^* > 0$. Then $q_1^* = 1$ indeed (when the consumer finds himself in the single-contender case, he would know for sure that the only contender is a good deal). Then, according to the claim in the above paragraph, we have $u_2^* = -\infty$. However, by setting $u_1 > 0$ and $u_2 = -\infty$, given $q_1^* = 1$, the monopolist's profit is $\overline{\pi} - u_1$ (the consumer will purchase deal 1 for sure regardless whether he can discern u_1 and 0), which is strictly decreasing in u_1. Hence the only possible candidate for u_1^* is 0.

It remains to prove that $u_2^* = -\infty$ given that $u_1^* = 0$. Note that $q_1^* = 0$ is not an equilibrium, otherwise the monopolist"s equilibrium profit would have been 0, and would have profited strictly from deviating to, say, $u_1 = u_2 = \overline{\pi}/2$. For any $q_1^* > 0$, the monopolist's profit is

$$\Pi(u_1^*, u_2) = \Pi(0, u_2) = q_1^* \overline{\pi} F(-u_2),$$

which is maximized by setting $u_2 = -\infty$. ■

Corollary 1 *In the case of a monopolist marketing up to two different deals, in any pure-seller-strategy equilibrium,* $(u_1^* + u_2^*)/2 = 0$.

Proof: By Lemma 5, it suffices to show that $(u_1^* + u_2^*)/2 < 0$ is impossible. By Lemma 6, if $(u_1^* + u_2^*)/2 < 0$, then $u_1^* = 0$, $u_2^* = -\infty$, and $q_1^* > 0$. Given $u_2^* = -\infty$ and $q_1^* > 0$, the monopolist's profit from setting any $u_1 \leq 0$ is

$$\Pi(u_1, u_2^*) = \Pi(u_1, -\infty) = q_1^*(\overline{\pi} - u_1)\left[1 - F(-u_1)\right].$$

By the same argument as in the proof of Proposition 1, there exists some $\underline{u} < 0$ such that $\Pi\left(\underline{u}, -\infty\right) > \Pi(0, -\infty) = \Pi(u_1^*, u_2^*)$, a contradiction. ■

Lemma 7 *Consider the case of a monopolist marketing up to two different deals. Consider the following constrained maximization problem: the monopolist is to maximize its profit by offering* (u_1, u_2)*, subject to* $u_2 \leq u_1 \leq 0$*, and given the consumer's strategy* (q_1, q_2) *where* $q_1 = 0$*. A solution of the monopolist's problem is* $(u_1, u_2) = \left(\underline{u}, \underline{u}\right)$*, where* $\underline{u} < 0$ *is as defined in Proposition 1.*

Proof: For any (u_1, u_2) such that $u_2 \leq u_1 \leq 0$, the monopolist's profit is

$$\begin{aligned}
\Pi(u_1, u_2) &= q_2\left(\overline{\pi} - \frac{u_1 + u_2}{2}\right)[1 - F(-u_2)] \\
&\leq q_2(\overline{\pi} - u_2)[1 - F(-u_2)] \\
&\leq q_2\left(\overline{\pi} - \underline{u}\right)\left[1 - F\left(-\underline{u}\right)\right] \\
&= \Pi\left(\underline{u}, \underline{u}\right),
\end{aligned}$$

where the second inequality follows from the definition of $\underline{u}$ in the proof of Proposition 1. ■

Proof of Proposition 6: By Corollary 1, all pure-seller-strategy equilibria resemble the one described in Proposition 4, in the sense that $(u_1^* + u_2^*)/2 = 0$. Therefore, if (q_1^*, q_2^*) is the consumers' equilibrium strategy, the monopolist's equilibrium strategy must also solve the constrained maximization problem described in Lemma 4. That is, any pure-seller-strategy equilibrium must take the form of $(u_1^*, u_2^*, q_1^*, q_2^*) = \left(u_1(q_2^*), -u_1(q_2^*), q_1^*, q_2^*\right)$.

Apparently, if the monopolist is to set (u_1, u_2) such that $(u_1 + u_2)/2 = 0$, any (q_1, q_2) would be the consumer's best response (see the proof of Proposition 4). Therefore, a candidate equilibrium $(u_1^*, u_2^*, q_1^*, q_2^*) = \left(u_1(q_2^*), -u_1(q_2^*), q_1^*, q_2^*\right)$ is a valid equilibrium as long as (u_1^*, u_2^*) is also the monopolist's *un*constrained optimal choice given (q_1^*, q_2^*).

Note that, if $\left(u_1(q_2^*), -u_1(q_2^*), q_1^*, q_2^*\right)$ is a pure-seller-strategy equilibrium, then $\left(u_1(q_2^*), -u_1(q_2^*), 0, q_2^*\right)$ will also be a pure-seller-strategy equilibrium. This is because (*i*) the single-contender case is an off-equilibrium event, and any q_1 can be supported by some pessimistic enough off-equilibrium beliefs (see the proof of Proposition 4), and (*ii*) lowering q_1 to 0 does not affect the monopolist's equilibrium profit, but weakly lowers its profit if it were to deviate. In what follows we shall hence focus on equilibria of the form

$(u_1^*, u_2^*, q_1^*, q_2^*) = \left(u_1(q_2^*), -u_1(q_2^*), 0, q_2^*\right)$.

Let $\mathbf{Q_2}$ denote the compact[23] set of q_2's such that $(u_1(q_2), -u_1(q_2), 0, q_2)$ is a pure-seller-strategy equilibrium. From Proposition 4 we know that $\mathbf{Q_2}$ is not empty, and contains the point 0.

For any $q_2, q_2' \in \mathbf{Q_2}$ such that $q_2 < q_2'$, the monopolist's equilibrium profits are

$$\begin{aligned}\Pi^*(q_2) &= (\overline{\pi} - u_1(q_2))\, F\left(2u_1(q_2)\right) + q_2\overline{\pi}\left[1 - F\left(2u_1(q_2)\right)\right] \\ &< (\overline{\pi} - u_1(q_2))\, F\left(2u_1(q_2)\right) + q_2'\overline{\pi}\left[1 - F\left(2u_1(q_2)\right)\right] \\ &\leq (\overline{\pi} - u_1(q_2'))\, F\left(2u_1(q_2')\right) + q_2'\overline{\pi}\left[1 - F\left(2u_1(q_2')\right)\right] \\ &= \Pi^*(q_2'),\end{aligned}$$

where the second inequality follows from the definition of $u_1(\cdot)$. Therefore, different equilibria are rankable in terms of the monopolist's equilibrium profit, with higher q_2^* implying higher profit.

On the other hand, in any pure-seller-strategy equilibrium, the consumer gets a positive surplus only when he has a type $d < 2u_1(q_2^*)$, which enables him to compare $u_1(q_2^*)$ and 0 and hence identify deal 1 out of the two deals. His expected surplus is hence

$$\overline{CS} = u_1(q_2^*)F\left(2u_1(q_2^*)\right), \tag{2}$$

which is strictly decreasing in q_2^*. Therefore, different equilibria are also rankable in terms of the consumer's expected surplus, with higher q_2^* implying lower expected surplus for the consumer.

Let $q_2^{max} := \sup \mathbf{Q_2}$. We shall now prove that $q_2^{max} < 1$.

Recall from the proof of Proposition 1 that $\underline{u}$ is the unique solution of $\max_{u \leq 0}(\overline{\pi} - u)[1 - F(-u)]$. Since $\underline{u} < 0$, we have

$$\left(\overline{\pi} - \underline{u}\right)\left[1 - F\left(-\underline{u}\right)\right] > \overline{\pi}.$$

Suppose $q_2^{max} = 1$, then by compactness of $\mathbf{Q_2}$ there is a pure-seller-strategy equilibrium featuring $q_2^* = 1$. In such an equilibrium, by Lemma 4, we must have $u_1^* = u_2^* = 0$. The monopolist's equilibrium profit is hence $\overline{\pi}$.

[23]Compactness follows from the usual continuity argument.

If the monopolist deviates to $u_1 = u_2 = \underline{u}$, its profit will increase to $\left(\overline{\pi} - \underline{u}\right)\left[1 - F\left(-\underline{u}\right)\right]$ thanks to $q_2^* = 1$, a contradiction.

In the remainder of this proof, suppose $f(0)\overline{\pi} \geq 1/2$. Then, by Lemmas 2, 3, 4, and 7, when the consumer's strategy (q_1, q_2) is such that $q_1 = 0$, the monopolist's profit is maximized either at $(u_1, u_2) = (u_1(q_2), -u_1(q_2))$, or at $(u_1, u_2) = \left(\underline{u}, \underline{u}\right)$.

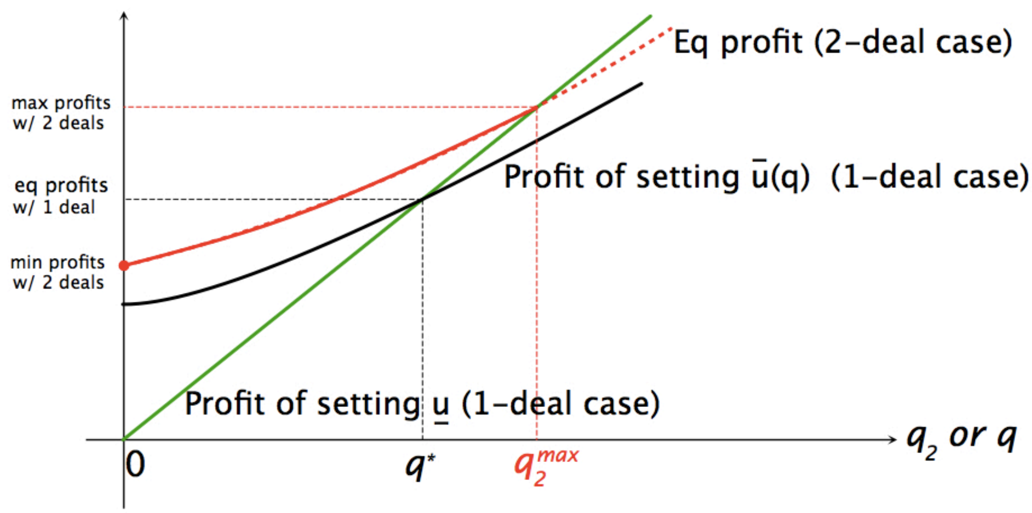

Figure 4: equilibrium profits in the one-deal and two-deal cases

In Figure 4, we plot the monopolist's profits at each of these two candidate maximizers as functions of q_2. The profit at $(u_1, u_2) = \left(\underline{u}, \underline{u}\right)$ is depicted by the straight line passing through the origin. Incidentally, it is the same straight line in Figure 1, with q_2 replacing q in the expression of $\Pi = q\left(\overline{\pi} - \underline{u}\right)\left[1 - F\left(-\underline{u}\right)\right]$.

The profit at $(u_1, u_2) = (u_1(q_2), -u_1(q_2))$ is depicted by the upper convex curve, and has the expression of $\Pi = (\overline{\pi} - u_1(q_2))\, F\,(2u_1(q_2)) + q_2\overline{\pi}\,[1 - F\,(2u_1(q_2))]$. That it is strictly increasing and convex can be seen by totally differentiating it wrt q_2 using the Envelope Theorem, yielding

$$\frac{d\Pi}{dq_2} = \overline{\pi}\,[1 - F\,(2u_1(q_2))]\,,$$

which is strictly positive and strictly increasing in q_2.

The convex curve is strictly above the straight line at $q_2 = 0$, and is strictly below at $q_2 = 1$ (recall that $u_1(1) = 0$). The shapes of the two profit functions dictate that the convex curve crosses the straight line once and only once, and crosses from above, at some

$q_2^{max} \in (0,1)$. Therefore, the candidate equilibrium $(u_1^*, u_2^*, q_1^*, q_2^*) = (u_1(q_2), -u_1(q_2), 0, q_2)$ is a valid equilibrium if and only if $q_2 \in \left[0, q_2^{max}\right]$.

We also superimpose onto Figure 4 the monopolist's profit if it offers the good deal $\overline{u}(q)$ in the one-deal case . It is depicted by the lower convex curve. Indeed, it is strictly below the upper convex curve at every $q < 1$, because it has the expression of

$$\begin{aligned}
\Pi &= (\overline{\pi} - \overline{u}(q))\left(F\left(\overline{u}(q)\right) + q\left[1 - F\left(\overline{u}(q)\right)\right]\right) \\
&\leq (\overline{\pi} - \overline{u}(q))\left(F\left(2\overline{u}(q)\right) + q\left[1 - F\left(2\overline{u}(q)\right)\right]\right) \\
&\leq (\overline{\pi} - \overline{u}(q))\,F\left(2\overline{u}(q)\right) + q\overline{\pi}\left[1 - F\left(2\overline{u}(q)\right)\right] \\
&\leq (\overline{\pi} - u_1(q))\,F\left(2u_1(q)\right) + q\overline{\pi}\left[1 - F\left(2u_1(q)\right)\right]
\end{aligned}$$

where the first two inequalities are strict if $\overline{u}(q) > 0$, and the third inequality is strict if $\overline{u}(q) = 0 \neq u_1(q)$, and hence at least one of these inequalities is strict for every $q < 1$. Note that the last expression is exactly the same as the monopolist's profit at $(u_1, u_2) = (u_1(q_2), -u_1(q_2))$ if we replace q with q_2, which proves that the upper and the lower convex curves touch only at $q = 1$.

It becomes apparent from Figure 4 that $q^* < q_2^{max}$, and that in the pure-seller-strategy equilibrium featuring $q_2^* = q_2^{max}$, the monopolist's profit is higher than its (unique) equilibrium profit in the one-deal case. ■

Appendix C: Omitted Proofs in Section 6

We prove Proposition 9 through a series of lemmas.

Lemma 8 *In the case of two sellers marketing up to only one deal each, in any pure-seller-strategy equilibrium, if* $u_1^* = u_2^* = u^*$*, then* $u^* > 0$.

Proof: Suppose $u^* < 0$. Then we must have $q_2^* = 0$ (when the consumer finds himself in the all-tied case, he would rationally refuse to purchase either brand), resulting in 0 equilibrium profit for both sellers. Seller 1, for example, can profit from deviating to $u_1 = \overline{\pi}/2 > 0$. After such deviation, seller 1 can make sales with probability at least $F(\overline{\pi}/2)$, a contradiction.

Suppose $u^* = 0$. Then $u_1 = 0$ must be a best response against $u_2^* = 0$. When $u_2^* = 0$, seller 1's profit as a function of u_1 is

$$\Pi_1(u_1, 0) = \begin{cases} (\overline{\pi} - u_1)[1 - F(-u_1)]q_2/2 & \text{if } u_1 \leq 0 \\ (\overline{\pi} - u_1)\left(F(u_1) + [1 - F(u_1)]q_2/2\right) & \text{if } u_1 \geq 0 \end{cases}.$$

which is exactly the same profit function as that in the one-deal case (with q replaced by $q_2/2$, and u replaced by u_1). However, from the proof of Proposition 1, we know that $u_1 = 0$ is never a best response regardless of $q_2/2$, a contradiction. ■

Lemma 9 *In the case of two sellers marketing up to only one deal each, in any pure-seller-strategy equilibrium, we have* $u_2^* < u_1^* < \overline{\pi}$.

Proof: By Lemma 8, it suffices to prove that there is no pure-seller-strategy equilibrium with $u_1^* = u_2^* = u^* > 0$. Suppose, on the contrary, such an equilibrium exists. Then we must have $q_2^* = 1$ (when the consumer finds himself in the all-tied case, he can guarantee a strictly positive surplus of u^* by purchasing randomly from one of the two sellers). Note that, since the single-contender case is an off-equilibrium event, any q_1^* can be supported by some off-equilibrium belief. It is wlog to set $q_1^* = 0$, because that leaves the sellers' equilibrium profits intact, while making their deviation profits weakly lower.

Given $(q_1^*, q_2^*) = (0, 1)$ and $u_1^* = u_2^* = u^*$, seller 1's equilibrium profit is

$$\Pi_1^* = (\overline{\pi} - u^*)/2.$$

If seller 1 deviates to $u_1 = 0$, its profit will become

$$\Pi_1 = \overline{\pi}[1 - F(u^*)]/2.$$

We shall show that $\Pi_1 > \Pi_1^*$, which will be a contradiction. Let $H(u^*) := 2(\Pi_1 - \Pi_1^*)$, and differentiate it wrt u^*, we have

$$H'(u^*) = 1 - \overline{\pi} f(u^*) \geq 1 - \overline{\pi} f(0) > 0,$$

where the first and the second inequalities follows from Assumptions 1 and 2, respectively. Therefore, $H(u^*) > H(0) = 0$ for any $u^* > 0$, and hence $\Pi_1 > \Pi_1^*$ as claimed.

If $u_1^* \geq \overline{\pi}$, seller 1 makes non-positive profit, and can strictly profit from deviating to $u_1 = (\overline{\pi} + u_2^*)/2$, a contradiction. ■

Lemma 10 *In the case of two sellers marketing up to only one deal each, in any pure-seller-strategy equilibrium, we have* $(u_1^* + u_2^*)/2 \geq 0$.

Proof: By Lemma 9, we have $u_2^* < u_1^* < \overline{\pi}$. Suppose $(u_1^* + u_2^*)/2 < 0$. Then $u_2^* < 0$, and hence the only chance that seller 2 can make any sales is when the consumer finds himself in the all-tied case. However, $(u_1^* + u_2^*)/2 < 0$ also implies that when the consumer finds himself in the all-tied case, he would rationally refuse to purchase; i.e., $q_2^* = 0$. Therefore, seller 2 makes 0 equilibrium profit, and can strictly profit from deviating to $u_2 = (u_1^* + \overline{\pi})/2$, a contradiction. ■

Lemma 11 *In the case of two sellers marketing up to only one deal each, in any pure-seller-strategy equilibrium, we have* $u_2^* < 0$.

Proof: By Lemma 9, we have $u_2^* < u_1^* < \overline{\pi}$. Suppose $u_2^* \geq 0$. Then $(u_1^* + u_2^*)/2 > 0$, and hence when the consumer finds himself in the all-tied case, he would rationally purchase for sure; i.e., $q_2^* = 1$.

Divide the consumer's type space into two (disjoint and exhaustive) subsets. The first subset are types who purchase from seller 1 for sure; i.e., those types in the set

$$D_1 := \left\{d \,\middle|\, d < u_1^* - u_2^*\right\} \cup \left\{d \,\middle|\, \max\{u_1^* - u_2^*, u_2^*\} \leq d < u_1^*\right\}.$$

The second subset are types who purchase from each seller with probability 1/2; i.e., those types in the set

$$D_2 := \left\{d \,\middle|\, u_1^* - u_2^* \le d < u_2^*\right\} \cup \left\{d \,\middle|\, d \ge u_1^*\right\}.$$

Seller 1's equilibrium profit is

$$\Pi_1^* = (\overline{\pi} - u_1^*)[Pr(D_1) + Pr(D_2)/2].$$

If seller 1 deviates to $u_1 = u_2^*$, it will share the market with seller 2, resulting in profit

$$\Pi_1 = (\overline{\pi} - u_2^*)/2 = (\overline{\pi} - u_2^*)[Pr(D_1)/2 + Pr(D_2)/2].$$

Equilibrium requires that such deviation is not profitable; i.e.,

$$(\overline{\pi} - u_1^*)[Pr(D_1) + Pr(D_2)/2] \ge (\overline{\pi} - u_2^*)[Pr(D_1)/2 + Pr(D_2)/2]. \tag{3}$$

On the other hand, seller 2's equilibrium profit is

$$\Pi_2^* = (\overline{\pi} - u_2^*)Pr(D_2)/2.$$

If seller 2 deviates to $u_2 = u_1^*$, it will share the market with seller 1, resulting in profit

$$\Pi_2 = (\overline{\pi} - u_1^*)/2 = (\overline{\pi} - u_1^*)[Pr(D_1)/2 + Pr(D_2)/2].$$

Equilibrium requires that such deviation is not profitable; i.e.,

$$(\overline{\pi} - u_2^*)Pr(D_2)/2 \ge (\overline{\pi} - u_1^*)[Pr(D_1)/2 + Pr(D_2)/2]. \tag{4}$$

Adding (3) and (4), we have $u_1^* \le u_2^*$, a contradiction. ■

Lemma 12 *In the case of two sellers marketing up to only one deal each, in any pure-seller-strategy* *equilibrium, we have* $(u_1^*, u_2^*) = (x^*, -x^*)$*, where* $x^* > 0$ *is the unique solution to*

$$\frac{1 - F(2x^*)}{f(2x^*)} = x^* + \overline{\pi}.$$

Proof: By Lemmas 10 and 11, we have $(u_1^* + u_2^*)/2 \geq 0$ and $u_2^* < 0$, which together imply $u_1^* > 0 > u_2^*$. Suppose $(u_1^* + u_2^*)/2 > 0$. Then, when the consumer finds himself in the all-tied case, he would rationally purchase for sure; i.e., $q_2^* = 1$. In addition, $(u_1^* + u_2^*)/2 > 0$ implies $u_1^* > -u_2^*$, and hence the single-contender case is off the equilibrium path. Fix any $q_1^* \in [0, 1]$. Against $u_1^* > 0$ and $(q_1^*, q_2^*) = (q_1^*, 1)$, seller 2's profit for any $u_2 < 0$ is

$$\Pi_2 = (\overline{\pi} - u_2)[1 - F(u_1^* - u_2)]/2,$$

which implies that u_2^* satisfies the FOC of

$$1 - F(u_1^* - u_2^*) = (\overline{\pi} - u_2^*)f(u_1^* - u_2^*). \tag{5}$$

Similarly, against $u_2^* < 0$ and $(q_1^*, q_2^*) = (q_1^*, 1)$, seller 1's profit for any $u_1 > -u_2^*$ is

$$\Pi_1 = (\overline{\pi} - u_1) - (\overline{\pi} - u_1)[1 - F(u_1 - u_2^*)]/2,$$

which implies that u_1^* satisfies the FOC of

$$1 - [1 - F(u_1^* - u_2^*)]/2 = (\overline{\pi} - u_1^*)f(u_1^* - u_2^*)/2. \tag{6}$$

Multiply (6) by 2, and subtract (5) from it, we have $2F(u_1^* - u_2^*) = (u_2^* - u_1^*)f(u_1^* - u_2^*) < 0$, a contradiction. This proves that $(u_1^* + u_2^*)/2 = 0$.

Recall that seller 2 must make strictly positive equilibrium profit (otherwise it can strictly profit from deviating to $u_2 = (\overline{\pi} + u_1^*)/2)$, hence we must have $q_2^* > 0$ (because the only chance that seller 2 makes any sales is when the consumer finds himself in the all-tied case). Against $u_1^* > 0$ and $q_2^* > 0$, for any $u_2 < 0$, seller 2's profit is

$$\Pi_2 = (\overline{\pi} - u_2)[1 - F(u_1^* - u_2)]q_2^*/2,$$

which implies that u_2^* satisfies the same FOC as (5). Rewrite (5) as

$$\frac{1 - F(2x^*)}{f(2x^*)} = x^* + \overline{\pi}, \tag{7}$$

where $x^* := u_1^* = -u_2^* > 0$. Since the LHS of (7) is weakly decreasing in x^* by Assumption 1, while the RHS is strictly increasing in x^* without bound, a solution to (7) is unique if it exists. Existence of a strictly positive solution follows from the fact that, by Assumption 2, we have the LHS strictly bigger than the RHS at $x^* = 0$. ■

Proof of Proposition 9: The first half of Proposition 9 follows from Lemma 12. It remains to prove the second half.

By the first half of Proposition 9, in any pure-seller-strategy equilibrium, the consumer gets a positive surplus iff he has a type $d < 2x^*$ (in which case he can compare utilities and is able to identify the positive-utility deal 1). His expected surplus is hence

$$CS = x^* F(2x^*).$$

In the equilibrium described in Proposition 4 for the case of a monopolist marketing up to two different deals, by (2) in the proof of Proposition 6, the consumer's expected surplus is

$$\overline{CS} = u_1(0)F(2u_1(0)) = \overline{x}F(2\overline{x}),$$

where $\overline{x} := u_1(0)$ is the unique solution to the first-order condition

$$\frac{F(2\overline{x})}{f(2\overline{x})} = 2(\overline{\pi} - \overline{x}) \tag{8}$$

by (1) in the proof of Lemma 4 (where we have simplified using $q_2 = 0$). To prove the second half of Proposition 9, it suffices to prove that $x^* < \overline{x}$.

Note that the LHS of (8) is strictly increasing and the RHS is strictly decreasing, and they are equal to each other when evaluated at $\overline{x}$. Therefore, to prove that $x^* < \overline{x}$, it suffices to prove that the LHS of (8) is strictly smaller than the RHS when they are evaluated at x^*. That is, it suffices to prove that

$$\frac{F(2x^*)}{f(2x^*)} < 2(\overline{\pi} - x^*). \tag{9}$$

Step 1: We first give a lower bound for $(\overline{\pi} - x^*)$.

Let $\Pi_1(u_1, u_2^*)$ and $\Pi_2(u_1^*, u_2)$ be sellers 1's and 2's profits as functions of their own offered utilities, where we have suppressed their dependence on the consumer's equilibrium strategy (q_1^*, q_2^*). Recall from Figure 3 that $\Pi_1(\cdot, -x^*)$ has a kink at $u_1 = x^*$. In order for $u_1 = x^*$ to be a local optimum for $\Pi_1(u_1, -x^*)$, the left derivative at $u_1 = x^*$ must be non-negative.

Whenever $u_2^* < 0$, for any $u_1 \in [0, u_2^*]$,

$$\begin{aligned}\Pi_1(u_1, u_2^*) &= (\overline{\pi} - u_1)\left(F(u_1) + q_1^* \left[F(-u_2^*) - F(u_1)\right] + \left[F(u_1 - u_2^*) - F(-u_2^*)\right] + \frac{q_2^*}{2}\left[1 - F(u_1 - u_2^*)\right]\right) \\ &= (\overline{\pi} - u_1)\left(F(u_1 - u_2^*) + \frac{q_2^*}{2}\left[1 - F(u_1 - u_2^*)\right] - (1 - q_1^*)\left[F(-u_2^*) - F(u_1)\right]\right).\end{aligned}$$

Therefore, for any $u_1 \in (0, -u_2^*)$,

$$\begin{aligned}\frac{\partial \Pi_1(u_1, u_2^*)}{\partial u_1} &= -\left(F(u_1 - u_2^*) + \frac{q_2^*}{2}\left[1 - F(u_1 - u_2^*)\right] - (1 - q_1^*)\left[F(-u_2^*) - F(u_1)\right]\right) \\ &\quad + (\overline{\pi} - u_1)\left[\left(1 - \frac{q_2^*}{2}\right) f(u_1 - u_2^*) + (1 - q_1^*) f(u_1)\right] \\ &\leq -\left(F(u_1 - u_2^*) + \frac{q_2^*}{2}[1 - F(u_1 - u_2^*)] - [F(-u_2^*) - F(u_1)]\right) \\ &\quad + (\overline{\pi} - u_1)\left[\left(1 - \frac{q_2^*}{2}\right) f(u_1 - u_2^*) + f(u_1)\right].\end{aligned}$$

Plugging in $u_2^* = -x^*$, local optimality of $u_1 = x^*$ hence requires that

$$\begin{aligned}0 &\leq \left.\frac{\partial \Pi_1(u_1, -x^*)}{\partial p_1}\right|_{u_1 = x^*-} \\ &\leq -\left(F(2x^*) + \frac{q_2^*}{2}\left[1 - F(2x^*)\right]\right) + \left[\left(1 - \frac{q_2^*}{2}\right) f(2x^*) + f(x^*)\right](\overline{\pi} - x^*) \\ &= -\left[F(2x^*) + \frac{q_2^*}{2} f(2x^*)(\overline{\pi} + x^*)\right] + \left[\left(1 - \frac{q_2^*}{2}\right) f(2x^*) + f(x^*)\right](\overline{\pi} - x^*) \\ &= -F(2x^*) - q_2^* f(2x^*)\overline{\pi} + \left[f(2x^*) + f(x^*)\right](\overline{\pi} - x^*),\end{aligned}$$

where the first equality follows from (7). This implies

$$q_2^* \leq \frac{[f(2x^*) + f(x^*)](\overline{\pi} - x^*) - F(2x^*)}{f(2x^*)\overline{\pi}}. \tag{10}$$

Rearranging terms, we can express (10) as a lower bound for $(\overline{\pi} - x^*)$:

$$\overline{\pi} - x^* \geq \frac{F(2x^*) + q_2^* f(2x^*)\overline{\pi}}{f(2x^*) + f(x^*)} \geq \frac{F(2x^*)}{f(2x^*) + f(x^*)}. \tag{11}$$

Step 2: We next give an upper bound for $F(2x^*)$.

In order for $u_2 = -x^*$ to be a global optimum for $\Pi_2(x^*, u_2)$, we must have

$$\begin{aligned} & \Pi_2(x^*, -x^*) \geq \Pi_2(x^*, x^*) \\ \iff \quad & \frac{q_2^*}{2}\left[1 - F(2x^*)\right](\overline{\pi} + x^*) \geq \left(\frac{1}{2}F(x^*) + \frac{q_2^*}{2}\left[1 - F(x^*)\right]\right)(\overline{\pi} - x^*) \\ \iff & \{\left[1 - F(2x^*)\right](\overline{\pi} + x^*) - \left[1 - F(x^*)\right](\overline{\pi} - x^*)\}\, q_2^* \geq F(x^*)(\overline{\pi} - x^*). \end{aligned}$$

Since the RHS is strictly positive, the term in the curly brackets is strictly positive. Dividing both sides by the term in the curly brackets, we have

$$q_2^* \geq \frac{F(x^*)(\overline{\pi} - x^*)}{\left[1 - F(2x^*)\right](\overline{\pi} + x^*) - \left[1 - F(x^*)\right](\overline{\pi} - x^*)}. \tag{12}$$

Combining (10) and (12), we have

$$\frac{F(x^*)(\overline{\pi} - x^*)}{\left[1 - F(2x^*)\right](\overline{\pi} + x^*) - \left[1 - F(x^*)\right](\overline{\pi} - x^*)} \leq \frac{[f(2x^*) + f(x^*)](\overline{\pi} - x^*) - F(2x^*)}{f(2x^*)\overline{\pi}}.$$

Rearranging terms, we have

$$
\begin{aligned}
F(2x^*) &\le [f(2x^*) + f(x^*)](\overline{\pi} - x^*) - \frac{F(x^*)(\overline{\pi} - x^*) f(2x^*)\overline{\pi}}{[1 - F(2x^*)]\,(\overline{\pi} + x^*) - [1 - F(x^*)]\,(\overline{\pi} - x^*)} \\
&= f(2x^*)(\overline{\pi} - x^*)\left(1 + \frac{f(x^*)}{f(2x^*)} - \frac{F(x^*)\overline{\pi}}{[1 - F(2x^*)]\,(\overline{\pi} + x^*) - [1 - F(x^*)]\,(\overline{\pi} - x^*)}\right) \\
&\le f(2x^*)(\overline{\pi} - x^*) \\
&\quad \times \left(1 + \frac{f(x^*)}{f(2x^*)} - \frac{x^* f(x^*)\overline{\pi}}{[1 - F(2x^*)]\,(\overline{\pi} + x^*) - [1 - F(2x^*) + x^* f(2x^*)]\,(\overline{\pi} - x^*)}\right) \\
&= f(2x^*)(\overline{\pi} - x^*)\left(1 + \frac{f(x^*)}{f(2x^*)} - \frac{x^* f(x^*)\overline{\pi}}{2x^*\,[1 - F(2x^*)] - x^* f(2x^*)(\overline{\pi} - x^*)}\right) \\
&= f(2x^*)(\overline{\pi} - x^*)\left(1 + \frac{f(x^*)}{f(2x^*)} - \frac{f(x^*)\overline{\pi}}{2f(2x^*)(\overline{\pi} + x^*) - f(2x^*)(\overline{\pi} - x^*)}\right) \\
&= f(2x^*)(\overline{\pi} - x^*)\left[1 + \frac{f(x^*)}{f(2x^*)}\left(1 - \frac{\overline{\pi}}{2(\overline{\pi} + x^*) - (\overline{\pi} - x^*)}\right)\right] \\
&= f(2x^*)(\overline{\pi} - x^*)\left[1 + \frac{f(x^*)}{f(2x^*)}\left(1 - \frac{\overline{\pi}}{\overline{\pi} + 3x^*}\right)\right],
\end{aligned}
$$

where the second inequality follows from Assumption 1,[24] and the third equality follows from (7). Therefore, to prove (9), it suffices to prove that

$$
\frac{f(x^*)}{f(2x^*)}\left(1 - \frac{\overline{\pi}}{\overline{\pi} + 3x^*}\right) < 1. \tag{13}
$$

Step 3: We shall now prove (13).

By (11), we have

$$
\begin{aligned}
\left[f(2x^*) + f(x^*)\right](\overline{\pi} - x^*) &\ge F(2x^*) \\
&= 1 - f(2x^*)(\overline{\pi} + x^*),
\end{aligned}
$$

where the equality follows from (7). Rearranging, we have

$$
\begin{aligned}
1 &\le 2f(2x^*)\overline{\pi} + f(x^*)(\overline{\pi} - x^*) \\
&\le 2f(0)\overline{\pi} + f(0)\overline{\pi} \\
&= 3f(0)\overline{\pi}, \qquad (14)
\end{aligned}
$$

[24] By Assumption 1, f is weakly decreasing. Hence $F(x^*) \ge x^* f(x^*)$ and $F(2x^*) - F(x^*) \ge x^* f(2x^*)$.

where the second inequality follows from Assumption 1.

By (7), we have

$$x^* = \frac{1 - F(2x^*)}{f(2x^*)} - \overline{\pi} \leq \frac{1}{f(0)} - \overline{\pi},$$

where the inequality follows from Assumption 1. Therefore,

$$\frac{x^*}{\overline{\pi} + x^*} \leq \frac{1/f(0) - \overline{\pi}}{\overline{\pi} + 1/f(0) - \overline{\pi}} = 1 - f(0)\overline{\pi} \leq \frac{2}{3}, \tag{15}$$

where the last inequality follows from (14).

By (11) again, we have

$$\overline{\pi} - x^* \geq \frac{F(2x^*)}{f(2x^*) + f(x^*)} \geq \frac{2x^* f(2x^*)}{f(2x^*) + f(x^*)},$$

where the second inequality follows from Assumption 1. Therefore,

$$\overline{\pi} \geq x^* \left(1 + \frac{2f(2x^*)}{f(2x^*) + f(x^*)}\right)$$

$$\frac{x^*}{\overline{\pi}} \leq \frac{f(2x^*) + f(x^*)}{3f(2x^*) + f(x^*)}. \tag{16}$$

By Assumption 1,

$$0 \leq \left(\frac{f}{1-F}\right)' = \frac{f'}{1-F} + \left(\frac{f}{1-F}\right)^2 = \frac{f}{1-F}\left(\frac{f'}{f} + \frac{f}{1-F}\right) = \frac{f}{1-F}\left((\ln f)' + \frac{f}{1-F}\right).$$

Therefore, for any $x \in [x^*, 2x^*]$,

$$\frac{d \ln f(x)}{dx} \geq -\frac{f(x)}{1 - F(x)} \geq -\frac{f(2x^*)}{1 - F(2x^*)},$$

where the second inequality follows from Assumption 1. Therefore,

$$\ln f(2x^*) - \ln f(x^*) = \int_{x=x^*}^{2x^*} \frac{d \ln f(x)}{dx} dx \geq -\frac{x^* f(2x^*)}{1 - F(2x^*)} = -\frac{x^*}{\overline{\pi} + x^*} \geq -\frac{2}{3},$$

where the second equality follows from (7), and the last inequality follows from (15). We hence have

$$\frac{f(x^*)}{f(2x^*)} \leq \exp\left(\frac{x^*}{\overline{\pi} + x^*}\right) \leq \exp(2/3), \tag{17}$$

and, by (16),

$$\frac{x^*}{\overline{\pi}} \leq \frac{f(2x^*) + f(x^*)}{3f(2x^*) + f(x^*)} = \frac{1 + f(x^*)/f(2x^*)}{3 + f(x^*)/f(2x^*)} \leq \frac{1 + \exp(2/3)}{3 + \exp(2/3)}. \tag{18}$$

The upper bound (18) allows us to give an even tighter upper bound for $f(x^*)/f(2x^*)$ than (17):

$$\frac{f(x^*)}{f(2x^*)} \leq \exp\left(\frac{x^*}{\overline{\pi} + x^*}\right) = \exp\left(\frac{x^*/\overline{\pi}}{1 + x^*/\overline{\pi}}\right) \leq \exp\left(\frac{1 + \exp(2/3)}{4 + 2\exp(2/3)}\right). \tag{19}$$

Plugging (16) and (19) into the LHS of (13), we then have

$$\begin{aligned} \frac{f(x^*)}{f(2x^*)}\left(1 - \frac{\overline{\pi}}{\overline{\pi} + 3x^*}\right) &= \frac{f(x^*)}{f(2x^*)}\left(\frac{3x^*/\overline{\pi}}{1 + 3x^*/\overline{\pi}}\right) \\ &\leq \exp\left(\frac{1 + \exp(2/3)}{4 + 2\exp(2/3)}\right) \times \left(\frac{3 + 3\exp(2/3)}{6 + 4\exp(2/3)}\right) \\ &= 0.9314 \\ &< 1, \end{aligned}$$

as claimed. ■

Appendix D: When Assumption 2 is Violated

This appendix contains the complementary analysis for the case when Assumption 2 is violated. We start with a monopolist marketing a single deal.

Proposition 11 *Consider a monopolist marketing up to only one deal. In the case of* $f(0)\overline{\pi} \geq 1$*, there exists a continuum of equilibria, indexed by different* $q^* \in [\overline{q}, 1]$ *with some* $\overline{q} \geq 1/2$*, where the monopolist offers the utility* $u^* = 0$*, and the consumer purchases with probability* q^* *whenever he cannot discern the utility from the deal and his reservation utility. Among these equilibria, the most efficient one is the one with* $q^* = 1$*, which also achieves the first best.*

Proof: Define $\underline{u}$ and $\overline{u}(q)$ as in the proof of Proposition 1; i.e., $\underline{u}$ is the unique maximizer of $\Pi(u;q)$ in the sub-range $u \in (-\infty, 0]$, while $\overline{u}(q)$ is that in the sub-range $u \in [0, \overline{\pi}]$. Since $\left.\frac{\partial \Pi}{\partial u}\right|_{u=0-} = f(0)\overline{\pi} - 1 \geq 0$, we have $\underline{u} = 0$, and hence $\overline{u}(q)$ is also the unique maximizer of Π in the whole range $u \in (-\infty, \overline{\pi}]$.

Define $\overline{q}$ as in the proof of Proposition 1; i.e., $\overline{q}$ is the point at which $\overline{u}(q)$ as a decreasing function of q first reaches 0 (in other words, $\overline{u}(q) = 0$ iff $q \geq \overline{q}$). Suppose $q^* < \overline{q} < 1$ in equilibrium. Then we must have $u^* = \overline{u}(q^*) > 0$ in equilibrium as well. But then when the consumer is unable to discern u^* and 0, he can still infer from the monopolist's equilibrium strategy that $u^* = \overline{u}(q^*) > 0$. His best response is hence to purchase the deal for sure (i.e., $q^* = 1$), contradicting the presumption that $q^* < \overline{q} < 1$.

On the other hand, any $q^* \in [\overline{q}, 1]$ can be part of an equilibrium, with the monopolist's best response being $u^* = 0$. ■

We then move on to the case of a monopolist marketing up to two different deals. When Assumption 2 is violated, there are more pure-seller-strategy equilibria. Not only that there are more pure-seller-strategy equilibria where the monopolist markets two different deals (and offers utilities that are mirror images of each other around 0), there is also a pure-seller-strategy equilibrium where the monopolist markets only one deal.

Proposition 12 *Consider a monopolist marketing up to two different deals. In the case of* $f(0)\overline{\pi} \geq 1$,

- *there exists a strictly decreasing function* $u_1(\cdot)$ *that maps* $[0,1]$ *into* $[0, \overline{\pi})$*, with* $u_1(1) = 0$*, such that, for every* $q_2^* \in [0,1]$*, there exists a pure-seller-strategy equilibrium featuring that specific* q_2^**, in which the monopolist offers deterministic utilities* $u_1^* = u_1(q_2^*)$ *and* $u_2^* = -u_1^*$*;*

- *there also exists a pure-seller-strategy equilibrium featuring* $(u_1^*, u_2^*) = (0, -\infty)$ *and* $(q_1^*, q_2^*) = (1, 0)$.

Proof: Note that Lemmas 2, 3, 4, 5, 6, 7 (with "$\underline{u} < 0$" replaced by "$\underline{u} = 0$"), and Proposition 4 remain valid, because their proofs do not rely on Assumption 2. However, the proof of Corollary 1 falls apart, because it relies on an argument made in the proof of Proposition 1, which in turn relies on Assumption 2. Therefore, we continue to have pure-seller-strategy equilibria of the form described in Proposition 6, but we can no longer use Corollary 1 to rule out pure-seller-strategy equilibria featuring $(u_1^*, u_2^*) = (0, -\infty)$.

Let q_2^{max} be defined as in the proof of Proposition 6. Since $f(0)\overline{\pi} > 1/2$ when Assumption 2 is violated, the same argument as in the last part of the proof of Proposition 6 suggests that (*i*) for every $q_2^* \in [0, q_2^{max}]$, there exists a pure-seller-strategy equilibrium featuring that specific q_2^*, in which the monopolist offers deterministic utilities $u_1^* = u_1(q_2^*)$ and $u_2^* = -u_1^*$ (where $u_1(\cdot)$ is defined in Lemma 4), and (*ii*), q_2^{max} is the maximum q_2 such that profit at $(u_1, u_2) = (u_1(q_2), -u_1(q_2))$ is weakly higher than profit at $(u_1, u_2) = (\underline{u}, \underline{u})$. However, when Assumption 2 is violated, $\underline{u} = 0$, and hence the former profit is weakly higher than the latter profit for every q_2 by the definition of $u_1(\cdot)$. Therefore, we have $q_2^{max} = 1$, which implies the first half of the proposition.

To prove the second half of the proposition, note that if $(u_1^*, u_2^*) = (0, -\infty)$, then any q_1 is a best response, and any q_2 is a best response against some off-equilibrium belief. So it suffices to prove that $(u_1, u_2) = (0, -\infty)$ is a best response against $(q_1^*, q_2^*) = (1, 0)$. The proof, however, is almost the same as that for $u = 0$ to be a best response against $q = 1$ in the one-deal case when Assumption 2 is violated (see the proof of Proposition 11), and hence is omitted. ■

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