Course Retaking and Related Policies
:An Economic Analysis

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Abstract

In this paper, we develop a simple model that can be helpful for explaining the course retaking and similar problems. We first analyze the situation when regeneration of signal is allowed. We find out some incentive implications and extend our model to analyze several policies regarding the course retaking. We show that those policies are effectively reducing the course retaking and restoring incentives. We also compare the two policies in some aspects and then provide a numerical example.

JEL Classification Code : D81, D82, I21

1. Introduction

Today, course retaking is a common practice in Korean universities. Numerous university students in Korea retake some of the courses they already have taken. A considerable number of them postpone their graduation to stay in school for (an) extra semester(s) in order to improve their GPA, retaking courses with bad grades. Without doubt, this phenomenon is largely due to growing significance of GPA as a signal to the economy indicating individual student's capability. Since Korean labor market lacks well-behaving system of hiring through

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reference calls, its reliance on public invitation is heavy, which makes publicly available and quantifiable signals like undergraduate GPA and nation-wide test scores tremendously crucial for job market. Then, there is no wonder we observe such a large demand for course retaking, provided this practice is sanctioned by universities.

In this paper, we develop a simple model that can be applied to analyzing the course retaking problem. Our model allows continuous signals stochastically produced by agents' effort. We take a few minimal assumptions and derive some interesting and implicative results regarding the regenerating signal problem. We extend the basic model to analyze effects of various policy options that is implemented by universities in the real world. Specifically, we focus on the two most widely used course retaking policies, namely the eligibility constraint and the grade limit for retakers. We find some real world implications for those policies and the course retaking itself. For illustration purpose, we also provide some simple numerical examples based on our model.

The remaining part of this paper is organized as follows. In Section 2, we briefly look at the course retaking practices in Korea and policies of major universities on the matter. In Section 3, we develop a simple stochastic signal production model that can be extended to the regeneration case. In Section 4, we incorporate the regeneration of signal into our model and derive some important features of signal regeneration. In Section 5, we extend our model to analyze two popular policies regarding the course retaking. In Section 6, we compare those policies in several aspects. In Section 7, we provide a numerical example for illustration. Finally, in Section 8, we make some concluding remarks.

2. Course Retaking Practice in Korea

Although the practice of course retaking is prevailing in Korea, there is no empirical study regarding the matter published yet. Lack of empirical data forced us to present the current situation in rather rough style. To show how course retaking problem is serious in some schools, we present an excerpt from a column in a school newspaper.2)

> "...... I felt uncomfortable when I saw the student roster of my class. It was because many students were retaking the course again. 30 out of total 58 students were re-takers. There may be few professors who would enjoy the situation that half the class are re-takers ....."

The story of 50% retakers seems to be exaggerated or extreme. However we argue that the story shows what is happening nowadays realistically. If a survey on the course taking were done, we assert with confidence, more than half of the subjects would report some experience of course retaking.

Recently, many Korean universities have been changing their policy regarding course retaking, mostly toward restriction of it. This shows that the issue of course retaking is now a major concern of many university administrations. The current policies of major universities in Korea are summarized in [Table 1].3)

As in the [Table 1], major universities are employing several different restrictive policies against the course retaking. The most widely used tools are eligibility constraint and the upper limit for retakers.

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2) By Seo, Dongyeop. From KAIST Times, 2006.3.28. Translated by authors.
3) It is interesting to note that universities place upper bound to the retake-eligible grade level, but never place lower bound to it. A possible guess might be that the society seeks to have accurate information on students with upper level of aptitudes more than that on the students with lower level of aptitudes.
4) Data obtained by authors from university websites.

[Table 1] Course-Retaking-Related Policies of Major Korean Universities (as of Fall, 2006)

University	Policy on Retaking[4)]	Before Change
ChoongAng U	Constraint on eligibility (≤C+ & junior or senior) Allowed only once for each course Only the highest grade remains	Not available
DongGuk U	No eligibility constraint or grade limit Only the last grade remains	Not available
Ewha Woman's U	Constraint on eligibility (≤C+) Grade upper limit for retaken courses (A-) Only the last grade remains	Not available
Hanyang U	No constraint and grade limit Only the last grade remains	None
Hongik U	No eligibility constraint or grade limit Only the last grade remains	None
Inha U	Constraint on eligibility (≤C+) Allowed only once for each course, up to 6 credit for each semester and 24 credit total Only the highest grade remains	No limit until 1998
KonKuk U	Not allowed	No limit until 2004
Korea U	Constraint on eligibility (≤C+) Grade upper limit for retaken courses (A) Only the highest grade remains	No limit until 2000
KAIST	Impose retaking fee (30,000 won per credit) Only the last grade remains, with a mark indicating retaken course.	Not allowed until 1996
KyungHee U	No eligibility constraint or grade limit Only the last grade remains	None
MyongJi U	No eligibility constraint or grade limit Only the last grade remains	None
PosTech	Grade upper limit for retaken courses (B+) Only the last grade remains	Not available
Seoul National U	Constraint on eligibility (≤C+) Only the last grade remains	No limit until 2005
Sogang U	Constraint on eligibility (≤B) Grade upper limit for retaken courses (B+) Only the last grade remains	Not available
SungKyunKwan U	Not allowed	No limit until 2004
SoongSil U	Constraint on eligibility (≤C+) Only the last grade remains	Not available
Yonsei U	Constraint on eligibility (≤D+) Only the last grade remains	No limit until 2004

3. Basic Model

In this section, we develop a framework analyzed throughout the paper. The signal space is defined as $\Sigma = [\underline{\omega}, \overline{\omega}]$[5]. This can be a GPA or SAT score and so on. There are homogeneous risk neutral agents with a stochastic production function that defines the relation between the signal and effort. The effort space is defined as $H = [\underline{e}, \overline{e}]$ and the stochastic production function defined as $f(\sigma|e)$, which denotes the conditional probability density function of the signal given some effort level. Also, we define the cost function of effort by $c(e)$, which is assumed to be increasing and convex in e. We assume the following properties of $f(\sigma|e)$:

(A1) $f(\sigma|e) = f(\sigma - e)$

(A2) $f(\sigma|e)$ is twice differentiable.

(A3) $f'(0) = 0$

(A4) $f(\sigma|e)$ shows monotone likelihood ratio property(MLRP) with respect to e.

Note that the assumptions are not too strong. Basically, we are assuming that the signal, σ is determined linearly in e with some noise around 0. For example, a state-space representation $\sigma = e + \theta$ where θ denotes a random variable with Normal distribution, which is widely used in many literature, is qualifying our assumption.

When a signal of an agent is realized, he or she receives some payoff according to the realized signal. The payoff function $\Pi(\sigma)$ assumed to be predetermined, and increasing and concave in σ. It can be interpreted as a long run estimate of productivity given certain level of signal. Therefore it can be

5) The support of the signal space may be $(-\infty, \infty)$.

treated to be stable in short run.

Under the setting above, we can now derive an agent's utility maximization problem and the following first order condition[6] as :

$$\max_e \int_{\underline{\omega}}^{\overline{\omega}} \Pi(\sigma) f(\sigma|e) d\sigma - c(e) \tag{a1}$$

$$\textit{(F.O.C)} \quad \int_{\underline{\omega}}^{\overline{\omega}} \Pi(\sigma) f_e(\sigma|e) d\sigma = c'(e) \tag{a2}$$

The meaning of the first order condition is straightforward. The optimal effort exerted by an agent is determined at where extra expected payoff by increasing his or her effort equals the marginal cost of the effort. We denote the optimal effort level determined by (a2) by e^*.

4. Regeneration of Signal

We now incorporate the regeneration of signal into the model. We explicitly assume that each agent's payoff is determined by the last signal he or she obtained. This is relevant to the real world problem of course retaking. Note that for the most of schools in [Table 1], only the last grade remains when a course is retaken. We here derive some incentive implications when the regeneration of signal is allowed.

We tackle the problem by the backward induction. Given an agent decided to regenerate his or her signal, his decision on the optimal effort is exactly the same as (a2). Therefore, his or her expected payoff when he or she regenerates

6) Note that the left hand side of (F.O.C) is decreasing in e.

the signal is defined as $R = \int_{\underline{\omega}}^{\bar{\omega}} \Pi(\sigma) f(\sigma|e^*) d\sigma - c(e^*)$.

Given this, an agent's utility maximization problem involves two decision variables. One is the optimal effort level he or she would exert in the first time and the other is the optimal threshold level where he or she decides to regenerate the signal. An agent's problem and the corresponding first order condition are :

$$\max_{\alpha, e_1} \int_{\alpha}^{\bar{\omega}} \Pi(\sigma) f(\sigma|e_1) d\sigma + R \int_{\bar{\omega}}^{\alpha} f(\sigma|e_1) d\sigma - c(e_1) \qquad \text{(b1)}$$

$$\textit{(F.O.C 1)} \quad -\Pi(\alpha) f(\alpha|e_1) + R f(\alpha|e_1) = 0 \qquad \text{(b2)}$$

$$\textit{(F.O.C 2)} \quad \int_{\alpha}^{\bar{\omega}} \Pi(\sigma) f_e(\sigma|e_1) d\sigma + R \int_{\bar{\omega}}^{\alpha} f_e(\sigma|e_1) d\sigma = c'(e_1) \qquad \text{(b3)}$$

Proposition 1. The solution of maximization problem under regeneration has the following properties.

$$\Pi(\alpha^*) = R \qquad \text{(c1)}$$

$$e_1^* < e^* \qquad \text{(c2)}$$

$$\int_{\alpha^*}^{\bar{\omega}} \Pi(\sigma) f(\sigma|e_1^*) d\sigma + R \int_{\bar{\omega}}^{\alpha} f(\sigma|e_1^*) d\sigma - c(e_1^*) \geqq R \qquad \text{(c3)}$$

(proof)

Since $f(\sigma|e) > 0 \;\; \forall e$, by (b2), $\Pi(\alpha) = R$ and we prove (c1).

By (a2) $c'(e^*) = \int_{\underline{\omega}}^{\bar{\omega}} \Pi(\sigma) f_e(\sigma|e^*) d\sigma$. Since $c'(e)$ is increasing in e, it is sufficient to show that :

$$\int_{\alpha^*}^{\bar{\omega}} \Pi(\sigma) f_e(\sigma|e) d\sigma + R \int_{\underline{\omega}}^{\alpha^*} f_e(\sigma|e) d\sigma < \int_{\underline{\omega}}^{\bar{\omega}} \Pi(\sigma) f_e(\sigma|e) d\sigma \quad \text{for } e = e^*$$

Note that $\Pi(\sigma) - R < 0 \;\; \forall \sigma \in [\underline{\omega}\,,\, \alpha^*]$. Also note that $f_e(\sigma|e^*) = 0$ at $\sigma = e^*$

and $f_e(\sigma|e*)<0\ \forall\sigma<e*$ and $f_e(\sigma|e*)>0\ \forall\sigma>e*$. We now show that $\alpha*<e*$, since $\Pi(\alpha*)<\int_{\underline{\omega}}^{\overline{\omega}}\Pi(\sigma)f(\sigma|e*)d\sigma<\Pi(\int_{\underline{\omega}}^{\overline{\omega}}\sigma f(\sigma|e*)d\sigma)=\Pi(e*)$ by the Jensen inequality and assumption (A1) and (A2).

Hence, the $RHS-LHS$ of the inequality yields :

$$\int_{\underline{\omega}}^{\alpha*}\Pi(\sigma)f_e(\sigma|e)d\sigma-R\int_{\underline{\omega}}^{\alpha*}f_e(\sigma|e)d\sigma=\int_{\underline{\omega}}^{\alpha*}[\Pi(\sigma)-R]f_e(\sigma|e)d\sigma$$

and using the facts derived above, we know that it has a positive value. This proves (c2).

Finally, the last inequality becomes an equality when we put $\alpha=\overline{w}$ and $e_1=\underline{e}$ instead of the optimal values. Therefore, by the optimality of $\alpha*$ and $e*$ the inequality holds generally.

$(Q.E.D)$

The result gives some interesting implications. Given the payoff function, agents are generally better off by allowing the regeneration of signal. However, they do not exert as much effort at the first time as they would when there is no regeneration opportunity. That is, they have less incentive to exert effort. This implies that in the real world, the existence of course retaking possibly undermines the studying incentives of students.

Also it is notable that agents choose their optimal threshold of regeneration at the point where their expected payoff when they regenerate signals equals their payoff at their first signal. If their signal is sufficiently high enough to exceed their expected regeneration value, they will stop at the first time and never regenerate their signals again.

However, it is not sure whether $e_1*>\alpha*$ or not from the analysis above. It is completely dependent upon the form of the cost function. What it means intuitively is that an agent exerts more effort to make his expected first time signal larger than the threshold. It is natural to think that a student won't choose his threshold even more than what he can actually achieve at the first time in an expected sense. Since this is more like the reality, we will explicitly add one more assumption.

(A5) We only consider cost functions that satisfies $\alpha^* < e_1^*$.

As discussed above, the result here seems to reflect the reality fairly well. In fact, the assumptions of our model is minimal and it can be applied to many different situations involving signal regeneration issue. In the next section, we will extend our model to analyze some policies against the course retaking that actually implemented in the real world.

5. Extension of the Model for Policy Analysis

As we showed at [Table 1], major universities in Korea are now starting to adopt several policies against the course retaking of their students. Most widely used tools are lower bounded eligibility constraints and upper bounded grade limit for retakers. The first tool is not allowing students to retake a course when they achieved more than certain level of grade. The second tool is imposing a grade ceiling for those who retakes.

In our model, even if there is a eligibility constraint, it can be unbinding. Therefore we denote an eligibility constraint by β and assume that $\beta < \alpha^*$ to make it a binding constraint. The following proposition characterizes the solution under an eligibility constraint for regeneration.

Proposition 2. Let β be the eligibility constraint. Let e_1^β denote the optimal effort choice at the first time when β is binding. Then we have,

$e_1{*} < e_1^\beta < e{*}$ (d1)

There is some utility loss by the constraint. (d2)

(proof)

As in the section 4, we can apply the backward induction again. We have $R = \int_{\underline{\omega}}^{\bar{\omega}} \Pi(\sigma) f(\sigma|e{*}) d\sigma - c(e{*})$ as the expected payoff when an agent regenerates his or her signal. Given this and the eligibility constraint β, the utility maximization problem of an agent involves only one decision variable, e_1 which is his optimal effort choice at the first time. That is :

$$\max_{e_1} \int_{\beta}^{\bar{\omega}} \Pi(\sigma) f(\sigma|e_1) d\sigma + R \int_{\bar{\omega}}^{\beta} f(\sigma|e_1) d\sigma - c(e_1)$$

The first order condition yields :

$$\int_{\beta}^{\bar{\omega}} \Pi(\sigma) f_e(\sigma|e_1^\beta) d\sigma + R \int_{\bar{\omega}}^{\beta} f_e(\sigma|e_1^\beta) d\sigma = c'(e_1^\beta)$$

By using $\int_{\underline{\omega}}^{\bar{\omega}} C f_e(\sigma|e) d\sigma = 0$ for a constant C,

$$\int_{\beta}^{\bar{\omega}} \Pi(\sigma) f_e(\sigma|e_1^\beta) d\sigma + R \int_{\bar{\omega}}^{\beta} f_e(\sigma|e_1^\beta) d\sigma = \int_{\beta}^{\bar{\omega}} [\Pi(\sigma) - R] f_e(\sigma|e_1^\beta) d\sigma$$

Similar to the proof of proposition 1, it is sufficient to show that :

$$\int_{\beta}^{\alpha{*}} [\Pi(\sigma) - R] f_e(\sigma|e) d\sigma > 0 \text{ for } e = e_1{*}$$

Since $\Pi(\sigma) \leqq R \quad \forall \sigma \in [\beta, \alpha{*}]$ and $f_e(\sigma|e_1{*}) < 0 \quad \forall \sigma < \alpha{*}$, the value is positive. Hence $e_1{*} < e_1^\beta$. $e_1^\beta < e{*}$ is proved exactly the same way as proving that $e_1{*} < e{*}$. Since optimal threshold is $\alpha{*}$, (d2) is straightforward.

$(Q.E.D)$

The result implies that the eligibility constraint policy can actually work only if the constraint set is a binding one. Also, the constraint policy gives students some additional incentive to study harder at the first time. However, it is still less than $e*$, which is the optimal effort when there is no retaking chance. Also note that imposing a binding eligibility constraint generally reduces student's utility.

The other possible way of restricting course retaking behavior mentioned above is to set the maximum grade that a retaker can achieve. This directly affects the effort choice at the regeneration process. The next proposition gives some pictures on this kind of policy.

Proposition 3. Let γ be the maximum limit of signal that can be achieved by regeneration. Let e_2^{γ} denote the optimal choice of effort at the second time and e_1^{γ} denote that of the first time. Let α_{γ} denote the optimal threshold when maximum limit is imposed. Then we have :

$$e_2^{\gamma} < e* \qquad \text{(e1)}$$

$$\alpha_{\gamma} < \alpha* \qquad \text{(e2)}$$

$$e_1* < e_1^{\gamma} \qquad \text{(e3)}$$

(proof)

We first derive the optimal choice at the second time. The utility maximization problem is :

$$\max_{e_2} \int_{\underline{\omega}}^{\gamma} \Pi(\sigma) f(\sigma|e_2) d\sigma + \int_{\gamma}^{\bar{\omega}} \Pi(\gamma) f(\sigma|e_2) d\sigma - c(e_2)$$

The first order condition yields :

$$\int_{\underline{\omega}}^{\gamma} \Pi(\sigma) f_e(\sigma|e_2^{\gamma}) d\sigma + \int_{\gamma}^{\bar{\omega}} \Pi(\gamma) f_e(\sigma|e_2^{\gamma}) d\sigma = c'(e_2^{\gamma})$$

Note that $\int_{\underline{\omega}}^{\bar{\omega}} \Pi(\sigma) f_e(\sigma|e^*) d\sigma = c'(e^*)$ and to show $c'(e_2^{\gamma}) < c'(e^*)$, it is sufficient to show that :

$$\int_{\underline{\omega}}^{\gamma} \Pi(\sigma) f_e(\sigma|e_2) d\sigma + \int_{\gamma}^{\bar{\omega}} \Pi(\gamma) f_e(\sigma|e_2) d\sigma < \int_{\underline{\omega}}^{\bar{\omega}} \Pi(\sigma) f_e(\sigma|e) d\sigma \quad \forall e$$

By taking difference of the both sides, we have :

$$\int_{\gamma}^{\bar{\omega}} [\Pi(\sigma) - \Pi(\gamma)] f_e(\sigma|e) d\sigma$$

Suppose that $\gamma > e$. Then $f_e(\sigma|e) > 0 \quad \forall \sigma \in [\gamma, \bar{\omega}]$. Since $\Pi(\sigma) - \Pi(\gamma) > 0 \quad \forall \sigma \in [\gamma, \bar{\omega}]$, the above term is positive.

Now suppose that $\gamma < e$. Since $\int_{\underline{\omega}}^{\bar{\omega}} \phi(x) f_e(x|e) dx > 0$ if $\phi'(x) > 0$ and $\int_{\gamma}^{\bar{\omega}} [\Pi(\sigma) - \Pi(\gamma)] f_e(\sigma|e) d\sigma$ is monotonic decreasing for $\gamma < e^*$, the above term is also positive. This proves (e1).

Let $Q = \int_{\underline{\omega}}^{\gamma} \Pi(\sigma) f(\sigma|e_2^{\gamma}) d\sigma + \int_{\gamma}^{\bar{\omega}} \Pi(\gamma) f(\sigma|e_2^{\gamma}) d\sigma - c(e_2^{\gamma})$. The maximization problem given the expected payoff obtained involves two decision variables, e_1 and α, the effort choice at the first time and the threshold, respectively. We have :

$$\max_{\alpha, e_1} \int_{\alpha}^{\bar{\omega}} \Pi(\sigma) f(\sigma|e_1) d\sigma + Q \int_{\bar{\omega}}^{\alpha} f(\sigma|e_1) d\sigma - c(e_1)$$

(F.O.C 1) $-\Pi(\alpha) f(\alpha|e_1) + Q f(\alpha|e_1) = 0$

(F.O.C 2) $\int_{\alpha}^{\bar{\omega}} \Pi(\sigma) f_e(\sigma|e_1) d\sigma + Q \int_{\bar{\omega}}^{\alpha} f_e(\sigma|e_1) d\sigma = c'(e_1)$

By solving the *(F.O.C 1)*, we have $\Pi(\alpha_{\gamma}) = Q$. Since $Q < R$ from the proof of (e1) and Π is an increasing function, $\alpha_{\gamma} < \alpha^*$ and this proves (e2).

By solving the *(F.O.C 2)*, we have :

$$\int_{\alpha}^{\bar{\omega}} \Pi(\sigma) f_e(\sigma|e_1^{\gamma}) d\sigma + Q\int_{\bar{\omega}}^{\alpha} f_e(\sigma|e_1^{\gamma}) d\sigma = c'(e_1^{\gamma})$$

Again it is sufficient to show that :

$$\int_{\alpha_\gamma}^{\bar{\omega}} \Pi(\sigma) f_e(\sigma|e) d\sigma + Q\int_{\bar{\omega}}^{\alpha_\gamma} f_e(\sigma|e) d\sigma > \int_{\alpha^*}^{\bar{\omega}} \Pi(\sigma) f_e(\sigma|e) d\sigma + R\int_{\bar{\omega}}^{\alpha^*} f_e(\sigma|e) d\sigma \ \text{ for } \ e = e_1{}^*$$

By taking the difference of both sides, we have :

$$(Q-R)\int_{\underline{\omega}}^{\alpha_\gamma} f_e(\sigma|e) d\sigma - R\int_{\alpha_\gamma}^{\alpha^*} f_e(\sigma|e) d\sigma \ \text{ for } \ e = e_1{}^*$$

Since $\int_{\underline{\omega}}^{\bar{\omega}} f_e(\sigma|e) d\sigma = 0$, $\int_{\underline{\omega}}^{\alpha_\gamma} f_e(\sigma|e) d\sigma < 0$ and the first term is positive. The second term is negative since $f_e(\sigma|e) < 0 \ \ \forall \sigma < \alpha^*$. This proves (e3).

$(Q.E.D)$

The result is consistent to general intuition. First, the limit of maximum signal for retakers reduces incentives when an agent tries for the second time. Moreover, by the backward induction, this affects the choice at the first time. Agents choose to exert more effort at the first time and they set their threshold less than before. That is, they are less willing to retake a course since the benefit of it is decreased by setting the limit. Therefore, we can say that the limit policy have both effects of lower incentives at the second time and de facto eligibility constraint.

6. Policy Comparison

In the previous section, we have analyzed the two most widely used policy regarding the course retaking. A natural question arises then. Which policy is better? To answer this question, we need to define some proper criterion first.

The next proposition provides one of them.

Proposition 4. Given that $e_1^{\beta} = e_1^{\gamma}$, $\beta < \alpha_{\gamma}$ holds. That is, given the same incentive at the first time, the eligibility constraint policy is better at lowering actual threshold of retaking than the limit policy.

(proof)

Using the fact that two first order conditions for e_1^{β} and e_1^{γ} must be equal, we have :

$$\int_{\beta}^{\bar{\omega}} [\Pi(\sigma) - R] f_e(\sigma | e_1^{\beta}) d\sigma = \int_{\alpha_{\gamma}}^{\bar{\omega}} [\Pi(\sigma) - Q] f_e(\sigma | e_1^{\gamma}) d\sigma$$

We can write this as :

$$\int_{\alpha_{\gamma}}^{\bar{\omega}} [R - Q] f_e(\sigma | e_1) d\sigma = \int_{\beta}^{\alpha_{\gamma}} [\Pi(\sigma) - R] f_e(\sigma | e_1) d\sigma$$

The left hand side is positive since $R - Q > 0$. Also, $\Pi(\sigma) - R < 0$ and $f_e(\sigma | e_1) < 0$ for $\sigma \in [\beta, \alpha_{\gamma}]$ or $[\alpha_{\gamma}, \beta]$. Therefore, for the right hand side to be positive, $\alpha_{\gamma} > \beta$ must hold.

(*Q.E.D*)

This result can be interpreted as follows. An university administration may want to find a better way to lower the retaking rate given the same incentive for students who take courses for the first time. Then the proposition 4 would suggest that the eligibility constraint policy is the better choice.

On the other hand, if the policy maker is not sure what exactly is the student's threshold α^* is, it is better to adopt the upper limit policy. By the proposition 3, we know that $\alpha_{\gamma} < \alpha^*$ regardless of the choice of γ. Therefore, any choice of γ will result in actual decrease in the course retaking rate and there is no uncertainty regarding whether the policy would be effective or not.

There are other possible criterion. One may want to compare the total social cost of the policies. While other want to maximize the student's welfare. However there are no tractable general comparison rule in terms of other criterion than one introduced in proposition 4. We, therefore, may give some points by presenting a numerical example that gives a clear picture of what our model says.

7. Numerical Example

We consider a very simple but quite illuminative example. Here we consider a setting where there are only three discrete signals: A, B, and C. The payoffs for these signals are exogenously given: *3*, *2*, and *1* respectively. As agents exert their effort, $e \in [0,1]$, at cost e^2 to improve expected gains. The probability of obtaining each signal alters, in a way that an agent becomes more likely to get better signals as he or she puts more effort. We let $\Pr(A|e)$, the probability of getting A when effort e is exerted, equal to e, $\Pr(B|e) \equiv e^{1/4} - e$ and $\Pr(C|e) \equiv 1 - e^{1/4}$. This is because we wanted to keep this example consistent with our assumption of *monotone likelihood ratio property* in the model.

Now, let's first think of the case where the retaking option is unavailable at all. The agent's maximization program is $Max_e \, Eu(e) = 3e + 2(e^{1/4} - e) + 1(1 - e^{1/4})$ $- e^2$. Its first order condition gives us that, e^*, the effort level optimal for the agent is 0.669, so $Eu(e^*)$=2.126. Next, we consider the case where retaking (once) is fully allowed with no restriction. As backward induction we first solve for e_2*, the optimal effort choice for retaking stage, and obviously this is same with $e^* = 0.669$. The maximization problem for the initial stage is the following,

$Max\{Max_{e_1}[3e_1+(1-e_1)2.126)-e_1^2],\ Max_{e_1}[3e_1+2(e_1^{1/4}-e_1)+(1-e_1^{1/4})2.126-e_1^2]\}$

involving the choice over the highest grade which would prompt the agent to retake. The left sub-maximization solves e_1*= 0.437, $Eu(e_1*,e_2*)$= 2.308 and the right one solves $\hat{e_1}$=0.472, $Eu(\hat{e_1},\ e_2^*)$= 2.271. Thus, he or she chooses to retake course only if its grade is lower than B (selecting e_1*= 0.437). The numbers are consistent with the result in our model in that $e_1^* < e_2{}^* = e^*$. With unrestricted retaking option, one does relax at the initial period yielding 43.7% chance of getting A, and when the grade has gone wrong, he or she retake the course and works more so that the A-probability goes up to 66.9%.

Let us also make cases for two different policy method in this example. First, if the constraint on eligibility of retaking is imposed at a level below B so that only C students can retake the course, the agent's optimization is exactly the same as the right sub-maximization program above,

that is, $Max_{e_1}[3e_1+2(e_1^{1/4}-e_1)+(1-e_1^{1/4})2.126-e_1^2]$. thus we get $\hat{e_1}$=0.472 and $E\hat{u}(\hat{e_1},\ e_2^*)$= 2.271 for the optimal decision. As we have also shown previously in the model, $\hat{e_1} > e_1^*$. In other words, one would work more at the first time taking course if he or she cannot retake it with grade B.

Secondly, if there is a grade upper limit, B, for the retaken course, optimization for the second period is modified:

$Max_{e_2}\widetilde{Eu_2}(e_2) = 2(e_2^{1/4}) + 1(1-e_2^{1/4}) - e_2^2$, thereby attaining the optimal effort $\widetilde{e_2}$= 0.305 and $\widetilde{Eu_2}(\widetilde{e}) = 1.650$. Similarly we can find that $\widetilde{e_1}$=0.567 > e_1^* and $E\widetilde{u}(\widetilde{e_1},\ \widetilde{e_2})$=2.199. This time, incentives for the retaking stage are sacrificed in return for incentive provision for the initial effort, so you work harder for the first time course-work than the second time. One might notice that expected agent payoff is higher under eligibility constraints than under grade upper limits, but this is neither general nor meaningful result, since in this numerical example there are only three signals and there is no room for the thresholds to be adjusted to level the policy effects; for illustrative purpose, we could not but let eligibility constraint level(β) and grade limit(γ) for a retaken course same at B. (whereas β should be smaller than γ to level the policy effect in our model with continuous signal space.)

8. Concluding Remarks

We have presented a simple model with uncertainty that explains and analyzes the course retaking and similar problems. We have found out that the course retaking generally reduces the initial incentives of students. We conjecture that this might be the main reason behind the recent policies against the course retaking. We also analyzed the two most popular policies regarding the issue. We found out that both policies effectively increase the initial incentives and make less students retaking courses. We have tried to compare the two policies, but only to find out that there is no general rule except for one. Finally, we have provided an illuminating numerical example to show a clear picture of our model and discussions.

Our model can be extended to several other ways. For example, we may allow the maximum of signal be reported. This would bring a different point to our discussion. Also, we may allow the information of whether an agent regenerated his or her signal become observable and paid differently according to that. This may result in different consequences since agents's willingness to regenerate signals will be deterred.

Finally, we propose that the issue of course retaking should be seriously dealt in human resources and labor economics literature. There is virtually no empirical survey on this matter that look deep inside the current situation of course retaking practice. Implications from our simple model strongly suggest that the issue is worth some careful attention from both academics and policy makers.

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