Arbitrage Comes Hand in Hand with the Risk of Market Crash*

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Abstract

This paper proposes a simple model in which hedge funds can initiate a sequence of arbitrage and a potential market crash even without any exogenous shock. When hedge fund managers share a concern that a rare event, not necessarily affecting the fundamentals, may occur, some hedge funds may opt out for fear of redemption risk, which leads to coordination failure. Our model demonstrates that the coordination failure generates an initial arbitrage opportunity but it comes with a chance of market crash. The model provides novel theoretical insights on the cause and amplification of the financial crisis. It also explains some empirically documented behavior of hedge funds in the existing literature and discusses policy implications.

JEL classification codes: G01, G23, G28

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1 Introduction

This paper proposes a simple theoretical model which explains how an asset market could collapse from coordination failure among hedge funds even *in the absence of any exogenous shock*. Such coordination failure arises from key features that characterize a hedge fund industry such as redemption risk, high leverage, synchronized collective actions of hedge funds, information asymmetry between investors and hedge fund managers and a fee structure of hedge funds. In the quest for arbitrage gains, hedge funds police and cleanse the market by eliminating mispricing. The effectiveness of arbitrage increases in proportion to the strength of unidirectional collective actions among hedge funds. When hedge fund managers share a concern that a rare event *may* occur, some hedge funds may opt out for fear of redemption risk, which leads to coordination failure. Once it takes place, the initial price of the asset deviates from its fundamental value, which delivers an arbitrage opportunity to the hedge funds. But concurrently it carries a chance of market crash in the future. Surprisingly, we find that this coordination failure can arise even without any exogenous shock.

The most notable feature of this coordination failure is that every arbitrage opportunity is spawned in tandem with the risk of market crash. Therefore the coordination failure is a double-edged sword to the hedge fund managers. This is the nature of the *crisis* equilibrium of our model. In contrast, if no hedge fund escapes from the coordination, the *calm* equilibrium holds such that there is neither mispricing in the initial price nor any chance of crash. Therefore, as typical in the model of coordination failure, our model demonstrates multiple equilibria, the calm equilibrium and the crisis equilibrium, depending on the success and failure of coordination, and the crisis equilibrium results in a self-fulfilling prophecy.

We show that a potential failure of coordination is an inherent nature of a hedge fund industry due to the redemption risk that each fund faces. Following the intuition of Shleifer and Vishny (1997) among many others, we assume that hedge fund investors redeem their investment once the value of the fund hits a certain threshold from above.[1]

[1] A key feature of this assumption is information asymmetry between hedge fund managers and investors. The investors may withdraw their investment even when the best opportunity of gains is available. Shleifer and Vishny (1997) validate this assumption by arguing that if the fund loses, the investors infer that the arbitragers are not as competent as they believed initially. They refer to the arbitrage constrained by such redemption risk as performance-based arbitrage. Gromb and Vayanos (2002) show that margin constraints have a similar effect of triggering redemption while Liu and Mello (2011) designate investor's concerns about coordination risk among themselves as a cause of redemption. All of these papers focus on why hedge funds withdraw their investment when the

Upon the redemption request from investors, a hedge fund should unwind its position and liquidate the fund.[2] On the alert for the redemption risk, a risk-neutral hedge fund manager *a priori* decides the optimal leverage ratio, i.e., how much she invests in the asset. In doing so, the fund manager who maximizes the hedge fund fee (similar to the so-called "two+twenty"), confronts a trade-off between redemption risk and return on investment. This involves choosing one of two diametrically opposite strategies[3]: an *aggressive* leverage strategy, which takes excess risk by maximally utilizing leverage capacity given the convex fee schedule and a *defensive* leverage strategy, which takes a trivial amount of leverage or a leverage only up to the level which eliminates any chance of liquidation. In equilibrium, how much portion of the ex-ante identical hedge fund managers chooses each type of strategy is endogenously determined and, as a result, the equilibrium price of the asset is derived. We demonstrate that a significant portion of funds is unnecessarily liquidated even when funding is sufficient enough to allow the calm equilibrium. Coordination failure is a driving force behind such a crisis equilibrium because if a particular fund is liquidated, the asset price plunges, which triggers a redemption request for another fund and so on.

This paper contributes to the literature on financial crisis by proposing an alternative cause. The existing studies have focused on explaining how an exogenous shock can be amplified within a financial system through a variety of frictions.[4] In contrast, our

market allows for the best opportunities. There is also ample evidence that redemption requests are induced by the performance of funds. According to the estimates of Buraschi et al. (2014), hedge funds experience sudden large outflows or forced deleveraging after experiencing 20% loss on average. Ben-David et al. (2012) find that during the 2008-2009 financial crisis, the redemption of hedge funds were three times more intense than that of mutual funds. Herein we do not intend to propose an alternative cause of redemption. Following the above mentioned theoretical papers coupled with the empirical evidence, we simply assume the redemption risk is given to the fund manager and focus on how such redemption risk may disrupt coordination among hedge funds and ultimately dislocate the market.

[2]Our main results do not change qualitatively even if a hedge fund is allowed to unwind only a part of its position to meet the redemption request and avoid the run. All we need is that the fund manager perceives a 'concern' about the possibility that the amount of redemption demanded may exceed a certain threshold. In fact, the frequent liquidation of hedge funds is an essential character of hedge fund industry; 1,057 (784) hedge funds were closed over the year 2016 (2017), 12.8% (9.4%) of the total number of hedge funds as of the year end in HFR database.

[3]Leverage plays an important role in hedge fund. Ang et al. (2011) estimate that the average gross leverage across all hedge funds from December 2004 to October 2009 is 2.1. The gross leverage increases to 4.8 after excluding equity sector which tend to use less leverage. Some funds such as fixed income arbitrage funds, show extremely large leverage, well above 30.

[4]For example, Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) among many others

model shows that a financial crisis is feasible even in the absence of an exogenous shock. The only required assumption is that hedge fund managers share a concern that a rare event *may* occur in the next period. The event itself is unsubstantial and if the event is realized, it is nothing but a self-fulfilling prophecy resulting from coordination failure among hedge funds in their response to the concern.

In addition, our model distinguishes itself from the existing models of coordination failures on the bank run, which was pioneered by Diamond and Dybvig (1983) and extended by others to incorporate the uncertainty on the fundamentals (Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), Allen and Gale (1998) and Goldstein and Pauzner (2005)). In these models, the bank runs resulting from coordination failure are caused directly by contractual linkages such as deposit contracts. In contrast, a series of fund liquidations are induced indirectly by a price mechanism in our model. As a result, the resulting crisis becomes market wide, much beyond institution-level crises analyzed in the bank run literature.

Our model also delivers some additional novel insights on the limits to arbitrage and the financial crisis. First, unlike the existing models of coordination failure, coordination failure in our model does not necessarily trigger a market crash. Upon the outbreak of coordination failure, hedge fund managers recognize only the fact that a market crash on the back of systemic redemptions *may* occur in the future. That is, in our model, a market crash is a probabilistic event, not a sure event even when the coordination failure arises.

Second and more importantly, our model implies that an arbitrage opportunity inherently accompanies the possibility of market crash. That is, they always come as a pair! In the calm equilibrium, the market price of the asset always equals its fair value and hedge fund managers earn only the fair rate of return from investment in the asset. In contrast, if the crisis equilibrium occurs, the market price deviates from its fair value and the hedge fund managers immediately recognize the occurrence of coordination failure and attempt to pounce on a resulting arbitrage opportunity. Moreover, they

show that adverse shocks can depress economic activity further through collateral channel in macroeconomics. In a context of financial markets, Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2008) study the amplification of shocks through margin constraints imposed on arbitrageurs. He and Krishnamurthy (2013) endogenize the equity constraints on financial intermediaries through moral hazard and show that the capital scarcity plays a critical part for price dynamics. In the literature on bank runs, many studies analyze how bad news on the fundamentals can be escalated into panics (e.g., Postlewaite and Vives (1987), Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988)). Global games have been also widely used in modeling the propagation of shocks through correlated signals (e.g., Morris and Shin (1998, 2000, 2004), Goldstein and Pauzner (2005), Liu and Mello (2011)).

all notice that in the next period, one of two states will occur; (i) a *good* state, in which the market price restores its fair value and (ii) a *bad* state, in which the market crashes due to redemption request from investors in the aggressively leveraged hedge funds. Which state will be realized in the next period is probabilistic. Expecting this redemption possibility, the hedge fund managers should decide their initial leverage ratios and, in turn, this demand from hedge funds determines the asset price at time zero, which is shown to deviate from the fair value. Notice that all of these results are endogenously derived in equilibrium. Therefore, our model demonstrates that a chance of market crash in the next period is a necessary and sufficient condition for the existence of arbitrage opportunity. This theoretical implication is in sharp contrast with the existing models on the limits to arbitrage, which assumes the arbitrage opportunity is exogenously given due to a fundamental shock or a demand shock. This novel feature of our model results from the fact that a single trigger, coordination failure among hedge funds, is a driving force behind the arbitrage opportunity as well as a market crash.

Third, the crisis equilibrium in our model is supported by heterogeneous strategies across ex-ante homogeneous hedge fund managers. Upon the outbreak of the crisis equilibrium, the aggressive leverage strategy speculates on arbitrage gains while taking liquidation risk. In contrast, the defensive leverage strategy safeguards against liquidation risk during the market crash and then buys the asset at a heavily discounted price with the maximum leverage. We calculate the expected payoff to the fund managers derived from each type of strategy. If one particular strategy delivers the higher expected payoff, some fund managers switch to that strategy. An overall equilibrium requires the two to have the same expected payoff and thus how much portion of the fund managers chooses each type of strategy is endogenously determined in the equilibrium.

Fourth, our model explains why *some* arbitrage trading strategies appear to 'pick up nickels in front of steamrollers.' Our model demonstrates that the crisis equilibrium is feasible only if the probability of the bad state is sufficiently small; otherwise, the crash equilibrium never occurs and only the calm equilibrium prevails. Therefore the aggressive leverage strategy resembles the steamroller analogy since it is characterized by a high probability of mediocre arbitrage gains coupled with a low probability of huge losses.

Besides the above theoretical predictions, our model also delivers some policy and empirical implications. First, our model predicts that hedge funds have a Jekyll and Hyde social role of enforcing market efficiency over a cycle of crisis. Hedge funds eliminate mispricing completely in the calm equilibrium and effectively do so even in the crisis equilibrium. In addition, we show that, in the crisis equilibrium, hedge funds are more effective in correcting mispricing than long-term funds, which is consistent with the argument of Stulz (2007) and the empirical findings by Kokkonen and Suominen (2015). This suggests that regulations such as the Volcker Rule may lead to more severe crisis by limiting bank investments in hedge funds.

Second, our model explains the behavior of hedge fund leverage empirically documented in Ang et al. (2011). They show that hedge fund leverage decreases prior to the start of the financial crisis in mid-2007. This empirical result is precisely what our model predicts. In the crisis equilibrium, coordination is disrupted and some funds choose the defensive leverage strategy in the pre-crisis period so that the overall hedge funds industry is relatively under-leveraged. Therefore, the empirical finding of Ang et al. (2011) is a 'must-be' in our model.

Our model can be considered an extension of the model proposed by Liu and Mello (2011), which investigates the redemption risk of a hedge fund through coordination failure *among hedge fund investors* in an individual hedge fund. Specifically, they assume that the investors in a particular hedge fund receive private signals about redemption requests and design the fragility of hedge fund capital through global games among hedge fund investors. Their model delivers a unique threshold equilibrium in which a hedge fund manager behaves conservatively once coordination risk is factored into their investment decisions. Thus they propose an alternative explanation for the limits to arbitrage by highlighting the investor's concerns about coordination risk. In contrast, our model assumes the redemption risk that an individual hedge fund confronts as given and rather focuses on the market-wide coordination risk among hedge funds. That is, we investigate hedge fund managers' concern about what *other fund managers* might decide to do whereas Liu and Mello (2011) explore hedge fund investors' concern about what *other investors* might decide to do. By doing so, we investigate the systematic risk of the market itself wherein the asset price in tandem with the individual fund's investment strategy is endogenously determined in equilibrium.[5] In addition, our model assumes only a dissemination of a public concern or signal, which may trigger coordination failure. Thus our model results in multiple equilibria, not a unique equilibrium unlike the model of Liu and Mello (2011).

This paper is also related to the prior studies which examine the indirect spillover through price mechanism in many other contexts, such as margin requirements (Brunnermeier and Pedersen (2008)) or performance-based fund flow (Shleifer and Vishny

[5]More broadly, our paper is related to the literature on fund flow decisions (Berk and Green (2004) and Chen et al. (2010)). We suggest an alternative underlying mechanism of fund flow solely due to the market-wide coordination failure among fund managers.

(1997)). Brunnermeier and Pedersen (2008) show that the firesale of collaterals can be induced by an increase in margin requirements requested by financiers, igniting a spiral between funding illiquidity and market illiquidity. In their model, the linkage between funding illiquidity and market illiquidity stems from the binding nature of the margin requirements of all investors. In contrast, the defensive funds discretionarily choose to be underleveraged in our model. Furthermore, because sufficient funds can be provided, the acute dry-up of liquidity during a crisis in our model is an equilibrium outcome rather than a cause for amplified shocks as in their paper.

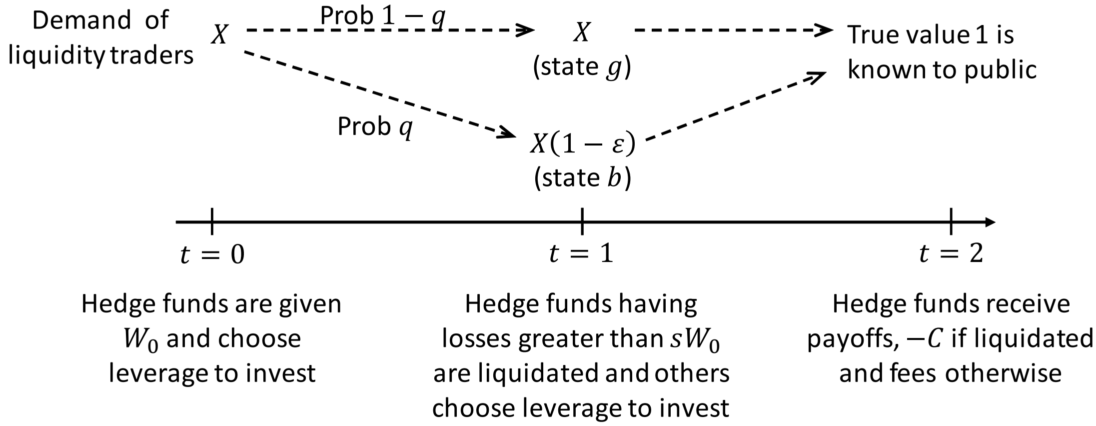
Finally, Shleifer and Vishny (1997) provide the key step that enables our model to connect coordination failure with a market-wide crisis. The resulting price dynamics in the crisis equilibrium of our model looks somewhat similar to those in their model. However, while they assume an exogenous demand shock to generate such a price dynamics, we fully endogenizes the creation and dissipation of an arbitrage opportunity through the coordination failure. Furthermore, the incentive of fund managers in our model is quite different from those in Shleifer and Vishny (1997). In their model, the potential redemption risk undermines significantly the aggressiveness of arbitrage. In contrast, the crisis equilibrium in our model demonstrates that aggressively leveraged funds coexist with the defensively leveraged funds. Besides, a key assumption underlying their model is that funding in the market is not sufficient to bring the asset price back to its fundamental value whereas mispricings in our model does not require such an assumption.

The rest of this paper proceeds as follows. Section 2 describes our model. Section 3 establishes the main result. Section 4 examines the policy implications. Section 5 concludes. All proofs are in the Appendix.

2 Setup

Our model is based on Shleifer and Vishny (1997). There are three participants: hedge fund managers, investors and long-term holders. Each investor deposits her wealth to a hedge fund at time 0 and the fund manager may invest it in a specific asset of our interest from time 0. At time 0 and 1, only fund managers know the true value of the asset, which is normalized to be 1. At time 2, everyone knows the true value and the price becomes the true value of 1. The asset supply is assumed to be 1. In our model, hedge fund managers play a role of marginal investors in setting the market price of the asset. We assume that hedge fund managers are risk-neutral. Hence, the fair rate of return equals the risk-free interest rate, which is normalized to be zero. Therefore,

Figure 1: Model Summary



in the absence of arbitrage trades, the hedge funds would earn the fair rate of return, which is zero.

Figure 1 illustrates the model setup. Long-term holders refer to those whose demand for the asset is stable over time. Examples include pension funds, insurance companies, banks and mutual funds. The demand of long-term holders for the asset is assumed to be $X < 1$ at time 0. There are two states at time 1. With probability $1-q$, the demand of long-term holders remains the same and is X. With probability q, long-term holders are hit by a shock and their demand reduces to $X(1-\varepsilon)$ for a given $\varepsilon \in [0,1]$. We call the first event a g (good) state and the latter a b (bad) state. As mentioned before, we are interested in an economy where the demand of long-term holders is stable. Hence, our main analysis is performed when a liquidity shock ε is small or even nonexistent in the b state.

Investors play the role of triggering redemption risk. There are a continuum of investors and fund managers, each with unit mass. At $t = 0$, the i-th investor, $i \in [0,1]$, deposits her wealth $W_{i,0} = W_0$ to the fund operated by the i-th fund manager.[6] The investors do not know the hedge fund's strategy but observe the net asset value of the fund at time 1. It is assumed that each investor requests redemption if the hedge fund experiences a loss greater than a given level, namely $W_0 - W_{i,1} > sW_0$ where $W_{i,1}$ is the asset value of hedge fund i (in each state) at time 1 and $s \geq 0$ is a given constant.

[6] The assumption of $W_{i,0} = W_0$ for all $i \in [0,1]$ can be relaxed. The required assumption is that $W_0 = \int_0^1 W_{i,0} di$ is well-defined and bounded. In this case, W_0 is interpreted as the ex-ante aggregate capital of hedge funds in the economy. Also, we simplify our model in the dimension of investors' strategy because our interest is on the coordination failure among fund managers. See Liu and Mello (2011) for coordination failure among investors in a single fund.

If $s=0$, investors do not tolerate any losses from hedge funds, and a larger level of s means smaller redemption risk to hedge funds.[7]

The incentive of a fund manager, who is assumed to be risk-neutral, is modeled as follows. If fund i survives till $t=2$, the i-th investor compensates the i-th fund manager and gives (i) $\alpha>0$ proportion of the gain, $\max\left(W_{i,2}-W_0,0\right)$, (performance fee) and (ii) $\beta>0$ proportion of the total fund size $W_{i,2}$ at $t=2$ (management fee). If the i-th fund is liquidated in the interim period, the i-th fund manager is penalized by C, where $C>0$. The liquidation cost, C, captures a reputation damage or a job search cost that a fund manager may face if his fund is liquidated.

Next, we characterize the strategy of a fund manager. We assume an external borrowing/lending market where the borrowing/lending rate is normalized to 0.[8] Fund managers can access this borrowing/lending market as well as invest in the asset. The strategy of fund manager i is a triplet $\left(l_{i,0},l_{i,1g},l_{i,1b}\right)$, where $l_{i,0}\in\left[-1,\bar{l}_0\right]$ is the leverage (borrowing divided by the fund capital) at $t=0$, and $l_{i,1g}$ and $l_{i,1b}\in\left[-1,\bar{l}_1\right]$ are the leverage levels in states g and b at $t=1$, respectively. A fund manager is constrained to use leverage at most $\bar{l}_0\left(\bar{l}_1\right)$ at $t=0\,(1)$ and is not allowed to take a short position on the asset.[9] We make the natural assumption that $\bar{l}_0>-1$ and $\bar{l}_1>-1$. The maximum leverage constraint can be interpreted as the margin requirements from the lender (e.g., a prime broker).[10] The demand of fund i for the asset is $W_0\left(1+l_{i,0}\right)/P_0$ at time 0, $W_{i,1g}\left(1+l_{i,1g}\right)/P_{1g}$ in state g and $W_{i,1b}\left(1+l_{i,1b}\right)/P_{1b}$ in state b if the fund survives. Here, P_0, P_{1g} and P_{1b} are the prices at time 0, in state g and in state b, respectively. The asset prices and the optimal leverage are discussed later.

The market clearing condition at $t=0$ is $1=X+\int_0^1 W_0\left(1+l_{i,0}\right)/P_0 di$, which gives

$$p_0\left(1-X\right)=W_0\int_0^1\left(1+l_{i,0}\right)di. \tag{1}$$

At $t=1$, we have

$$p_{1g}\left(1-X\right)=\int_0^1 W_{i,1g}\left(1+l_{i,1g}\right)\cdot\mathbf{1}\left(W_{i,1g}\geq W_0\left(1-s\right)\right)di, \tag{2}$$

[7]The assumption of constant s across investors is not necessary. However, the homogeneous s emphasizes that the heterogeneous behavior of fund managers in an equilibrium is not driven by the ex-ante heterogeneity of fund manager types.

[8]The most of the hedge fund leverage is provided through short-term funding. See Section 2.2 of Ang et al. (2011).

[9]The short-sale constraints are innocuous because we focus on the case that the asset is underpriced once its price deviates from its fundamental value.

[10]This assumption can be relaxed to the case that the funding rate is proportional to the leverage ratio.

where $W_{i,1g} = W_0\left(\frac{p_{1g}}{p_0}\left(1+l_{i,0}\right)-l_{i,0}\right)$, and

$$p_{1b}\left(1-X\left(1-\varepsilon\right)\right)=\int_0^1 W_{i,1b}\left(1+l_{i,1b}\right)\cdot\mathbf{1}\left(W_{i,1b}\geq W_0\left(1-s\right)\right)di, \tag{3}$$

where $W_{i,1b} = W_0\left(\frac{p_{1b}}{p_0}\left(1+l_{i,0}\right)-l_{i,0}\right)$. The asset is *mispriced* if any of p_0, p_{1g} and p_{1b} is different from the fundamental value of 1. Similarly, it is overpriced if the prices are greater than 1, and underpriced if smaller than 1.

Finally, we define an equilibrium of this economy as follows.

Definition 1. *An equilibrium of the economy is the leverage profile* $\left(l_{i,0}, l_{i,1g}, l_{i,1b}\right)_{i\in[0,1]}$ *and the price* (p_0, p_{1g}, p_{1b})*, such that*
(i) given the price (p_0, p_{1g}, p_{1b}) *and the leverage* $l_{i,0}$ *at* $t=0$*,* $l_{i,1g}$ *solves the maximization problem,*

$$\begin{aligned} U_{i,1g} = &\max_{l_{i,1g}}\left[\alpha\max\left(W_{i,2g}-W_0,0\right)+\beta W_{i,2g}\right]\cdot\mathbf{1}\left(W_{i,1g}\geq W_0\left(1-s\right)\right) \\ &-C\cdot\mathbf{1}\left(W_{i,1g}<W_0\left(1-s\right)\right) \end{aligned} \tag{4}$$

where

$$W_{i,2g} = W_{i,1g}\left(\frac{1}{p_{1g}}\left(1+l_{i,1g}\right)-l_{i,1g}\right), \text{ and} \tag{5}$$

$$W_{i,1g} = W_0\left(\frac{p_{1g}}{p_0}\left(1+l_{i,0}\right)-l_{i,0}\right), \tag{6}$$

and $l_{i,1b}$ *solves the maximization problem,*

$$\begin{aligned} U_{i,1b} = &\max_{l_{i,1b}}\left[\alpha\max\left(W_{i,2b}-W_0,0\right)+\beta W_{i,2b}\right]\cdot\mathbf{1}\left(W_{i,1b}\geq W_0\left(1-s\right)\right) \\ &-C\cdot\mathbf{1}\left(W_{i,1b}<W_0\left(1-s\right)\right) \end{aligned} \tag{7}$$

where

$$W_{i,2b} = W_{i,1b}\left(\frac{1}{p_{1b}}\left(1+l_{i,1b}\right)-l_{i,1b}\right), \tag{8}$$

$$W_{i,1b} = W_0\left(\frac{p_{1b}}{p_0}\left(1+l_{i,0}\right)-l_{i,0}\right); \tag{9}$$

(ii) given the price (p_0, p_{1g}, p_{1b})*,* $l_{i,0}$ *maximizes*

$$U_0\left(l_{i,0}\right)=\left(1-q\right)U_{i,1g}+qU_{i,1b}, \tag{10}$$

where $U_{i,1g}$ and $U_{i,1b}$ are given by (4) and (7); and
(iii) given the strategy profile $(l_{i,0}, l_{i,1g}, l_{i,1b})_{i\in[0,1]}$, the price (p_0, p_{1g}, p_{1b}) satisfies the market clearing conditions, (1)-(3).

Condition (i) gives the optimization problem of fund manager i at time 1. Given the price (p_0, p_{1g}, p_{1b}) and the leverage $l_{i,0}$ at $t = 0$, the value of fund i at $t = 1$, is evaluated by (6) in state g and (9) in state b. If $W_{i,1} < W_0(1-s)$, the i-th fund manager is penalized by the liquidation cost C. When $W_{i,1} \geq W_0(1-s)$, fund manager i maximizes his compensation $\alpha \max(W_{i,2} - W_0, 0) + \beta W_{i,2}$ where $W_{i,2}$, the final value of the i-th fund at $t = 2$, is evaluated by (5) following state g and (8) following state b. Thus, the i-th fund manager finds the optimal leverage $l_{i,1}$ at $t = 1$ by solving (4) in state g and (7) in state b.

Condition (ii) specifies the optimization problem of the i-th fund manager at $t = 0$. Note that from (10), $U_0(l_{i,0})$ is the expected utility of fund manager i at $t = 0$. Hence, the optimal leverage of the i-th fund manager at $t = 0$ is the maximizer of the expected utility. Condition (iii) states that the asset market clears.

In the next section, we provide detailed procedures for finding equilibria described in Definition 1 for an economy with parameters of our interest. Then, our main theoretical results follow.

3 Model Prediction

We show that there are two equilibria under certain conditions. In one equilibrium, a financial crisis is generated without any exogenous shock, and in the other, the fundamental value of the asset is fully reflected in the market price.

For the rest of the paper, we make the following assumption.

Assumption 1. $W_0\left(1+\bar{l}_0\right) + X \geq 1$ *and* $W_0\left(1+\bar{l}_1\right) + X(1-\varepsilon) \geq 1$.

Assumption 1 posits that funding liquidity is large enough to support the fair prices. We intentionally assume sufficient funding liquidity to emphasize that the crisis in our economy is not caused by the exogenous shortage in funding liquidity. When the asset has the fair price 1 at time 0, the maximally possible market demand equals the amount of all available funding which is $W_0\left(1+\bar{l}_0\right) + X$, and this is greater than the supply, 1, by the first condition of Assumption 1. Similarly, the second condition implies sufficient funding liquidity in both states at time 1.

Before solving for equilibria, we discuss some useful properties. First, overpricing is not possible, and therefore we focus only on underpricing.

Lemma 1. *In any equilibrium,* $p_{1g} \leq 1$*,* $p_{1b} \leq 1$ *and* $p_0 \leq 1$*.*

The intuition behind this lemma is as follows. Fund managers know that the price will be 1 eventually and do not hold an overpriced asset because short selling is not allowed. Since the long-term holders' demand is smaller than the supply, any price greater than 1 cannot be an equilibrium price.

Next, due to the following lemma, we can restrict our attention to an equilibrium where the mispricing in state b is always at least larger than that in state g.

Lemma 2. *Under Assumption 1, if* q *and* s *are sufficiently small, it holds that* $p_{1g} \geq p_{1b}$*.*

Small q means that state b is considered a rare event. Also, small s describes an asset market where redemption risk cannot be ignored.[11]

Furthermore, the following theorem provides additional restrictions on equilibrium prices.

Theorem 1. *Under Assumption 1, if* q *and* s *are sufficiently small, the followings hold:*

(i) there is no liquidation if and only if $p_0 = p_{1g} = p_{1b} = 1$*, and*

(ii) there are some liquidations if and only if $p_{1b} < p_0 < p_{1g} = 1$*.*

Because the existence and non-existence of liquidations are collectively exhaustive and mutually exclusive, the above theorem implies that there are only two possibilities of equilibria, $p_0 = p_{1g} = p_{1b} = 1$ and $p_{1b} < p_0 < p_{1g} = 1$. Accordingly, we adopt the following definitions.

Definition 2. A *calm equilibrium* is an equilibrium with $p_0 = p_{1g} = p_{1b} = 1$. A *crisis equilibrium* is an equilibrium with $p_{1b} < p_0 < p_{1g} = 1$.

When $p_0 = p_{1g} = p_{1b} = 1$, the information of the fund managers is fully reflected in the market price at $t = 0$, before the fundamental value of the two-period asset is known to the public. Because the price does not fluctuate around the fundamental value, we say the equilibrium is calm. In contrast, a crisis equilibrium generates underpricing at time 0 and allows even worse mispricing in state b.

The key feature of a crisis equilibrium is that an arbitrage opportunity ($p_0 < 1$) comes hand in hand with the risk of market crash ($p_{1b} < p_0$). Hence, a crisis equilibrium can be a double-edged sword to the fund managers. They may bet on state g aggressively at $t = 0$ by taking high leverage but will suffer a large loss from the market crash in state b. On the other hand, they may opt out and avoid the risk of the market

[11] Upper bounds for q and s are provided in (22) and (23), respectively.

crash by being defensive initially and waiting for the later opportunity. We will show that the crisis equilibrium stands at the balance of tensions between the two trading strategies.

If both calm and crisis equilibria exist, a crisis equilibrium may be considered a coordination failure. In a crisis equilibrium, a hedge fund is liquidated because other funds are liquidated and thus the asset price plummets. However, Assumption 1 guarantees sufficient funding in the market and no hedge funds need to be liquidated if no other funds are liquidated. In this sense, the crisis in our model is distinctive from the one in Brunnermeier and Pedersen (2008), in that funding liquidity or interaction between funding liquidity and market liquidity is not the reason for the crisis in our model. A crisis materializes in our economy because fund managers' incentives and redemption risk can make agents in the economy fail to coordinate their strategies to achieve a calm equilibrium. Our model is distinctive also from Diamond and Dybvig (1983) because coordination failure in our model does not necessarily trigger a market crash. Even in the crisis equilibrium where fund managers fail to coordinate on their strategies, the market crashes only in state b which occurs probabilistically.

Existence of one equilibrium is shown first.

Theorem 2. *A calm equilibrium exists if Assumption 1 holds.*

Given the existence of a calm equilibrium, we solve for a crisis equilibrium to show the existence of multiple equilibria.

3.1 Time 1

In this subsection, we solve for a crisis equilibrium at $t = 1$, where p_0 and $(l_{i,0})_{i\in[0,1]}$ are given. Then, we express $U_{i,1g}$, $U_{i,1b}$, p_{1g}, and p_{1b} as functions of p_0 and $(l_{i,0})_{i\in[0,1]}$.

We start with state g. First, we confirm that no fund is liquidated in state g.

Lemma 3. *If Assumption 1 holds and q and s are sufficiently small, no fund is liquidated in state g in any equilibrium.*

The intuition underlying the above lemma is as follows. From Lemma 2, the AUM of a hedge fund is greater in state g than state b. Hence, if a fund is liquidated in state g, it will be liquidated in state b, too. For this case, the fund manager gets the utility $-C$. Because he can always deviate to the minimum leverage -1 and avoid liquidation, this cannot occur in any equilibrium.

Furthermore, the following lemma pins down the equilibrium price in state g.

Lemma 4. *If Assumption 1 holds and q and s are sufficiently small, $p_{1g} = 1$ in any equilibrium.*

Knowing that $p_2 = 1$, a fund manager will take the maximum leverage to invest in the asset as long as $p_{1g} < 1$. Then, we see $p_{1g} = 1$ because every hedge fund survives, funding liquidity is sufficient and overpricing is not possible.

It is straightforward to compute the equilibrium utility in state g as a function of p_0 and $l_{i,0}$ because $p_{1g} = p_2 = 1$ and the AUM will not change from stage g to time 2.

Lemma 5. *Suppose Assumption 1 holds and q and s are sufficiently small. Given p_0 and $l_{i,0}$, the equilibrium utility of fund manager i in stage g is*

$$U_{i,1g} = \alpha \max\left(W_{i,1g} - W_0, 0\right) + \beta W_{i,1g}, \tag{11}$$

where $W_{i,1g} = W_0 \left(\frac{1}{p_0}\left(1 + l_{i,0}\right) - l_{i,0}\right)$.

Now, we examine state b. For state b, those who are liquidated exit the market, and only the remaining fund managers demand the asset. Because the i-th fund is liquidated if

$$W_{i,1b} < W_0 \left(1 - s\right),$$

we can define the critical leverage l^* at $t = 0$ as

$$W_{i,1b} = W_0 \left(\frac{p_{1b}}{p_0} + \left(\frac{p_{1b}}{p_0} - 1\right) l^*\right) = W_0 \left(1 - s\right).$$

This gives

$$l^* = \frac{\frac{p_{1b}}{p_0} - (1 - s)}{1 - \frac{p_{1b}}{p_0}}, \tag{12}$$

and it follows that

$$l_{i,0} \leq l^* \Leftrightarrow W_{i,1b} \geq W_0 \left(1 - s\right). \tag{13}$$

Then, we can express the i-th fund manager's compensation at $t = 2$ as

$$\begin{aligned} -C \text{ if } l_{i,0} > l^* \text{(liquidated) and} \\ \alpha \max\left(W_{i,2b} - W_0, 0\right) + \beta W_{i,2b} \text{ if } l_{i,0} \leq l^* \text{(not liquidated).} \end{aligned} \tag{14}$$

Note that the leverage decision in state b is relevant only for the fund managers who have survived in state b.

The following Lemma describes the optimal leverage $l_{i,1b}$ for fund manager i who has survived in state b.

Lemma 6. *If $p_{1b} < 1$, the optimal $l_{i,1b}$ for the i-th fund that survives in state b is $\bar{l}_1$.*

Because the asset is underpriced ($p_{1b} < 1$), a fund manager who has survived will take the maximum leverage $\bar{l}_1$.

Using this optimal leverage in state b, we can express the final wealth of hedge fund i as

$$\begin{aligned} W_{i,2b} &= W_{i,1b}\left(\frac{1}{p_{1b}}\left(1+\bar{l}_1\right)-\bar{l}_1\right) \\ &= W_0\left(\frac{p_{1b}}{p_0}\left(1+l_{i,0}\right)-l_{i,0}\right)\left(\frac{1}{p_{1b}}\left(1+\bar{l}_1\right)-\bar{l}_1\right). \end{aligned} \tag{15}$$

Combining Lemma 6 and the condition (3), we describe the equilibrium price in state b in the following proposition:

Proposition 1. *The equilibrium price in state b is determined by*

$$p_{1b} = \frac{1}{1-X\left(1-\varepsilon\right)}\int_0^1 W_{i,1b}\left(1+\bar{l}_1\right)\cdot\mathbf{1}\left(W_{i,1b}\geq W_0\left(1-s\right)\right)di. \tag{16}$$

This proposition reveals the key mechanism of mispricing amplification. When the long-term funds sell ε proportion of their holdings, it directly reduces the price as shown in $\frac{1}{1-X(1-\varepsilon)}$ of (16). Although the size of ε may be minuscule, the ultimate effect is amplified because the value of fund, $W_{i,1b} = W_0\left(\frac{p_{1b}}{p_0}\left(1+l_{i,0}\right)-l_{i,0}\right)$, is quoted by the market price of the asset. Note that $W_{i,1b}$ determines the size of the dollar demand at $t = 1$, $W_{i,1b}\left(1+\bar{l}_1\right)$, and more importantly the liquidation decision by investors, $\mathbf{1}\left(W_{i,1b}\geq W_0\left(1-s\right)\right)$. Not only is the direct impact of the decrease in p_{1b} on $W_{i,1b}$ negative, but the decrease in $W_{i,1b}$ also ignites the liquidations such that $W_{i,1b} < W_0\left(1-s\right)$, forcing fund managers to sell more, which in turn causes a further price decline and so on.

By plugging $W_{i,1b} = W_0\left(\frac{p_{1b}}{p_0}\left(1+l_{i,0}\right)-l_{i,0}\right)$ into (16), we obtain p_{1b} as an implicit function of p_0 and $(l_{i,0})_{i\in[0,1]}$:[12]

$$p_{1b} = \frac{1}{1-X\left(1-\varepsilon\right)}\int_0^1 W_0\left(\frac{p_{1b}}{p_0}\left(1+l_{i,0}\right)-l_{i,0}\right)\left(1+\bar{l}_1\right)\cdot\mathbf{1}\left(\frac{p_{1b}}{p_0}\left(1+l_{i,0}\right)-l_{i,0}>1-s\right)di. \tag{17}$$

Then, l^* defined in (12) is also an implicit function of p_0 and $(l_{i,0})_{i\in[0,1]}$.

Finally, we can express the i-th fund manager's expected utility in state b as a function of p_0 and $(l_{i,0})_{i\in[0,1]}$.

[12] We show the existence of p_{1b} in (17) given p_0 and $(l_{i,0})_{i\in[0,1]}$ with parameters of our interests in the proof of Theorems 2 and 3.

Lemma 7. *Given p_0 and $(l_{i,0})_{i\in[0,1]}$, the equilibrium utility of fund manager i in state b is*

$$U_{i,1b} = (\alpha \max(W_{i,2b} - W_0, 0) + \beta W_{i,2b}) \cdot \mathbf{1}(l_{i,0} \leq l^*) - C \cdot \mathbf{1}(l_{i,0} > l^*), \tag{18}$$

where $W_{i,2b}$ is written as a function of p_0 and $(l_{i,0})_{i\in[0,1]}$ by (15) and (17).

3.2 Time 0

This subsection analyzes a crisis equilibrium in time 0. As we have seen in the previous subsection, a high leverage decision at time 0 leads to liquidation in state b ($l_{i,0} > l^*$). As will be shown later, it turns out that some fund managers take high leverage at time 0 to be liquidated in state b and the others decide to survive. We consider the liquidation case first.

Lemma 8. *If $p_0 < p_{1g} = 1$, the optimal $l_{i,0}$ for the i-th fund which is liquidated in state b, is $\bar{l}_0$.*

If a hedge fund is to be liquidated in state b at time 1, it is optimal at time 0 for the fund to bet on state g. Because the asset price will rise in state g, the fund manager chooses to lever up his position as much as possible, that is, $l_{i,0} = \bar{l}_0$.

Resorting to Lemmas 6 and 8, we restrict our attention to the following strategy profile in finding a crisis equilibrium.[13] We do not fix the proportion h and l_* now but let them be determined endogenously in equilibrium.

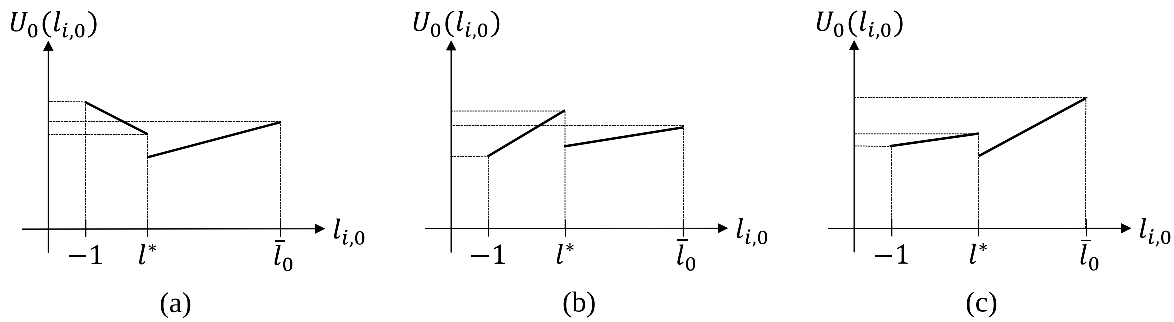
Definition 3. The *bang-bang strategy profile* refers to a strategy profile in which $h \in (0,1)$ proportion of fund managers take $l_{i,0} = \bar{l}_0 > l^*$ and the other fund managers take $(l_{j,0}, l_{j,1b}) = \left(l_*, \bar{l}_1\right)$, where $l_* = -1$ or l^*.

In the bang-bang strategy profile, the h proportion of the total fund managers use the aggressive strategy that takes the maximum leverage $\bar{l}_0 (> l^*)$ to earn high profits from $t = 0$ to $t = 1$ if state g realizes. The remaining $(1-h)$ proportion use the defensive leverage $l_* (\leq l^*)$ so that they can survive in state b to exploit the arbitrage opportunity.

The coexistence of aggressive and defensive strategies in the bang-bang strategy profile reflects that arbitrage is inherently associated with the risk of market crash. Due to the defensive strategy of some funds, the market price does not support the

[13] We do not explicitly consider the strategy in state g because it does not affect the fund manager's utility.

Figure 2: Optimal Leverage Decision



This figure shows how the i-th fund manager maximizes $U_0(l_{i,0})$ at $t=0$ by choosing the leverage at $t=0$ over the admissible range of $\left[-1,\bar{l}_0\right]$ using the piecewise linearity of $U_0(l_{i,0})$ as expressed in (19). The i-th fund manager takes -1, l^*, or $\bar{l}_0$ as the optimal leverage, as shown in panels (a), (b) and (c), respectively.

fair value initially, $p_0 < 1$, and arbitrage opportunities arise, which in turn makes the aggressive strategy attractive. Once state b is realized, investors of aggressive funds request redemption, triggering the firesale. As a result, the market crash is realized, $p_{1b} < p_0$.

The objective of fund managers at $t=0$ is to maximize $U_0(\cdot)$ in (10),

$$U_0(l_{i,0}) = (1-q)\,U_{i,1g} + qU_{i,1b}, \tag{19}$$

where $U_{i,1g}$ and $U_{i,1b}$ are now given by (11) and (18), the equilibrium utilities at $t=1$, respectively.

In the bang-bang strategy profile, we consider only $\left\{-1,l^*,\bar{l}_0\right\}$ as possible leverage at $t=0$. To show why this is justifiable intuitively, consider the case of $\alpha=0$ and $\beta=1$.[14] Then, $U_0(l_{i,0})$ in (19) becomes a piecewise linear function of $l_{i,0}$ with a discontinuity at l^*. The piecewise linearity of $U_0(l_{i,0})$ makes the optimal leverage decision at $t=0$ over the admissible range of $\left[-1,\bar{l}_0\right]$ quite simple; the optimal leverage should be found among $\left\{-1,l^*,\bar{l}_0\right\}$ as illustrated in Figure 2.

We discuss how h is determined in equilibrium. Suppose h is close to 0. Then, almost all hedge funds go defensive and wait for a bigger shock (state b) which gives an arbitrage opportunity. But survival of many hedge funds in state b pushes up the price P_{1b} and the arbitrage profit will be small. Thus the defensive strategy is less attractive

[14] Lemmas 11 and 15 provide the general case $\alpha,\beta > 0$.

and hedge funds will switch to the aggressive strategy, which increases h. On the other hand, if h is large, only a small group of hedge funds $(1-h)$ survive in state b. This implies the underpricing of P_{1b} is large and so are arbitrage profits. Then, hedge funds have incentives to go defensive, which decreases h.

In short, h is determined in equilibrium when the aggressive strategy is indifferent to the defensive strategy. This implies

$$U_0\left(l_*\right) = U_0\left(\bar{l}_0\right), \tag{20}$$

where $U_0(\cdot)$ is from (19). In addition, recall that $p_{1b} < p_0 < p_{1g}$ from Theorem 1. Then, the market clearing prices of p_0 and p_{1b} given by (1) and (3), respectively, need to satisfy

$$p_{1b} = \frac{W_{*,1b}}{1-X\left(1-\varepsilon\right)}\left(1-h\right)\left(1+\bar{l}_1\right) < p_0 = \frac{W_0}{1-X}\left(1+h\bar{l}_0+\left(1-h\right)l_*\right) < p_{1g} = 1, \tag{21}$$

where $W_{*,1b} = W_0\left(\frac{p_{1b}}{p_0}\left(1+l_*\right)-l_*\right)$. If (20) and (21) hold for some $h \in (0,1)$, a crisis equilibrium exists. The following theorem shows such h exists under some conditions.[15]

Theorem 3. *Assume that $\varepsilon > 0$. Under Assumption 1, if q, s and W_0 are sufficiently small, a crisis equilibrium exists.*

The theorem implies multiple equilibria because a calm equilibrium exists under weaker conditions by Theorem 2. A remarkable prediction of the multiple equilibria is that the asset may be underpriced and some funds are liquidated even when liquidity can be sufficiently provided in the market to support the calm equilibrium. In the crisis equilibrium, long-term holders initiates a small shock by selling a small fraction $\varepsilon > 0$ of their asset holdings, and the selling pressure ignites the liquidation of some hedge funds when some conditions are met.

The conditions for a crisis equilibrium are that the probability for state b is small, the initial capital size of hedge funds is not too large (but large enough so that the fair pricing equilibrium exists) and there is substantial risk of redemption. The required conditions are economically justifiable in the following context. First, with small q, our model intends to describe an unusual crisis rather than a normal phenomenon. Second, for small s, after such a rare event occurs, investors in panic will compete to exit the market before others and they will request redemption under a tight condition. Many existing models describe how such an amplification is triggered once a crisis is

[15]Upper bounds for parameters are provided in the proof of the theorem in the appendix.

initiated (see, e.g., Diamond and Dybvig (1983), Morris and Shin (2004), Goldstein and Pauzner (2005), Brunnermeier and Pedersen (2008) and Liu and Mello (2011)). Third, with small W_0, we stress that the asset market in our model is an example of a highly specialized market. Hence, although there is a large amount of outside capital, many potential investors are reluctant to participate, limiting the initial capital of funds. While there is a potentially large amount of capital in the economy (Assumption 1), the arbitrage trades in the asset market are heavily dependent on the short-term funding. Theorem 3 shows that, under these circumstances, mispricing coupled with the risk of a crisis may arise as a result of coordination failure.

In the next theorem, we assume $\varepsilon = 0$ and find a striking result that the selling pressure from the long-term holders is not essential for a crisis. Thus, a crisis can arise completely endogenously without any demand shock.

Theorem 4. *Assume that $\varepsilon = 0$. Under Assumption 1, if q, s and W_0 are sufficiently small, a crisis equilibrium exists.*

A calm equilibrium still exists and the theorem shows existence of multiple equilibria even when the long-term holders do not initiate any shock. It also implies that an arbitrage opportunity is spawned in tandem with the risk of market crash because the crisis equilibrium demonstrates both an arbitrage opportunity and the risk of market crash whereas the calm one does none of them.

Note that when $\varepsilon = 0$, state b is essentially identical to state g. While there is no fundamental difference between states g and b, the state realization is common knowledge. If state b turns out to be the true one, a significant mass of fund managers exit the market simultaneously only because others do so. At $t = 0$, the fund managers expect this coordination failure to happen in state b and decide their time-0 positions; Some bet on the coordination failure by going defensive and the others do the opposite. The defensive strategy reduces demand for the asset at $t = 0$ and invites an arbitrage opportunity which may not disappear quickly.

The comparison of our crisis equilibrium to the limits of arbitrage by Shleifer and Vishny (1997, SV) follows. First, note that the price in SV's model economy behaves like our price dynamics $p_{1b} < p_0 < p_{1g} = 1$ in the crisis equilibrium. However, they need to assume an exogenous demand shock dynamics to generate such a price pattern. In their model, arbitrageurs do not fully exploit the given mispricing ($p_0 < 1$) due to the possibility of the deeper mispricing ($p_{1b} < p_0$) driven by accentuated demand shocks. In contrast, our model fully endogenizes the creation and dissipation of an arbitrage opportunity. We find that coordination failure can explain how an arbitrage opportunity arises ($p_0 < 1$) as well as why it becomes more severe ($p_{1b} < p_0$) or disappears ($p_{1g} = 1$).

It is worth emphasizing that our approach to a crisis is clearly different from crises in global games in two ways. First, our model does not need exogenous demand shocks but requires only a public concern or signal so that fund managers can coordinate on it. The asset prices are determined endogenously through coordination failure and its expectation. In contrast, a standard global game requires correlated private signals through which a unique equilibrium is derived when the exogenously given prices are not affected by the behavior of the agents. Second, a crisis in our model can arise without any fundamental shocks while a crisis in a global game is ignited by bad fundamentals on real macro outcomes or economy-wide preference shocks. In our crisis equilibrium, it is hedge funds' coordination failure, not the economic fundamentals reflected in the correlated private information, that results in a mispricing.

Another distinctive feature of our model is that the crisis equilibrium is supported by heterogeneous strategies across ex-ante homogeneous hedge fund managers. Recall that mispricing in our economy exists at the balance of aggressive and defensive fund managers. In addition, as we previously explained, more (less) players with the aggressive leverage make the payoff from the aggressive strategy less (more) attractive. That is, in state b, a hedge fund has a strong incentive to survive if all the other funds are liquidated. This leads to the following lemma which is not true in bank run models.

Lemma 9. *In any equilibrium, there are some funds which survive in state b.*

In particular, a bank run in Diamond and Dybvig (1983) is triggered by the strategic complementarity of early withdrawals among depositors—if a depositor believes that other depositors will withdraw, it would be better for him or her to withdraw early. Hence, in a bank run, all depositors withdraw their funds altogether. In contrast, the key amplifier in our crisis equilibrium is the price feedback mechanism, which equalizes the attractiveness of the two strategic substitutes in the bang-bang strategy profile. The introduction of this price feedback generates a market-wide crisis through heterogeneous strategies, distinctive from an institution-level crisis of Diamond and Dybvig, Goldstein and Pauzner (2005) and Liu and Mello (2011).

Lastly, we close this section by showing that the crisis equilibrium is feasible only if the probability of the bad state is sufficiently small.

Theorem 5. *Under Assumption 1, only a calm equilibrium exists if q is sufficiently large.*

Our model provides one potential explanation of why some arbitrage trading strategies appear to 'pick up nickels in front of steamrollers.' Note that the aggressive strategy in the crisis equilibrium is characterized by a high probability of small arbitrage gains coupled with a low probability of huge losses.

4 Policy Implications

In this section, we examine policy implications of our model economy. We focus on the scenario of a crisis equilibrium to learn about the effect of a certain policy on the evolution of a given crisis. In particular, the responses to three episodes are analyzed; (i) the aggregate capital size of the fund industry, (ii) the liquidation cost on a fund hedge manager and (iii) the beliefs on the future prospect of the economy.

First, we examine the effect of the fund industry size on mispricing. Recall that W_0 is the aggregate capital size of hedge funds and X is the aggregate holdings by the long-term holders in our model economy at the beginning of time 0. We assume that before time 0, during which the asset is fairly priced, long-term holders' capital X may be transferred to hedge funds. The following theorem describes how mispricing in the crisis equilibrium responds to the capital allocations in fund industry.

Theorem 6. *Under Assumption 1, if q, s and W_0 are sufficiently small, it holds that $\frac{dp_0}{dW_0} > \frac{dp_0}{dX} > 0$ and $\frac{dp_{1b}}{dW_0} > \frac{dp_{1b}}{dX} > 0$ in a crisis equilibrium.*

We find that the hedge fund capital W_0 is more effective in alleviating the mispricing than other types of capital X. More interestingly, the hedge fund capital has a larger impact on price efficiency than the long-term holders' demand. This implies that transferring capital from long-term holders to hedge funds may induce higher efficiency. This theoretical finding supports the argument by Stulz (2007) that hedge funds can reduce mispricing more effectively than other funds. Agreeing to his view, Kokkonen and Suominen (2015) empirically demonstrate that the aggregate size of hedge funds is more important than that of mutual funds in reducing the misvaluation of U.S. individual stocks. Furthermore, this theorem suggests that regulators need to be cautious in implementing policies. For example, regardless of the necessity of regulating speculative investments through the newly adopted rules after the financial crisis such as the Volcker Rule, our model shows the possibility that it may lead to more severe crisis by limiting bank investments in hedge funds.

However, Theorem 6 needs to be interpreted carefully. As is shown in the previous section, hedge funds can destabilize a financial market because hedge funds generate a crisis equilibrium in which a crisis may arise without any exogenous shock. Theorem 6 assumes a crisis equilibrium realizes and then shows hedge funds mitigate the mispricing. When a calm equilibrium realizes, Theorem 6 is silent.

The two opposing effects of hedge funds on financial markets, as described in Theorems 4 and 6, reflect controversial views on hedge funds in literature. While Stulz (2007) and Kokkonen and Suominen (2015) take a stance that hedge funds reduce market inefficiency, Brunnermeier and Nagel (2004) and Griffin et al. (2011) suggest that hedge funds might accentuate mispricing of technology stocks in the tech bubble\burst from 1997 to 2002.[16] Ahn et al. (2018) propose this dual-effect of arbitrageurs as their inherent nature with the supporting evidence in fixed-income arbitrage markets.

Next, we examine the reaction to liquidation costs. In particular, we are interested in the monetary and non-monetary incentives such as legal damages or job search frictions, which is captured by C in our model economy. Recalling general public's criticism on that bankers or traders did not suffer much but also receive bonuses from bailout money given to financial institutions in the middle of the financial crisis,[17] we consider the effect of C in our model as a highly topical issue.

As manifested in the following theorem, our model predicts that as the liquidation cost C increases, the proportion h of aggressive hedge funds decreases and hence more hedge funds survive in a crisis (state b), pushing the price in a crisis toward the fair value. However, the pre-crisis price will diverge away from the fair value.

Theorem 7. *Under Assumption 1, if q, s and W_0 are sufficiently small, it holds that $\frac{dh}{dC} < 0$, $\frac{dp_0}{dC} < 0$ and $\frac{dp_{1b}}{dC} > 0$ in a crisis equilibrium.*

The theorem means that if the collapse of a hedge fund is costlier to the hedge fund manager, a crisis will be less severe at the cost of more severe mispricing in a pre-crisis period. If managers of liquidated funds do not suffer much but receive even bonuses during a crisis, C may be viewed as small and Theorem 7 implies that the crisis mispricing will be severe but, in a pre-crisis period, the asset price will be closer to the fair value. Another instance of regulations making C larger is to make it harder to start another hedge fund after shutting down one. Under this regulation, the market may be depressed in the pre-crisis period but the benefit from such a strict policy will realize in the case of a crisis.

[16] Khandani and Lo (2011) point out the Quant Meltdown in August 2007 was caused by the interaction among hedge funds adopting similar strategies. Ang et al. (2011) track the deleveraging process of hedge funds over the financial crisis from 2007 to 2009.

[17] For example, see the article in the following link: `https://www.nytimes.com/2009/07/31/business/31pay.html`.

Lastly, our model predicts that the aggregate leverage of hedge funds at time 0 is *lower* in a crisis equilibrium than in a calm equilibrium. Recall that $p_0 = 1$ in a calm equilibrium and $p_0 < 1$ in a crisis equilibrium. Then, in conjunction with multiple equilibria (Theorems 2-4), the market clearing condition at time 0 yields the following corollary.

Corollary 1. *The aggregate leverage of hedge funds at time 0,* $\int l_{i,0} di$, *is lower in a crisis equilibrium than in a calm equilibrium.*

This corollary predicts low leverage of hedge funds prior to a financial crisis. It implies that in a crisis equilibrium, the hedge funds do not take enough leverage to support the fair price when a financial crisis (state b) did not happen yet and no hedge fund is liquidated (time 0). Interestingly, this finding provides one of the potential causes for the empirical observations by Ang et al. (2011). They show leverage in the hedge fund industry decreases prior to the start of the financial crisis in mid-2007.[18] According to the prediction of our model, lower leverage, especially driven by some defensive hedge funds, may indicate an immediate crisis.

5 Conclusion

There are two unique characteristics of hedge funds driving the main results of our model: the redemption risk and the heavy reliance on leverage. As argued by Liu and Mello (2011), equity in hedge funds is fragile, because it can be redeemed at the request of investors, whereas equity in banks is locked in permanently. According to the empirical study of Buraschi et al. (2014), hedge funds experience large outflows of capital after experiencing 20% loss on average. In contrast, banks' capital do not show such a tendency.

In addition, leverage plays a central role in the hedge fund industry. Many hedge funds rely heavily on leverage to enhance returns on assets which would not be sufficiently high to attract funding, on an unlevered basis. Ang et al. (2011) estimate that the average gross leverage across all hedge funds from December 2004 to October 2009 is 2.1. Excluding hedging funds in equity sector which rely less on leverage, this figure increases to 4.8. Some funds such as fixed income arbitrage funds, show extremely large leverage, well above 30.[19]

[18] See Fig.4 of Ang et al. (2011).

[19] At the beginning of 1998, the leverage ratio of the LTCM was greater than 25. See Lowenstein (2000).

In this paper, we demonstrate that hedge funds can initiate a financial crisis due to these two unique features even in the absence of any exogenous shock. Such a crisis arises as a result of coordination failure in the sense that a significant mass of fund managers may exit the market simultaneously only because others do so.

In addition to the implications of our model stated in the section of introduction, we want to stress the implication of our model for a market design. The crisis-proofness of a particular security market could be associated with the relative portion of arbitragers in the market. If the security is designed to attract a larger portion of arbitragers, the security becomes rarely and less mispriced thanks to the active move of the arbitragers. However, they may initiate the crisis itself. Note that the arbitragers are extremely similar in identifying a trade opportunity and implementing it. They seize a trade opportunity when the gap between the market price and its fair value widens and unwind their positions if the gap contracts. Simply put, their investment strategies are homogeneous and lack diversity. Furthermore, to monetize the arbitrage gains, they need to act as a huddled mass. Thus, the payoff to each hedge fund is positively related to how much portion of the hedge funds take the same action. In that sense, this is a coordination game with strategic complementarities. This requirement for collective actions is destructive to the market once coordination is disrupted.[20] As such, our model delivers an important implication for the sustainability of the security market; the security should be well designed to attract demand from diverse sets of investors who have different investment strategies, different investment horizons, different redemption risk and different leverages; the viability of the market depends on the balanced composition of market participants.

[20]Chung et al. (2018) empirically investigate how the Japanese Floater (JF) market collapsed during the financial crisis. The JF market is composed mainly of foreign relative value driven hedge funds so that it is substantially lack of diversity in market participants. Even after the crisis, the market fails to resurrect.

A Proofs

We need the following lemma for the proofs below.

Lemma 10. *In any equilibrium,* $U_0(l_{i,0})$ *in* (10) *satisfies that*

$$U_0(l_{i,0}) \geq \beta W_0$$

for any $i \in [0,1]$.

Proof Note that any fund manager can choose the strategy of the minimum leverage, $l_{i,0} = l_{i,1g} = l_{i,1b} = -1$, yielding $W_{i,2g} = W_{i,2b} = W_0$, which in turn implies that $U_{i,1g} = U_{i,1b} = \beta W_0$. Hence, for $U_0(l_{i,0})$ with the optimal leverage $l_{i,0}$, it holds that

$$U_0(l_{i,0}) \geq (1-q)\beta W_0 + q\beta W_0 = \beta W_0,$$

proving the lemma. $\square$

Proof of Lemma 1 Assume that $p_{1b} > 1$. Since p_{1b} is positive, some fund managers have survived in state b. (Otherwise, $p_{1b} = 0$ by (3).) Suppose the i-th fund manager is one of them. He knows that for sure the price will decrease from state b in $t = 1$ to $t = 2$. Since the i-th fund manager's compensation, $\alpha \max(W_{i,2b} - W_0, 0) + \beta W_{i,2b}$, is strictly increasing in $W_{i,2b}$ and, from (8), $W_{i,2b}$ is strictly decreasing in $l_{i,1b}$ by the assumption that $p_{1b} > 1$, he will take $l_{i,1b} = -1$, the lowest leverage. This optimal leverage is applied to every manager who has survived, implying $p_{1b} = 0$ by (3). This contradicts $p_{1b} > 1$. Hence, $p_{1b} \leq 1$.

Assume that $p_{1g} > 1$. From the identical logic above, it leads to a contradiction. Hence, $p_{1g} \leq 1$.

Assume that $p_0 > 1$. Since $p_{1g} \leq 1$ and $p_{1b} \leq 1$, the i-th fund manager knows that the price will decrease from $t = 0$ to both states in $t = 1$. Note that if $l_{i,0} > -1$, by lowering $l_{i,0}$, he can strictly increase both $W_{i,1g}$ and $W_{i,2b}$, in turn strictly increasing his expected utility in (10). Hence, he will take the lowest leverage, -1. This leverage choice is applied to all fund managers at $t = 0$, implying $p_0 = 0$ by (1). This contradicts $p_0 > 1$. Hence, $p_0 \leq 1$. $\square$

Proof of Theorem 2 Consider a strategy profile that all fund managers take the leverage of $l_{i,0} = l_{i,1g} = \frac{1-X}{W_0} - 1 \leq \min\left(\bar{l}_0, \bar{l}_1\right)$ and $l_{i,1b} = \frac{1-X(1-\varepsilon)}{W_0} - 1 \leq \bar{l}_1$. Given this strategy profile, the market clearing conditions (1)-(3) imply $p_0 = p_{1g} = p_{1b} = 1$. Note that the i-th fund manager's investment decision does not affect the prices. Plugging $p_0 = p_{1g} = p_{1b} = 1$ into (10) gives

$$U_0 \left(l_{i,0}\right) = \left(1 - q\right) \beta W_0 + q\beta W_0 = \beta W_0,$$

which does not depend on the i-th fund manager's decision. Thus, he does not have an incentive to deviate from his strategy. Therefore, we have a calm equilibrium. This completes the proof. □

Proof of Lemma 2 First, assume that q and s are small enough so that

$$q < \min \left(\frac{1}{2}, \frac{1 + \bar{l}_1}{2 + \bar{l}_0 + \bar{l}_1} \right) \tag{22}$$

and

$$s < 1 - \frac{\left(1 + \bar{l}_1\right) W_0}{1 - X}. \tag{23}$$

Assume $p_{1b} > p_{1g}$ to show contradiction in the following steps.

Step 1. $p_{1g} < 1$: This follows from $p_{1b} \leq 1$ (Lemma 1) and $p_{1g} < p_{1b}$ (assumption).

Step 2. There exists fund i which is liquidated in state g: Because $p_{1g} < 1$ and $p_2 = 1$, $W_{i,2g} = W_{i,1g} \left(\frac{p_2}{p_{1g}} \left(1 + l_{i,1g}\right) - l_{i,1g} \right)$ is strictly increasing in $l_{i,1g}$, implying that any fund that has survived in state g takes $l_{i,1g} = \bar{l}_1$. Assume that there does not exists any liquidation in state g. Then, it follows that

$$p_{1g} \left(1 - X\right) = \left(1 + \bar{l}_1\right) \int_0^1 W_{i,1g} di \geq \left(1 + \bar{l}_1\right) W_0 \left(1 - s\right) > \left(1 - X\right),$$

where the first equality is due to the assumption of no liquidation, the first inequality is due to the redemption rule and the last inequality is from (23). Hence, it holds that $p_{1g} \left(1 - X\right) > 1 - X$, which contradicts $p_{1g} < 1$. Thus, there should exists some liquidation in state g.

Step 3. The fund i in Step 1 survives in state b: If the fund i in Step 2 is liquidated also in state b, the payoff to the fund manager is $-C$, a contradiction to Lemma 10.

Step 4. There is no liquidation in state b: Assume that there exists fund j which is liquidated in state b. Then, the fund j is also liquidated in state g because $W_{j,1g} = \left(W_0 \frac{p_{1g}}{p_0} \left(1 + l_{i,0}\right) - W_0 l_{i,0} \right) \leq W_{j,1b} = \left(W_0 \frac{p_{1b}}{p_0} \left(1 + l_{i,0}\right) - W_0 l_{i,0} \right) \leq W_0 \left(1 - s\right)$. This contradicts to Steps 2 and 3.

Step 5. $p_{1b} = 1$: From Lemma 1, it suffices to show that $p_{1b} < 1$ leads to a contradiction. Assume that $p_{1b} < 1$. Because $p_{1b} < 1$ and $p_2 = 1$, $W_{i,2b} =$

$W_{i,1b}\left(\frac{p_2}{p_{1b}}\left(1+l_{i,1b}\right)-l_{i,1b}\right)$ is strictly increasing in $l_{i,1b}$, implying that any fund that has survived in state b takes $l_{i,1b}=\bar{l}_1$. By Step 4, there is no liquidation in state b. Then, it follows that

$$p_{1b}\left(1-X\right)=\left(1+\bar{l}_1\right)\int_0^1 W_{i,1b}di\geq\left(1+\bar{l}_1\right)W_0\left(1-s\right)>\left(1-X\right),$$

where the first equality is due to Step 4, the first inequality is due to the redemption rule and the last inequality is from (23). It gives $p_{1b}\left(1-X\right)>\left(1-X\right)$, a contradiction.

Step 6. $p_{1g}<p_0<p_{1b}=1$: If $p_0\leq p_{1g}$, there is no liquidation in state g, contradicting Step 2. Hence, $p_0>p_{1g}$. If $p_0=p_{1b}=1$, $U_{i,2g}=U_{i,1g}=-C$ and $U_{i,2b}=U_{i,1b}=\beta W_0$ for the fund i in Step 2. Hence, the i-th fund manager has an incentive to deviate to $l_{i,0}=-1$, which guarantees $U_{i,1g}=U_{i,1b}=\beta W_0$. Hence, $p_0<1$.

Step 7. For the fund i in Step 2, $l_{i,0}=\bar{l}_0$: Because $U_{i,2g}=U_{i,1g}=-C$, the i-th fund manager in Step 2 maximizes only $W_{i,2b}=W_{i,1b}=W_0\left(\frac{p_{1b}}{p_0}\left(1+l_{i,0}\right)-l_{i,0}\right)$, which is strictly increasing in $l_{i,0}$ when $p_0<p_{1b}=1$ (Step 6).

Step 8. The lemma holds: We show that Step 7 induces a contradiction. With $l_{i,0}=\bar{l}_0$, the fund i in Step 2 has the expected utility,

$$\widetilde{U}=-qC+(1-q)\left(\alpha\left(W_0\frac{1}{p_0}\left(1+\bar{l}_0\right)-W_0\bar{l}_0-W_0\right)+\beta\left(W_0\frac{1}{p_0}\left(1+\bar{l}_0\right)-W_0\bar{l}_0\right)\right).$$

Consider an alternative strategy $l_{i,0}=-1$, $l_{i,1g}=\bar{l}_1$, which yields the expected utility

$$\widehat{U}=q\left(\alpha\left(W_0\frac{1}{p_{1g}}\left(1+\bar{l}_1\right)-W_0\bar{l}_1-W_0\right)+\beta\left(W_0\frac{1}{p}\left(1+\bar{l}_1\right)-W_0\bar{l}_1\right)\right)+(1-q)\,\beta W_0.$$

Some algebras show that the two inequalities $p_{1g}<p_0$ (Step 6) and (22) imply $\widehat{U}>\widetilde{U}$, contradicting the optimality of $l_{i,0}=\bar{l}_0$ (Step 7). □

Proof of Lemma 3 Assume that fund i is liquidated in state g, $W_0\left(1-s\right)>W_{i,1g}$. From Lemma 2,

$$W_{i,1g}=W_0\left(\frac{p_{1g}}{p_0}\left(1+l_{i,0}\right)-l_{i,0}\right)\geq W_0\left(\frac{p_{1b}}{p_0}\left(1+l_{i,0}\right)-l_{i,0}\right)=W_{i,1b}.$$

Hence, the i-th fund is liquidated in both states, resulting in $U_0\left(l_{i,0}\right)=-C$. This contradicts Lemma 10. Hence, the i-th fund is not liquidated in state b. □

Proof of Lemma 4 Assume $p_{1g} \neq 1$ by contradiction. Lemma 1 implies $p_{1g} < 1$. Pick an arbitrary fund i. Recall that fund i is not liquidated in state g by Lemma 3, i.e., $W_{i,1g} \geq W_0 (1-s)$. Then, $U_{i,1g}$ in (4) is a strictly increasing function of $W_{i,2g}$. Furthermore, with $p_{1g} < 1$, $W_{i,2g}$, given by (5), strictly increases with $l_{i,1g}$, which in turn leads to $l_{i,1g} = \bar{l}_1$ for all $i \in [0,1]$. Because no fund is liquidated in state g by Lemma 3 and the fair pricing is not supported, we have that

$$\int_0^1 W_0 \left(\frac{1}{p_0} (1 + l_{i,0}) - l_{i,0} \right) \left(1 + \bar{l}_1 \right) di < 1 - X.$$

With the relationship $\int_0^1 l_{i,0} di = \frac{p_0(1-X)}{W_0} - 1$ by (1), the inequality above can be written as

$$((1 - p_0)(1 - X) + W_0) \left(1 + \bar{l}_1 \right) < 1 - X. \tag{24}$$

Then,

$$1 - X < W_0 \left(1 + \bar{l}_1 \right) \leq ((1 - p_0)(1 - X) + W_0) \left(1 + \bar{l}_1 \right) < 1 - X.$$

The first inequality holds by Assumption 1, the second inequality is due to Lemma 1 and the last inequality is from (24). This is a contradiction and we have proved $p_{1g} = 1$. $\square$

Proof of Lemma 5 Because $p_{1g} = 1$ (Theorem 1) and $p_2 = 1$, the i-th fund manager knows that the fund size will not change at $t = 2$. Hence, $W_{i,2g} = W_{i,1g}$, implying that

$$U_{i,1g} = \alpha \max (W_{i,2g} - W_0, 0) + \beta W_{i,2g} = \alpha \max (W_{i,1g} - W_0, 0) + \beta W_{i,1g},$$

where $W_{i,1g} = W_0 \left(\frac{1}{p_0} (1 + l_{i,0}) - l_{i,0} \right)$. $\square$

Proof of Lemma 6 Note that $U_{i,1b} = \alpha \max (W_{i,2b} - W_0, 0) + \beta W_{i,2b}$ from (7) is strictly increasing in $W_{i,2b}$ and that $W_{i,2b} = W_{i,1b} \left(\left(\frac{1}{p_{1b}} - 1 \right) l_{i,1b} + \frac{1}{p_{1b}} \right)$ is strictly increasing in $l_{i,1b}$ when $p_{1b} < 1$. Hence, the i-th fund which survives in state b will take the optimal leverage of $\bar{l}_1$, proving the lemma. $\square$

Proof of Lemma 8 For the i-th fund to be liquidated in state b, it holds that $W_{i,1b} < W_0 (1-s)$, implying that $U_{i,1b} = -C$. Note that $p_{1g} = 1$ from Lemma 4. Then, the i-th fund manger's expected utility (10) can be written as follows:

$$\begin{aligned} U_0(l_{i,0}) &= (1-q) U_{i,1g} + qU_{i,1b} \\ &= (1-q)(\alpha \max(W_{i,2g} - W_0, 0) + \beta W_{i,2g}) - qC \\ &= (1-q)((\alpha+\beta) W_{i,2g} - \alpha W_0) - qC \\ &= (1-q)\left((\alpha+\beta) W_0 \left(\frac{1}{p_0} - 1\right)(1 + l_{i,0}) + \beta W_0\right) - qC \end{aligned}$$

where the third and fourth equalities hold because $W_{i,2g} = W_{i,1g} = W_0 \left(\frac{1}{p_0}(1+l_{i,0}) - l_{i,0}\right) > W_0$. Note that the above is strictly increasing in $l_{i,0}$ with $p_0 < 1$, proving the lemma. □

Proof of Lemma 9 Assume that no fund survives in state b in some equilibria. Consider two cases of (i) $p_{1g} = 0$ and (ii) $p_{1g} > 0$.

Step 1. $p_0 > 0$: From (1), $p_0 = 0$ implies almost every fund takes $l_{i,0} = -1$, no position in the asset at $t = 0$. This contradicts the assumption that no fund survives in state b. Hence, $p_0 > 0$.

Step 2. $p_{1b} = 0$: Since no fund survives in state b, this is implied by (3).

Step 3. $U_{i,1b} = \infty$ if the i-th fund manager survives: Note that for any $l_{i,1b} > -1$, (8) implies $W_{i,2b} = \infty$ by Step 2 and hence $U_{i,1b} = \infty$ from (7).

Step 4. If $p_{1g} = 0$, every fund is liquidated in state g : Assume that some funds survive in state g. That is, there exists a fund i such that $W_{i,1g} \geq W_0 (1-s)$. Fund i maximizes the utility of (4), which is a strictly increasing in $W_{i,2}$ given by (5). Because $W_{i,2g}$ is infinite with any $l_{i,1b} > -1$, there should be a strictly positive demand. This contradicts $p_{1g} = 0$. Hence, no fund survives in state g.

Step 5. The lemma holds: If $p_{1g} = 0$, $U_0(l_{i,0}) = -C$ for all $i \in [0,1]$ because every fund is liquidated in state g (Step 4) and in state b (assumption). If $p_{1g} > 0$, $U_0(l_{i,0})$ is finite for all $i \in [0,1]$ in the equilibrium because $p_{1g} > 0$ (assumption), $p_0 > 0$ (Step 1) and every fund is liquidated in state b. Hence, for any nonnegative p_{1g}, $U_0(l_{i,0})$ is finite for all $i \in [0,1]$. However, if the i-th fund manager deviates and takes $l_{i,0} = -1$, he will survive in state b, which gives $U_0(-1) = (1-q) U_{i,1g} + qU_{i,1b} \geq -(1-q) C + qU_{i,1b} = \infty$ by Step 3. This is a contradiction to the assumption that no fund survives in state b. Hence, the lemma holds. □

Proof of Theorem 1 (i) $\Rightarrow$: Consider an equilibrium without liquidation. It will be verified that $p_0 = p_{1b} = p_{1g} = 1$. From Lemma 4, $p_{1g} = 1$.

We show that $p_{1b} = 1$. Assume $p_{1b} \neq 1$ by contradiction. Lemma 1 implies $p_{1b} < 1$, which in turn leads to $l_{i,0} = \bar{l}_1$ from Lemma 6. Hence, we have that

$$\int_0^1 W_0 \left(\frac{1}{p_0} (1 + l_{i,0}) - l_{i,0} \right) \left(1 + \bar{l}_1\right) di < 1 - X (1 - \varepsilon).$$

Because $\int_0^1 l_{i,0} di = \frac{p_0(1-X)}{W_0} - 1$ by (1), the inequality can be written as

$$((1 - p_0)(1 - X) + W_0) \left(1 + \bar{l}_1\right) < 1 - X (1 - \varepsilon). \tag{25}$$

Then,

$$1 - X (1 - \varepsilon) \leq W_0 \left(1 + \bar{l}_1\right) \leq ((1 - p_0)(1 - X) + W_0) \left(1 + \bar{l}_1\right) < 1 - X (1 - \varepsilon).$$

The first inequality holds by Assumption 1, the second inequality is due to Lemma 1 and the last inequality is from (25). This is a contradiction and we have proved $p_{1b} = 1$.

Lastly, we show $p_0 = 1$. Using Lemma 1, assume $p_0 < 1$ by contradiction. Then, by (1) and Assumption 1, $\int_0^1 (1 + l_{i,0})\, di = \frac{p_0(1-X)}{W_0} < \frac{1-X}{W_0} \leq \left(1 + \bar{l}_0\right)$. Because $l_{i,0} \leq \bar{l}_0$ for all $i \in [0, 1]$, it should hold that $l_{i,0} < \bar{l}_0$ for some $i \in [0, 1]$. Fix this i. Recalling that $p_{1b} = p_{1g} = 1$, implying no liquidation at $t = 1$, we compute (10), the expected utility of fund manager i. Some algebras give

$$U_0 (l_{i,0}) = (\alpha + \beta) W_0 \left(\frac{1}{p_0} (1 + l_{i,0}) - l_{i,0} \right) - \alpha W_0.$$

Because $p_0 < 1$, from Lemma 8, $l_{i,0} = \bar{l}_0$ is optimal. This is a contradiction, which proves $p_0 = 1$.

(i) $\Leftarrow$: Because $p_0 = p_{1g} = p_{1b}$, it holds that $W_{i,1b} = W_{i,1g} = W_0 \geq W_0 (1 - s)$, implying that there is no liquidation.

(ii) $\Rightarrow$: Consider an equilibrium where some funds are liquidated. Let the i-th fund be one of the liquidated funds. We will show that $p_{1b} < p_0 < p_{1g} = 1$. From Lemma 4, $p_{1g} = 1$. Hence, it suffices to show that $p_{1b} < p_0 < 1$.

We compute his expected utility at time 0 defined in (10). From Lemma 3, fund i is liquidated only in state b. Because $U_{i,1b} = -C$, the expected utility of the i-th fund manager is expressed as:

$$U_0 (l_{i,0}) = (1 - q) (\alpha \max (W_{i,2g} - W_0, 0) + \beta W_{i,2g}) - qC \geq \beta W_0,$$

where the last inequality is from Lemma 10. The inequality implies $W_{i,2g} > W_0$ because otherwise the inequality does not hold. Furthermore, because $p_{1g} = 1$ implies $W_{i,2g} = W_{i,1g}$, we have that

$$W_0 < W_{i,2g} = W_{i,1g} = W_0 \left(\frac{1}{p_0} (1 + l_{i,0}) - l_{i,0} \right),$$

implying that

$$(1 + l_{i,0}) \left(\frac{1}{p_0} - 1 \right) > 0.$$

Because $l_{i,0} \geq -1$,

$$p_0 < 1. \tag{26}$$

Furthermore, if $p_{1b} \geq p_0$, no fund will face a loss in state b, contradicting the assumption that the i-th fund is liquidated. Hence, it holds that

$$p_{1b} < p_0. \tag{27}$$

From (26) and (27), we have that $p_{1b} < p_0 < 1$. With Lemma 4, this completes the proof.

(ii) $\Leftarrow$: This is implied by the contrapositive of (i) $\Rightarrow$. □

We need following lemmas to prove Theorems 3 and 4. Recall that $l^* = \frac{\frac{p_{1b}}{p_0} - (1-s)}{1 - \frac{p_{1b}}{p_0}}$ and fund manager i is not liquidated if $l_{i,0} \leq l^*$.

Lemma 11. *Under Assumptions 1-2 and for a sufficiently small s or $s = 0$, any equilibrium satisfies*

$$\max \left(W_{i,2g} - W_0, 0 \right) = W_{i,2g} - W_0 \textit{ for any } l_{i,0} \in \left[-1, \bar{l}_0 \right] \text{ and}$$
$$\max \left(W_{i,2b} - W_0, 0 \right) = W_{i,2b} - W_0 \textit{ for any } l_{i,0} \in \left[-1, l^* \right].$$

Proof We prove the lemma in two parts by classifying equilibria into those without any liquidation and those with some liquidation.

For the rest of the proof, we assume that

$$0 \leq s < \frac{1}{\left(\frac{\alpha+\beta}{\beta + \frac{C}{W_0}} \right) \left(\frac{1-q}{q} \right) \left(\frac{1+\bar{l}_0}{1+\bar{l}_1} \right) + 1}, \tag{28}$$

implying that

$$\frac{1 + \bar{l}_0}{\left(\frac{\beta + \frac{C}{W_0}}{\alpha+\beta} \right) \frac{q}{1-q} + 1 + \bar{l}_0} < \frac{1 + \bar{l}_1}{\frac{s}{1-s} + 1 + \bar{l}_1}. \tag{29}$$

Clearly, the RHS of (28) is positive.

First, consider an equilibrium where no fund is liquidated. Theorem 1 shows that $p_0 = p_{1g} = p_{1b} = 1$, implying $W_{i,2g} = W_{i,2b} = W_0$ for any $l_{i,0} \in \left[-1, \bar{l}_0\right]$. Hence, the two equations in the lemma are valid.

Second, consider an equilibrium where some funds are liquidated. Due to Lemma 3, those funds are liquidated only in state b. From Theorem 1, $p_0 < p_{1g} = 1$. Hence, for any $i \in [0,1]$ and $l_{i,0} \in \left[-1, \bar{l}_0\right]$,

$$W_{i,1g} = W_0 \left(\frac{1}{p_0} (1 + l_{i,0}) - l_{i,0} \right) = W_0 \left(\frac{1}{p_0} - 1 \right) (1 + l_{i,0}) + W_0 \geq W_0,$$

proving the first equation in Lemma 11.

Turn to the second equation in the lemma. By assumption, some funds are liquidated, and by Lemma 9, some funds survive in state b. Also, from Theorem 1, it holds that $p_{1b} < p_0 < p_{1g} = 1$. Take any $i, j \in [0,1]$ such that the i-th fund is liquidated and the j-th fund survives in state b. Then, $l_{j,0} \in [-1, l^*]$ and it suffices to show that $W_{j,2b} \geq W_0$.

Because fund manager j survives in state b, $W_{j,1b} \geq (1-s)\, W_0$. Also from Lemma 6, $l_{j,1b} = \bar{l}_1$ is the optimal choice and hence we need to prove that

$$(1-s) \left(\frac{1}{p_{1b}} \left(1 + \bar{l}_1\right) - \bar{l}_1 \right) \geq 1, \tag{30}$$

which implies that $W_{j,2b} = W_{j,1b} \left(\frac{1}{p_{1b}} \left(1 + \bar{l}_1\right) - \bar{l}_1 \right) \geq W_0$.

For the i-th fund manager which is liquidated, $l_{i,0} = \bar{l}_0$ from Lemma 8. Furthermore, Lemma 10 implies

$$U_0 \left(l_{i,0}\right) = (1-q) \left((\alpha + \beta)\, W_0 \left(\frac{1}{p_0} - 1 \right) \left(1 + \bar{l}_0\right) + \beta W_0 \right) - qC \geq \beta W_0.$$

Rearranging the terms in the inequality, we have

$$p_0 \leq \frac{1 + \bar{l}_0}{\left(\frac{\beta + \frac{C}{W_0}}{\alpha + \beta} \right) \frac{q}{(\alpha+\beta)(1-q)} + 1 + \bar{l}_0}. \tag{31}$$

Then, combining (29), (31) and Theorem 1 yields

$$p_{1b} < \frac{1 + \bar{l}_1}{\frac{s}{1-s} + 1 + \bar{l}_1}.$$

This implies (30) and the proof is completed. □

Lemma 12. *If*

$$W_0 < \overline{W}_0 \equiv \frac{1 - X\left(1-\varepsilon\right)}{1+\bar{l}_1} + \frac{1-X}{1+\bar{l}_0}, \tag{32}$$

it holds that

$$\frac{1}{1-X\left(1-\varepsilon\right)}\left(1-h\right)W_0\left(1+\bar{l}_1\right) < \frac{1}{1-X}hW_0\left(1+\bar{l}_0\right) < 1$$

for any $h \in \left[\underline{h}, \overline{h}\right]$ *where* $\underline{h}$ *and* $\overline{h}$ *are given by*

$$\underline{h} \equiv \frac{\frac{1-X}{1-X(1-\varepsilon)}\left(\frac{1+\bar{l}_1}{1+\bar{l}_0}\right)}{1+\frac{1-X}{1-X(1-\varepsilon)}\left(\frac{1+\bar{l}_1}{1+\bar{l}_0}\right)} > 0 \tag{33}$$

and

$$\overline{h} \equiv \frac{1-X}{W_0\left(1+\bar{l}_0\right)}. \tag{34}$$

Moreover, $\underline{h} < \overline{h}$.

Proof Note that

$$\frac{1}{1-X\left(1-\varepsilon\right)}\left(1-h\right)W_0\left(1+\bar{l}_1\right) < \frac{1}{1-X}hW_0\left(1+\bar{l}_0\right)$$

if and only if

$$h > \frac{\frac{1-X}{1-X(1-\varepsilon)}\left(\frac{1+\bar{l}_1}{1+\bar{l}_0}\right)}{1+\frac{1-X}{1-X(1-\varepsilon)}\left(\frac{1+\bar{l}_1}{1+\bar{l}_0}\right)} = \underline{h}.$$

Also, $\frac{1}{1-X}hW_0\left(1+\bar{l}_0\right) < 1$ if and only if

$$h < \frac{1-X}{W_0\left(1+\bar{l}_0\right)} = \overline{h}. \tag{35}$$

Lastly, from (32),

$$W_0 < \frac{1 - X\left(1-\varepsilon\right)}{1+\bar{l}_1} + \frac{1-X}{1+\bar{l}_0},$$

implying

$$\underline{h} = \frac{\frac{1-X}{1-X(1-\varepsilon)}\left(\frac{1+\bar{l}_1}{1+\bar{l}_0}\right)}{1+\frac{1-X}{1-X(1-\varepsilon)}\left(\frac{1+\bar{l}_1}{1+\bar{l}_0}\right)} < \frac{1-X}{W_0\left(1+\bar{l}_0\right)} = \overline{h}.$$

Hence, the interval of $\left(\underline{h}, \overline{h}\right)$ is well-defined, which completes the proof. $\square$

Lemma 13. *Assume that $W_0 < \overline{W}_0$ where $\overline{W}_0$ is defined in (32). Then, it holds that*

$$\overline{q} \equiv \min \frac{\left(\frac{1}{W_0}\left(\frac{1-X(1-\varepsilon)}{1+\bar{l}_1} + \frac{1-X}{1+\bar{l}_0}\right) - 1\right)\left(\bar{l}_0 + 1\right)}{\left(\frac{1}{W_0}\left(\frac{1-X(1-\varepsilon)}{1+\bar{l}_1} + \frac{1-X}{1+\bar{l}_0}\right) - 1\right)\left(2 + \bar{l}_0 + \bar{l}_1\right) + \frac{\frac{C}{W_0}+\beta}{\alpha+\beta}} > 0. \tag{36}$$

In addition, if $q < \overline{q}$, the following holds:

$$q\left(\frac{1}{g\left(\underline{h}\right)}\left(1+\bar{l}_1\right) - \bar{l}_1 + \frac{\frac{C}{W_0}+\beta}{\alpha+\beta}\right) - (1-q)\left(\frac{1}{f\left(\underline{h}\right)} - 1\right)\left(\bar{l}_0 + 1\right) < 0,$$

where $f(h) = \frac{1}{1-X} h W_0 \left(1+\bar{l}_0\right)$, $g(h) = \frac{1}{1-X(1-\varepsilon)}(1-h) W_0 \left(1+\bar{l}_1\right)$, and $\underline{h}$ is given in (33).

Proof First, we show that $\overline{q}$ in (36) is strictly positive. Because $W_0 < \overline{W}_0 = \frac{1-X(1-\varepsilon)}{1+\bar{l}_1} + \frac{1-X}{1+\bar{l}_0}$ from (32), it follows that

$$\frac{1}{W_0}\left(\frac{1-X(1-\varepsilon)}{1+\bar{l}_1} + \frac{1-X}{1+\bar{l}_0}\right) > 1, \tag{37}$$

which implies $\overline{q}$ in (36) is strictly positive.

Next, turn to the latter inequality. Some algebras give

$$\begin{aligned}
& q\left(\frac{1}{f\left(\underline{h}\right)}\left(1+\bar{l}_1\right) - \bar{l}_1 + \frac{\frac{C}{W_0}-\alpha}{\alpha+\beta}\right) - (1-q)\left(\frac{1}{g\left(\underline{h}\right)} - 1\right)\left(\bar{l}_0 + 1\right) \\
=& q\left[(\lambda-1)\left(2+\bar{l}_0+\bar{l}_1\right) + \frac{\frac{C}{W_0}+\beta}{\alpha+\beta}\right] + (\lambda-1)\left(\bar{l}_0+1\right) \\
<& \overline{q}\left[(\lambda-1)\left(2+\bar{l}_0+\bar{l}_1\right) + \frac{\frac{C}{W_0}+\beta}{\alpha+\beta}\right] + (\lambda-1)\left(\bar{l}_0+1\right) = 0,
\end{aligned}$$

where $\lambda = \frac{1}{W_0}\left(\frac{1-X(1-\varepsilon)}{1+\bar{l}_1} + \frac{1-X}{1+\bar{l}_0}\right) > 1$ and the last inequality and equality are from (37). This completes the proof of the second claim in the lemma. $\square$

Lemma 14. *Suppose f, g and v be functions on $(0,1)\times[0,\infty)$ that satisfy*

$$\begin{aligned}
f(h,s) &= \frac{1}{1-X} W_0 \left(1 + h\bar{l}_0 + (1-h)\, v(h,s)\right) \\
g(h,s) &= \frac{1}{1-X(1-\varepsilon)} W_0 (1-h)(1-s)\left(1+\bar{l}_1\right) \text{ and} \\
v(h,s) &= \frac{\frac{g(h,s)}{f(h,s)} - (1-s)}{1 - \frac{g(h,s)}{f(h,s)}}.
\end{aligned}$$

If $0 < g(h_0,0) < f(h_0,0) < 1$, then $f(h,s)$ and $g(h,s)$ are continuously differentiable and $0 < g(h,s) < f(h,s) < 1$ in a neighborhood of $(h_0,0)$.

Proof It is obvious that $g(h,s)$ is continuously differentiable. Plugging $v(h,s)$ to $f(h,s)$ yields the equation,

$$f(h,s) = \frac{W_0}{1-X}\left(1 + h\bar{l}_0 + (1-h)\frac{f(h,s)(1-s) - g(h,s)}{g(h,s) - f(h,s)}\right).$$

Under the assumption that $f(h,s) > g(h,s)$, $f(h,s)$ has a unique solution,

$$f(h,s) = \frac{B + \sqrt{B^2 - \frac{4W_0}{1-X}h\left(1+\bar{l}_0\right)g(h,s)}}{2}, \tag{38}$$

where $B = \frac{W_0}{1-X}\left((1-h)s + h\left(1+\bar{l}_0\right)\right) + g(h,s)$. (In fact, some algebras show that $B^2 - \frac{4W_0}{1-X}h\left(1+\bar{l}_0\right)g(h,s) \geq 0$ for any $s \geq 0$.) Hence, the expression (38) along with the continuously differentiability of $g(h,s)$ confirms that $f(h,s)$ is continuously differentiable in a neighborhood of $(h_0, 0)$. Also, the inequalities $0 < g(h,s) < f(h,s) < 1$ follow from the continuity of $f(h,s)$ and $g(h,s)$ and the inequalities $0 < g(h_0,0) < f(h_0,0) < 1$. □

Lemma 15. *Assume that s is sufficiently small. Given p_0 and p_{1b}, the expected utility of the i-th fund manger at $t=0$ can be written as*

$$U(l_{i,0}) = (\delta_0 + \delta_1 l_{i,0}) \cdot \mathbf{1}(l_{i,0} \leq l^*) + (\gamma_0 + \gamma_1 l_{i,0}) \cdot \mathbf{1}(l_{i,0} > l^*), \tag{39}$$

where l^* is given in (12),

$$\delta_0 = W_0\left[(1-q)\frac{1}{p_0} + q\left(\frac{1}{p_{1b}} + \left(\frac{1}{p_{1b}} - 1\right)\bar{l}_1\right)\frac{p_{1b}}{p_0}\right], \tag{40}$$

$$\delta_1 = W_0\left[(1-q)\left(\frac{1}{p_0} - 1\right) + q\left(\frac{1}{p_{1b}} + \left(\frac{1}{p_{1b}} - 1\right)\bar{l}_1\right)\left(\frac{p_{1b}}{p_0} - 1\right)\right], \tag{41}$$

$$\gamma_0 = W_0\left[(1-q)\frac{1}{p_0} - q\frac{\frac{C}{W_0} - \alpha}{\alpha + \beta}\right] \quad \textit{and} \tag{42}$$

$$\gamma_1 = W_0(1-q)\left(\frac{1}{p_0} - 1\right). \tag{43}$$

Proof Assume that s is sufficiently small so that Lemma 11 applies.

Note that $(1-q)U_{i,1g} + qU_{i,1b}$ is equivalent to $U(l_{i,0})$, where

$$U(l_{i,0}) \equiv \frac{1}{\alpha+\beta}\left((1-q)U_{i,1g} + qU_{i,1b} + \alpha W_0\right). \tag{44}$$

In addition, from that

$$
\begin{aligned}
U_{i,1g} &= (\alpha+\beta)\, W_{i,2g} - \alpha W_0, \\
W_{i,2g} &= W_{i,1g} = W_0 \left(\left(\frac{1}{p_0} - 1 \right) l_{i,0} + \frac{1}{p_0} \right), \\
U_{i,1b} &= \left((\alpha+\beta)\, W_{i,2b} - \alpha W_0 \right) \cdot \mathbf{1}\left(l_{i,0} \le l^*\right) - C \cdot \mathbf{1}\left(l_{i,0} > l^*\right), \\
W_{i,2b} &= W_{i,1b} \left(\left(\frac{1}{p_{1b}} - 1 \right) \bar{l}_1 + \frac{1}{p_{1b}} \right) \\
&\quad \text{and} \\
W_{i,1b} &= W_0 \left(\left(\frac{p_{1b}}{p_0} - 1 \right) \bar{l}_1 + \frac{p_{1b}}{p_0} \right),
\end{aligned}
$$

some algebras show that (39) holds. □

Proof of Theorems 3 and 4 The proof strategy is as follows. First, the existence of a crisis equilibrium will be proved when $s=0$. Then, it will be shown that the claim holds in a neighborhood of $s=0$, or $s \in [0, \bar{s})$ for a sufficiently small $\bar{s} > 0$ which is to be determined below.

For the rest of the proof, we assume that $0 < W_0 \le \overline{W}_0$ where $\overline{W}_0$ is given by (32). Besides, we assume that $0 < q < \overline{\overline{q}}$ where $\overline{\overline{q}} = \min\left(\frac{1}{2}, \frac{1+\bar{l}_1}{2+\bar{l}_0+\bar{l}_1}, \overline{q}\right)$ and $\overline{q}$ is given by (36). Note that such q satisfies the conditions for Lemmas 2 and 13. Resorting to Lemma 3, we do not consider the liquidation in state g. Also, from Lemma 4, we take the fair pricing $p_{1g} = 1$ in state g as given and use the expression (11) for $U_{i,1g}$. For each case of $s=0$ and $s>0$, we will verify $p_{1g}=1$ in a crisis equilibrium.

Set $s=0$. Note that from Lemma 15, we consider the expected payoff of (39) as the maximizing objective of fund managers. We show that a crisis equilibrium can be supported by the bang-bang strategy profile in Definition 3 with $l_* = -1$ and a properly chosen h.

If the bang-bang strategy profile is implemented, the market clearing conditions (1) and (3) allow us to view the prices as functions of h:

$$
p_0(h) = \frac{1}{1-X} h W_0 \left(1+\bar{l}_0\right) \tag{45}
$$

and

$$
p_{1b}(h) = \frac{1}{1-X(1-\varepsilon)} (1-h)\, W_0 \left(1+\bar{l}_1\right). \tag{46}
$$

We will show that $\Delta(h) = 0$ for some $h \in (0,1)$, where $\Delta(h) \equiv U(-1; p_0(h), p_{1b}(h)) - U\left(\bar{l}_0; p_0(h), p_{1b}(h)\right)$ and $U(\cdot)$ is defined in (39) and here we indicate dependence on $p_0(h)$ and $p_{1b}(h)$ explicitly. The bang-bang strategy profile with h_0 such that $\Delta(h_0) = 0$ will turn out to constitute a crisis equilibrium in the following steps.

Recall that $\underline{h}$ and $\overline{h}$ are defined in Lemma 12.

Step 1. $0 < p_{1b}(h) < p_0(h) < 1$ if $h \in \left(\underline{h}, \overline{h}\right)$: Apply the definition of $p_0(h)$ and $p_{1b}(h)$ in (45) and (46), respectively, to Lemma 12.

Step 2. We have

$$\Delta(h) = W_0\left(q\left(\frac{1}{p_{1b}(h)}\left(1+\bar{l}_1\right) - \bar{l}_1 + \frac{\frac{C}{W_0}-\alpha}{\alpha+\beta}\right) - (1-q)\left(\frac{1}{p_0(h)} - 1\right)\left(\bar{l}_0+1\right)\right)$$

for $h \in [0,1]$: Clear by (40)-(43).

Step 3. $\Delta\left(\underline{h}\right) < 0 < \Delta\left(\overline{h}\right)$: In Lemma 13, $f(h) = p_{1b}(h)$ and $g(h) = p_0(h)$. Thus, the same lemma implies that $\Delta\left(\underline{h}\right) < 0$. On the other hand, setting $h = \overline{h}$ and utilizing Lemma 12 give

$$p_0\left(\overline{h}\right) = \frac{1}{1-X}\overline{h}W_0\left(1+\bar{l}_0\right) = 1 > \frac{1}{1-X(1-\varepsilon)}\left(1-\overline{h}\right)W_0\left(1+\bar{l}_1\right) = p_{1b}\left(\overline{h}\right).$$

Then,

$$\begin{aligned}
&\Delta\left(\overline{h}\right)\\
=&W_0\left(q\left(\frac{1}{p_{1b}\left(\overline{h}\right)}\left(1+\bar{l}_1\right) - \bar{l}_1 + \frac{\frac{C}{W_0}-\alpha}{\alpha+\beta}\right) - (1-q)\left(\frac{1}{p_0\left(\overline{h}\right)} - 1\right)\left(\bar{l}_0+1\right)\right)\\
=&W_0 q\left(\left(\frac{1}{p_{1b}\left(\overline{h}\right)} - 1\right)\left(1+\bar{l}_1\right) + \frac{C}{W_0(\alpha+\beta)} + \frac{\beta}{\alpha+\beta}\right) > 0. \qquad (47)
\end{aligned}$$

The second equality holds because $p_0\left(\overline{h}\right) = \frac{1}{1-X}\overline{h}W_0\left(1+\bar{l}_0\right) = 1$. To see the inequality, note that

$$1 > \frac{1}{1-X(1-\varepsilon)}\left(1-\overline{h}\right)W_0\left(1+\bar{l}_1\right) = p_{1b}\left(\overline{h}\right)$$

by Lemma 12.

Step 4. $\Delta(h_0) = 0$ for some $h_0 \in \left(\underline{h}, \overline{h}\right)$ and such h_0 is unique: By Step 3 and the continuity of $\Delta(h)$, there exists $h_0 \in \left(\underline{h}, \overline{h}\right)$ such that $\Delta(h_0) = 0$. Furthermore, since $p_0(h)$ is strictly increasing in h and $p_{1b}(h)$ is strictly decreasing in h, Step 2 implies

$$\frac{\partial\Delta(h)}{\partial h} > 0 \text{ where } h \in \left[\underline{h}, \overline{h}\right]. \qquad (48)$$

Thus, uniqueness is proved.

Step 5. The strategy profile is a crisis equilibrium: Set $h = h_0$. Since $\Delta(h_0) = 0$, $U(-1; p_0(h_0), p_{1b}(h_0)) = U\left(\bar{l}_0; p_0(h_0), p_{1b}(h_0)\right)$. Note that $p_0(h_0) < 1$ from Step 1 implies that $\gamma_1 > 0$ from (43). Because $l^* = -1$ for $s = 0$, $U(l_{i,0}) = (\gamma_0 + \gamma_1 l_{i,0}) \cdot \mathbf{1}(l_{i,0} > l^*)$ is strictly increasing in $l_{i,0}$ for $l_{i,0} \in \left(-1, \bar{l}_0\right]$. Thus, $U(l_{i,0}) \leq U(-1) = U\left(\bar{l}_0\right)$ for any $l_{i,0} \in \left[-1, \bar{l}_0\right]$, showing that fund manager i does not have an incentive to deviate. Finally, we show that the fair pricing can hold in state g with the bang-bang strategy profile with $h = h_0$. If the fair pricing holds in state g, the aggregate wealth in state g is

$$\int_0^1 W_{i,1g} di = h_0 W_0 \left(\frac{1}{p_0(h_0)} \left(1 + \bar{l}_0\right) - \bar{l}_0 \right) + (1 - h_0) W_0 \geq W_0,$$

where the inequality is from $p_0(h_0) < 1$ (Step 1). Hence, the fair price can be supported by taking the uniform leverage $l = \frac{1}{\int_0^1 W_{i,1g} di} - 1 < \frac{1}{W_0} - 1 \leq \bar{l}_1$ across all fund managers. Therefore, the strategy profile is an equilibrium and Step 1 implies that $0 < p_{1b}(h) < p_0(h) < 1$, which in turn shows that it is a crisis equilibrium by Theorem 1.

Next, we examine the existence of a crisis equilibrium when $s > 0$. We analyze a crisis equilibrium around the neighborhood of $s = 0$ such that Lemmas 2 and 11 still hold. From Lemma 11, we consider the payoff of (39).

For the ease of analysis, we classify the equilibria at $s = 0$ into two cases, $\delta_1 > 0$ and $\delta_1 \leq 0$ where δ_1 is given by (41).

First, we consider the case where $\delta_1 \leq 0$ at $(h, s) = (h_0, 0)$. We will show that the equilibrium strategy profile and prices for $s = 0$ are also an equilibrium for $s > 0$. For $s = 0$, we have shown that the bang-bang strategy profile with $h = h_0$ is an equilibrium. Let p_0 and p_{1b} denote the equilibrium prices for $s = 0$.

We show that p_0 and p_{1b} clear the markets for a sufficiently small $s > 0$. Because (1) does not involve s, p_0 satisfies (1). Take $\bar{s} > 0$ such that $l^* = \frac{\frac{p_{1b}}{p_0} - (1-s)}{1 - \frac{p_{1b}}{p_0}} = -1 + \frac{s}{1 - \frac{p_{1b}}{p_0}} < \bar{l}_0$ for any $s < \bar{s}$. Then, the h_0 proportion liquidate in state b, which is the same as in $s = 0$. Thus, p_{1b} satisfies (3).

We show that $U(-1) = U\left(\bar{l}_0\right) \geq U(l_{i,0})$ for $l_{i,0} \in \left[-1, \bar{l}_0\right]$ to claim the fund managers do not have incentives to deviate. We use (39). The assumption $\delta_1 \leq 0$ implies that $U(-1) \geq U(l_{i,0})$ for $l_{i,0} \leq l^*$. Since $p_0 < 1$, we have $\gamma_1 > 0$, implying that $U\left(\bar{l}_0\right) \geq U(l_{i,0})$ for $l_{i,0} \in \left(l^*, \bar{l}_0\right]$. The property $U(-1) = U\left(\bar{l}_0\right)$ comes from the construction of h_0.

Therefore, if $\delta_1 \leq 0$ at $(h, s) = (h_0, 0)$, there is a crisis equilibrium for $s > 0$.

The second case is $\delta_1 > 0$ at $(h, s) = (h_0, 0)$. The proof idea is as follows. We restrict our attention to the bang-bang strategy profile in Definition 3 with $l_* = l^*$: the h proportion of fund managers (indexed by i) take $l_{i,0} = \bar{l}_0$ and liquidate in state b, and the remaining $1-h$ proportion (indexed by j) take $(l_{j,0}, l_{j,1}(b)) = \left(l^*, \bar{l}_1\right)$. Like what we did in the case $s = 0$, we define

$$\Delta(h,s) = U(l^*; p_0, p_{1b}) - U\left(\bar{l}_0; p_0, p_{1b}\right).$$

Then, we show h_s satisfying $\Delta(h_s, s) = 0$ exists and the suggested strategy profile with $h = h_s$ yields a crisis equilibrium.

We express dependence of l^*, p_0 and p_{1b} on (h, s) explicitly. Denoting l^* in (12) as a function of prices gives

$$l^*(h,s) = \frac{\frac{p_{1b}(h,s)}{p_0(h,s)} - (1-s)}{1 - \frac{p_{1b}(h,s)}{p_0(h,s)}}. \tag{49}$$

The prices satisfy the market clearing conditions (1) and (3):

$$p_0(h,s) = \frac{1}{1-X} W_0 \left(1 + h\bar{l}_0 + (1-h) l^*(h,s)\right) \text{ and} \tag{50}$$

$$p_{1b}(h,s) = \frac{1}{1-X(1-\varepsilon)} (1-h)(1-s) W_0 \left(1 + \bar{l}_1\right). \tag{51}$$

The equality (51) holds because, in state b, the $(1-h)$ fund managers survive with $W_{i,1b} = W_0 \left(\frac{p_{1b}}{p_0}(1+l^*) - l^*\right) = (1-s) W_0$.

Then, we can write $\Delta(h,s)$ as a function of (h,s) explicitly. Use (39)-(43) along with the above expressions to write $\Delta(h,s)$ as follows:

$$\begin{aligned}
&\Delta(h,s) \\
=&U(l^*(h,s)) - U\left(\bar{l}_0\right) = (\delta_0 + \delta_1 l^*(h,s)) - \left(\gamma_0 + \gamma_1 \bar{l}_0\right) && (52) \\
=&(\delta_0 - \gamma_0) + (\delta_1 - \gamma_1) l^*(h,s) - \gamma_1 \left(\bar{l}_0 - l^*(h,s)\right) \\
=&W_0 q \left(\frac{1}{p_{1b}(h,s)} + \left(\frac{1}{p_{1b}(h,s)} - 1\right)\bar{l}_1\right)\left(\frac{p_{1b}(h,s)}{p_0(h,s)} + \left(\frac{p_{1b}(h,s)}{p_0(h,s)} - 1\right) l^*(h,s)\right) \\
&+ W_0 q \left(\frac{\frac{C}{W_0} - \alpha}{\alpha + \beta}\right) - W_0 (1-q)\left(\frac{1}{p_0(h,s)} - 1\right)\left(\bar{l}_0 - l^*(h,s)\right). && (53)
\end{aligned}$$

The following steps show that there exists a crisis equilibrium.

Step 1. $\Delta(h,s)$ and $l^*(h,s)$ are continuously differentiable and $0 < p_{1b}(h,s) < p_0(h,s) < 1$ in a neighborhood of $(h_0, 0)$: We have shown that $0 < g(h_0, 0) < f(h_0, 0) <$

1 from a crisis equilibrium with $s=0$. Lemma 14 implies $p_0(h,s)$ and $p_{1b}(h,s)$ are continuously differentiable with respect to (h,s) and $0<p_{1b}(h,s)<p_0(h,s)<1$ in a neighborhood of $(h_0,0)$. Thus, $l^*(h,s)$ is a continuously differentiable function of (h,s) and so is $\Delta(h,s)$.

Step 2. $l^*(h,s)\in\left[-1,\bar{l}_0\right]$ in a neighborhood of $(h_0,0)$: Note that the continuously differentiability of $p_{1b}(h,s)$ and $p_0(h,s)$ from Lemma 14 implies that $\frac{\partial l^*(h,s)}{\partial s}|_{s=0}=\frac{1}{1-\frac{p_{1b}(h,s)}{p_0(h,s)}}>0$ when h is close to h_0. Also, since $\bar{l}_0>-1$, the continuity of $l^*(h,s)$ (Step 1) implies that $-1\leq l^*(h,s)<\bar{l}_0$ in a neighborhood of $(h_0,0)$.

Step 3. $U(l^*)\geq U(l_{i,0})$ for $l_{i,0}\in[-1,l^*]$ in a neighborhood of $(h_0,0)$: Note that $[-1,l^*]$ is non-empty by Step 2. We utilize the property that $\delta_1>0$ at $(h_0,0)$. From the expression of δ_1 in (41), we find that δ_1 is continuous in p_0 and p_{1b} if $p_0>0$ and $p_{1b}>0$. By the continuity of $p_0(h,s)$ and $p_{1b}(h,s)$ in (h,s) around $(h_0,0)$, verified in Lemma 14, we can always find a neighborhood of $(h_0,0)$ such that $\delta_1>0$. This implies $U(l^*)\geq U(l_{i,0})$ for $l_{i,0}\in[-1,l^*]$.

Step 4. There is $\overline{s}>0$ such that there exists a continuously differentiable function h_s of s such that $\Delta(h_s,s)=0$ for all $s\in[0,\overline{s})$: By invoking the implicit function theorem, we conclude that there exist a continuously differentiable function h_s and $\overline{s}>0$ such that

$$\Delta(h_s,s)=0$$

for any $s\in[0,\overline{s})$.

Step 5. The suggested strategy profile constitutes a crisis equilibrium: Consider the bang-bang strategy profile in Definition 3 with $h=h_s$ and $l_*=l^*$. From Step 4, we know that $U(l^*)=U\left(\bar{l}_0\right)$. Furthermore, Step 3 shows that $U(l^*)\geq U(l_{i,0})$ for $l_{i,0}\leq l^*$. Also, since $p_0(h_s,s)<1$ from $p_0(h_0,0)<1$ and the continuity of $p_0(h,s)$ in Lemma 14, we have $\gamma_1>0$, implying that $U\left(\bar{l}_0\right)>U(l_{i,0})$ for $l_{i,0}\in\left(l^*,\bar{l}_0\right)$. Combining these findings, we have $U(l^*)=U\left(\bar{l}_0\right)\geq U(l_{i,0})$ for $l_{i,0}\in\left[-1,\bar{l}_0\right]$, verifying that no fund manager has an incentive to deviate from the bang-bang strategy profile. Finally, we show that the fair pricing can hold with the bang-bang strategy profile with $h=h_s$. If the fair pricing holds in state g, the aggregate wealth in state g is

$$\int_0^1 W_{i,1g}di=h_0W_0\left(\frac{1}{p_0(h_s,s)}\left(1+\bar{l}_0\right)-\bar{l}_0\right)+(1-h_0)W_0\left(\frac{1}{p_0(h_s,s)}(1+l^*)-l^*\right)\geq W_0,$$

where the inequality is from $p_0(h_s,s)<1$ (Step 1). Hence, the fair price can be supported by taking the uniform leverage $l=\frac{1}{\int_0^1 W_{i,1g}di}-1<\frac{1}{W_0}-1\leq\bar{l}_1$ across all fund managers. From Step 1, we have $0<p_{1b}(h_s,s)<p_0(h_s,s)<1$, implying that there exist liquidations by Theorem 1. Hence, the bang-bang strategy profile constitutes a crisis equilibrium. □

Proof of Theorem 5 Theorem 2 guarantees the existence of a calm equilibrium. Hence, it suffices to show a crisis equilibrium does not exist under when q is sufficiently large.

Assume that a crisis equilibrium exists and set q is close to 1 to show contradiction in the following steps.

Step 1. If no fund is liquidated in state b, $p_{1b} = 1$: Set $l_{i,1} = -1+1/\frac{W_0}{1-X(1-\varepsilon)} \int_0^1 \left(\frac{1}{p_0}\left(1+l_{i,0}\right) - l_{i,0}\right) di < \bar{l}_1$, where the equality is from $p_0 < 1$ and Assumption 1. With the given $l_{i,1}$, $p_{1b} = 1$.

Step 2. There exists $\delta > 0$ such that $p_0 > \delta$: Otherwise, $p_{1b} = 1$ from Step 1.

Step 3. No fund is liquidated in state b: If fund i is liquidated, $U_{i,0}$ is close to $-C$ because q is close to 1 and p_0 is bounded away from zero (Step 2).

Step 4. $p_{1b} = 1$: Combine Steps 1 and 3.

Step 5. The theorem holds: Step 4 contradicts that a crisis equilibrium exists. □

The following lemma is needed for the proof of the remaining theorems.

Lemma 16. *Assume $s = 0$. Let θ be any parameter in the model. Then, it holds that*

$$\frac{\partial \Delta(h)}{\partial \theta} + \frac{\partial \Delta(h)}{\partial h}\frac{dh}{d\theta} = 0,$$

where

$$\Delta(h) = q\left(\frac{1}{p_{1b}(h)}\left(1+\bar{l}_1\right) - \bar{l}_1 + \frac{\frac{C}{W_0} - \alpha}{\alpha+\beta}\right) - (1-q)\left(\frac{1}{p_0(h)} - 1\right)\left(\bar{l}_0 + 1\right)$$

$$p_0(h) = \frac{W_0 h\left(1+\bar{l}_0\right)}{1-X}$$

$$p_{1b}(h) = \frac{W_0(1-h)\left(1+\bar{l}_1\right)}{1-X(1-\varepsilon)}.$$

and that

$$\frac{\partial \Delta(h)}{\partial h} = q\frac{1-X(1-\varepsilon)}{(1-h)^2 W_0} + (1-q)\frac{1-X}{h^2 W_0} > 0. \tag{54}$$

Proof In the proof of Theorem 3, it is shown that the bang-bang strategy profile with h such that $\Delta(h) = 0$ constitutes a crisis equilibrium. From the implicit function theorem, it holds that

$$\frac{\partial \Delta(h)}{\partial \theta} + \frac{\partial \Delta(h)}{\partial h}\frac{dh}{d\theta} = 0.$$

Also, from the chain rule, it follows that

$$\begin{aligned}\frac{\partial \Delta(h)}{\partial h} &= -q\frac{1}{p_{1b}(h)^2}\left(1+\bar{l}_1\right)\frac{\partial p_{1b}(h)}{\partial h} + (1-q)\frac{1}{p_0^2(h)}\left(\bar{l}_0+1\right)\frac{\partial p_0(h)}{\partial h}. \\ &= q\frac{1-X(1-\varepsilon)}{(1-h)^2 W_0} + (1-q)\frac{1-X}{h^2 W_0} > 0.\end{aligned}$$

This completes the proof of the lemma. □

Lemma 17. *Under Assumption 1, if q, s and W_0 are sufficiently small, it holds that $\frac{dp_0}{dX} > 0$ and $\frac{dp_{1b}}{dX} > 0$ in a crisis equilibrium.*

Proof First, we prove the lemma at $s = 0$. Set $s = 0$. Also, assume that q is small enough to satisfy

$$q < 1 - \overline{h}, \tag{55}$$

where $\overline{h}$ is defined in (34). Note that the RHS of (55) is strictly positive. Also, recall that $h < \overline{h}$ at $s = 0$ from the proof of Theorem 3. From Lemma 16, it holds that

$$\frac{\partial \Delta(h)}{\partial X} + \frac{\partial \Delta(h)}{\partial h}\frac{dh}{dX} = 0. \tag{56}$$

Some algebras show that

$$\begin{aligned}\frac{\partial \Delta(h)}{\partial X} &= -q\frac{1}{p_{1b}(h)^2}\left(1+\bar{l}_1\right)\frac{\partial p_{1b}(h)}{\partial X} + (1-q)\frac{1}{p_0^2(h)}\left(\bar{l}_0+1\right)\frac{\partial p_0(h)}{\partial X} \\ &= -q\frac{(1-\varepsilon)}{W_0(1-h)} + (1-q)\frac{1}{W_0 h}.\end{aligned} \tag{57}$$

We find that $\frac{\partial \Delta(h)}{\partial X} > 0$ because

$$\begin{aligned}\frac{\partial \Delta(h)}{\partial X} &\geq -q\frac{1}{W_0(1-h)} + (1-q)\frac{1}{W_0 h} = \frac{1}{W_0}\left(-q\frac{1}{(1-h)} + (1-q)\frac{1}{h}\right) \\ &\geq \frac{1}{W_0}\left(\frac{-q}{1-\overline{h}} + \frac{1-q}{\overline{h}}\right) = \frac{1}{\overline{h}W_0}\left(1 - \frac{q}{1-\overline{h}}\right) > 0,\end{aligned} \tag{58}$$

where the first inequality is from $(1-\varepsilon) \leq 1$, the second inequality is from $h < \overline{h}$ and the last inequality is from (55).

Next, we show that $-\frac{dh}{dX} < \frac{h}{1-X}$. By plugging (57) and (54) to (56) and rearranging terms, we have that

$$\begin{aligned}-\frac{dh}{dX} &= \frac{\frac{\partial \Delta(h)}{\partial X}}{\frac{\partial \Delta(h)}{\partial h}} = \frac{-q\frac{(1-\varepsilon)}{(1-h)} + (1-q)\frac{1}{h}}{q\frac{1-X(1-\varepsilon)}{(1-h)^2} + (1-q)\frac{1-X}{h^2}} \\ &= \frac{h}{1-X}\cdot\left(\frac{-q\frac{(1-X)(1-\varepsilon)}{h(1-h)} + (1-q)\frac{1-X}{h^2}}{q\frac{1-X(1-\varepsilon)}{(1-h)^2} + (1-q)\frac{1-X}{h^2}}\right) < \frac{h}{1-X}.\end{aligned} \tag{59}$$

Finally, we determine the sign of $\frac{dh}{dX}$, $\frac{dp_0}{dX}$ and $\frac{dp_{1b}}{dX}$. From the inequalities of (54) and (58), (56) implies that

$$\frac{dh}{dX} < 0. \tag{60}$$

Recall that $p_0 = \frac{W_0\left(1+\bar{l}_0\right)h}{1-X}$. Hence,

$$\begin{aligned}\frac{dp_0}{dX} &= \frac{\partial p_0}{\partial X} + \frac{\partial p_0}{\partial h}\frac{dh}{dX} = \frac{W_0\left(1+\bar{l}_0\right)h}{\left(1-X\right)^2} + \frac{W_0\left(1+\bar{l}_0\right)}{1-X}\frac{dh}{dX} \\ &= \frac{W_0\left(1+\bar{l}_0\right)}{1-X}\left(\frac{h}{1-X} + \frac{dh}{dX}\right) > 0,\end{aligned} \tag{61}$$

where the last inequality is from (59). Recall that $p_{1b} = \frac{W_0\left(1+\bar{l}_1\right)(1-h)}{1-X(1-\varepsilon)}$. Hence,

$$\begin{aligned}\frac{dp_{1b}}{dX} &= \frac{\partial p_{1b}}{\partial X} + \frac{\partial p_{1b}}{\partial h}\frac{dh}{dX} = \frac{W_0\left(1+\bar{l}_1\right)(1-h)}{\left(1-X\left(1-\varepsilon\right)\right)^2} - \frac{W_0\left(1+\bar{l}_1\right)}{1-X\left(1-\varepsilon\right)}\frac{dh}{dX} \\ &= \frac{W_0\left(1+\bar{l}_1\right)}{1-X\left(1-\varepsilon\right)}\left(1-h-\frac{dh}{dX}\right) > 0,\end{aligned} \tag{62}$$

where the last inequality is from (60).

Because the economy is continuous, the inequalities of (60), (61) and (62) still hold when s is sufficiently small. This completes the proof of the lemma. □

Lemma 18. *Under Assumption 1, if q, s, ε and W_0 are sufficiently small, it holds that $\frac{dp_0}{dW_0} - \frac{dp_0}{dX} > 0$ and $\frac{dp_{1b}}{dW_0} - \frac{dp_{1b}}{dX} > 0$ in a crisis equilibrium.*

Proof First, we prove the lemma at $s = \varepsilon = 0$. Set $s = \varepsilon = 0$. In addition, assume that W_0 is small enough to satisfy

$$1 - X > W_0. \tag{63}$$

Utilizing Lemma 16, we have that

$$\frac{\partial\Delta\left(h\right)}{\partial W_0} - \frac{\partial\Delta\left(h\right)}{\partial X} + \frac{\partial\Delta\left(h\right)}{\partial h}\left(\frac{dh}{dW_0} - \frac{dh}{dX}\right) = 0. \tag{64}$$

From (57) and (58) in the proof of Lemma 16, we have

$$\frac{\partial\Delta\left(h\right)}{\partial X} = -\frac{q}{W_0\left(1-h\right)} + \frac{1-q}{W_0 h} > 0. \tag{65}$$

Also, some algebras show that

$$\frac{\partial \Delta (h)}{\partial W_0} = \frac{1-X}{W_0^2}\left(-\frac{q}{W_0 (1-h)} + \frac{1-q}{W_0 h}\right). \tag{66}$$

From the expressions of (65) and (66), we have that

$$\frac{\partial \Delta (h)}{\partial W_0} - \frac{\partial \Delta (h)}{\partial X} > \frac{\partial \Delta (h)}{\partial W_0} - \frac{1-X}{W_0}\frac{\partial \Delta (h)}{\partial X} = 0, \tag{67}$$

where the first inequality is from (63) and (65). Next, we show that $h - h\frac{W_0}{1-X} + W_0 \frac{dh}{dW}_0 - W_0 \frac{dh}{dX} > 0$. Using Lemma 16, we find that after some algebras,

$$\begin{aligned} h - h\frac{W_0}{1-X} + W_0\frac{dh}{dW_0} - W_0\frac{dh}{dX} &= h - h\frac{W_0}{1-X} - W_0\frac{\frac{\partial \Delta(h)}{\partial W_0}}{\frac{\partial \Delta(h)}{\partial h}} + W_0\frac{\frac{\partial \Delta(h)}{\partial X}}{\frac{\partial \Delta(h)}{\partial h}} \\ =& h\left(1 - \frac{W_0}{1-X}\right)\left(\frac{2q\frac{1-X}{(1-h)^2}}{q\frac{1-X}{(1-h)^2} + (1-q)\frac{1-X}{h^2}}\right) > 0, \end{aligned} \tag{68}$$

where the last equality is from (63).

Finally, we determine the sign of $\frac{dh}{dW_0} - \frac{dh}{dX}$, $\frac{dp_0}{dW_0} - \frac{dp_0}{dX}$ and $\frac{dp_{1b}}{dW_0} - \frac{dp_{1b}}{dX}$. From the inequalities of (54) and (67), (64) implies that

$$\frac{dh}{dW}_0 - \frac{dh}{dX} < 0. \tag{69}$$

Turn to $\frac{dp_0}{dW_0} - \frac{dp_0}{dX}$. Recall that $p_0 = \frac{W_0\left(1+\bar{l}_0\right)h}{1-X}$. Hence, it holds that

$$\begin{aligned} \frac{dp_0}{dW_0} - \frac{dp_0}{dX} &= \frac{\partial p_0}{\partial W_0} - \frac{\partial p_0}{\partial X} + \frac{\partial p_0}{\partial h}\left(\frac{dh}{dW}_0 - \frac{dh}{dX}\right) \\ &= \frac{\left(1+\bar{l}_0\right)h}{1-X} - \frac{W_0\left(1+\bar{l}_0\right)h}{(1-X)^2} + \frac{W_0\left(1+\bar{l}_0\right)}{1-X}\left(\frac{dh}{dW}_0 - \frac{dh}{dX}\right) \\ &= \frac{\left(1+\bar{l}_0\right)}{1-X}\left(h - h\frac{W_0}{1-X} + W_0\frac{dh}{dW}_0 - W_0\frac{dh}{dX}\right) > 0, \end{aligned} \tag{70}$$

where the last inequality is from (68).

Next, we examine $\frac{dp_{1b}}{dW_0} - \frac{dp_{1b}}{dX}$. Recall that $p_{1b} = \frac{W_0\left(1+\bar{l}_1\right)(1-h)}{1-X}$. Hence,

$$\begin{aligned} \frac{dp_{1b}}{dW_0} - \frac{dp_{1b}}{dX} &= \frac{\partial p_{1b}}{\partial W_0} - \frac{\partial p_{1b}}{\partial X} + \frac{\partial p_{1b}}{\partial h}\left(\frac{dh}{dW}_0 - \frac{dh}{dX}\right) \\ &= \frac{\left(1+\bar{l}_1\right)(1-h)}{1-X} - \frac{W_0\left(1+\bar{l}_1\right)(1-h)}{(1-X)^2} - \frac{W_0\left(1+\bar{l}_1\right)}{1-X}\left(\frac{dh}{dW}_0 - \frac{dh}{dX}\right) \\ &= \frac{\left(1+\bar{l}_1\right)(1-h)}{1-X}\left(1 - \frac{W_0}{1-X}\right) - \frac{W_0\left(1+\bar{l}_1\right)}{1-X}\left(\frac{dh}{dW}_0 - \frac{dh}{dX}\right) > 0, \end{aligned} \tag{71}$$

where the last inequality is from (63) and (69).

Because the economy is continuous, the inequalities of (69), (70) and (71) still hold when s and ε are sufficiently small. This completes the proof of the lemma. □

Proof of Theorem 6 The theorem directly follows from Lemmas 17 and 18. □

Proof of Theorem 7 First, we prove Theorem 7 at $s = 0$. Set $s = 0$. From Lemma 6, it holds that

$$\frac{\partial \Delta(h)}{\partial C} + \frac{\partial \Delta(h)}{\partial h}\frac{dh}{dC} = 0. \tag{72}$$

Note that

$$\frac{\partial \Delta(h)}{\partial C} = \frac{q}{\alpha + \beta} > 0. \tag{73}$$

Next, we determine the sign of $\frac{dh}{dC}$, $\frac{dp_0}{dC}$ and $\frac{dp_{1b}}{dC}$. From the inequalities of (54) and (73), (72) implies that

$$\frac{dh}{dC} < 0. \tag{74}$$

Recall that $p_0 = \frac{W_0\left(1+\bar{l}_0\right)h}{1-X}$. Hence,

$$\frac{dp_0}{dC} = \frac{\partial p_0}{\partial h}\frac{dh}{dC} = \frac{W_0\left(1+\bar{l}_0\right)}{1-X}\frac{dh}{dC} < 0, \tag{75}$$

where the last inequality is from (74). Recall that $p_{1b} = \frac{W_0\left(1+\bar{l}_1\right)(1-h)}{1-X(1-\varepsilon)}$. Hence,

$$\frac{dp_{1b}}{dC} = \frac{\partial p_{1b}}{\partial h}\frac{dh}{dC} = -\frac{W_0\left(1+\bar{l}_1\right)}{1-X(1-\varepsilon)}\frac{dh}{dC} > 0, \tag{76}$$

where the last inequality is from (74).

Because the economy is continuous, the inequalities of (74), (75) and (76) still hold when s is sufficiently small. This completes the proof of the theorem. □

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