Endogenous Monetary-Fiscal Regime Change in the United States

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Outline

Part I: Background

- Chang, Choi and Park (2017), A New Approach to Model Regime Switching, *Journal of Econometrics*, 196, 127-143.

Part II: Monetary and Fiscal Policy Interactions

- Chang and Kwak (2016), Endogenous Monetary-Fiscal Regime Change in the United States.

Mean Switching Model

The basic mean model with regime switching is given by

$$(y_t - \mu_t) = \gamma(y_{t-1} - \mu_{t-1}) + u_t$$

with

$$\mu_t = \mu(s_t),$$

where

- (s_t) is a state process specifying a binary state of regime
- $s_t = 0$ and 1 are referred respectively to as low and high mean regimes

in the model.

Volatility Switching Model

The basic volatility model with regime switching is given by

$$y_t = \sigma_t u_t$$

with

$$\sigma_t = \sigma(s_t),$$

where

- (s_t) is a state process specifying a binary state of regime
- $s_t = 0$ and 1 are referred respectively to as low and high volatility regimes

in the model.

Conventional Regime Switching Model

The state process (s_t) is assumed to be entirely independent of other parts of the underlying model, and specified as a two state markov chain.

Therefore, the two transition probabilities

$$a = \mathbb{P}\{s_t = 0 | s_{t-1} = 0\}$$
$$b = \mathbb{P}\{s_t = 1 | s_{t-1} = 1\},$$

completely specify the state process (s_t).

Shortcomings

The standard markov switching models have the following shortcomings.

- The regime switching is strictly exogenous, being completely independent from observed time series
- The state process (s_t) is strictly stationary and does not allow for persistency
- The computation becomes too complicated as the number of states increases
- The state process (s_t) itself takes discrete values and cannot be directly related to other relevant economic variables

All these shortcomings seriously limit the applicability and usefulness of the standard markov switching models.

References

- Hamilton (1989), A New Approach to the Economic Analysis of Nonstationary Time Series and The Business Cycle, *Econometrica* 57, 357-384
- Kim (1994), Dynamic Linear Models with Markov-Switching, *Journal of Econometrics*, 60, 1-22
- Kim and Nelson (1999), State-Space Models with Regime Switching. MIT Press, Cambridge, MA
- Kim, Piger and Startz (2008), Estimation of Markov Regime-Switching Regression Models with Endogenous Switching, *Journal of Econometrics* 143, 263-273

A New Approach to Regime Switching

Chang, Choi and Park (2017), A New Approach to Model Regime Switching, *Journal of Econometrics*, 196, 127-143.

Present a regime switching model whose regime is determined by an endogenous autoregressive latent factor.

A simple estimation and factor extraction method using a modified markov switching filter is developed.

Provide a strong evidence on endogenous regime switching in all economic models commonly analyzed by regime switching models.

The presence of endogeneity allows us to more effectively extract the information revealed by the observed time series on the unobserved states.

New Regime Switching Model

Specify a model

$$\begin{aligned} y_t &= m_t + \sigma_t u_t \\ &= m(x_t, y_{t-1}, \ldots, y_{t-k}, s_t, \ldots, s_{t-k}) + \sigma(x_t, s_t, \ldots, s_{t-k}) u_t \\ &= m(x_t, y_{t-1}, \ldots, y_{t-k}, w_t, \ldots, w_{t-k}) + \sigma(x_t, w_t, \ldots, w_{t-k}) u_t \end{aligned}$$

where

- covariate (x_t) is exogenous,
- state process (s_t) is driven by $s_t = 1\{w_t \geq \tau\}$,
- latent factor (w_t) is specified as $w_t = \alpha w_{t-1} + v_t$,

and

$$\begin{pmatrix} u_t \\ v_{t+1} \end{pmatrix} =_d \mathbb{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right)$$

with endogeneity parameter ρ.

New Mean Switching Model

The mean model with autoregressive latent factor is given by

$$\gamma(L)(y_t - \mu_t) = u_t$$

where $\gamma(z) = 1 - \gamma_1 z - \cdots - \gamma_k z^k$ is a $k-$th order polynomial, $\mu_t = \mu(s_t)$, $s_t = 1\{w_t \geq \tau\}$, $w_t = \alpha w_{t-1} + v_t$ and

$$\begin{pmatrix} u_t \\ v_{t+1} \end{pmatrix} =_d \mathbb{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right)$$

Again a shock (u_t) at time t affects the regime at time $t+1$, and the regime switching becomes endogenous. The endogeneity parameter ρ represents the reversion of mean in our mean model.

Mean Reversion

In the new model, the mean reversion of the observed time series (y_t) occurs in two different levels.

First level: If the lag polynomial $\gamma(z)$ satisfies the stationarity condition, the observed time series (y_t) reverts to the state dependent mean (μ_t).

Second level: If the endogeneity parameter $\rho < 0$, the state dependent mean (μ_t) moves to offset the effect of a shock to the observed time series (y_t). When $\rho > 0$, on the other hand, the movement of (μ_t) at the second level would entail an unstabilizing effect on (y_t).

New Volatility Switching Model

The volatility model with autoregressive latent factor is given by

$$y_t = \sigma_t u_t$$

where $\sigma_t = \sigma(s_t) = \sigma(w_t)$, $s_t = 1\{w_t \geq \tau\}$, $w_t = \alpha w_{t-1} + v_t$ and

$$\begin{pmatrix} u_t \\ v_{t+1} \end{pmatrix} =_d \mathbb{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right)$$

A shock (u_t) at time t affects the regime at time $t+1$, and the regime switching becomes endogenous. The endogeneity parameter ρ, which is expected to be negative, represents the leverage effect in our volatility model.

Advantages

The new regime switching models have the following advantages.

- The regime switching is endogenous and systematically affected by the observed time series
- The state process (s_t) is allowed to be nonstationary as well as stationary
- The latent factor (w_t) has continuous values and, once extracted, can be directly related to many other relevant economic variables taking continuous values
- Easily extended to models with multiple regimes

These advantages greatly improve the applicability and usefulness of markov switching models.

Relationship with Conventional Switching Model

- The new model reduces to the conventional markov switching model when the underlying autoregressive latent factor is stationary and exogenous, i.e., independent of the model innovation.
- Assume $\rho = 0$, and obtain transition probabilities of (s_t). In our approach, they are given as functions of the autoregressive coefficient α of the latent factor and the level τ of threshold.
- Note that

$$\mathbb{P}\left\{s_t = 0 \,\middle|\, w_{t-1}\right\} = \mathbb{P}\left\{w_t < \tau \,\middle|\, w_{t-1}\right\} = \Phi(\tau - \alpha w_{t-1})$$
$$\mathbb{P}\left\{s_t = 1 \,\middle|\, w_{t-1}\right\} = \mathbb{P}\left\{w_t \geq \tau \,\middle|\, w_{t-1}\right\} = 1 - \Phi(\tau - \alpha w_{t-1})$$

from $w_t = \alpha w_{t-1} + v_t$ and $v_t \sim \mathbb{N}(0, 1)$.

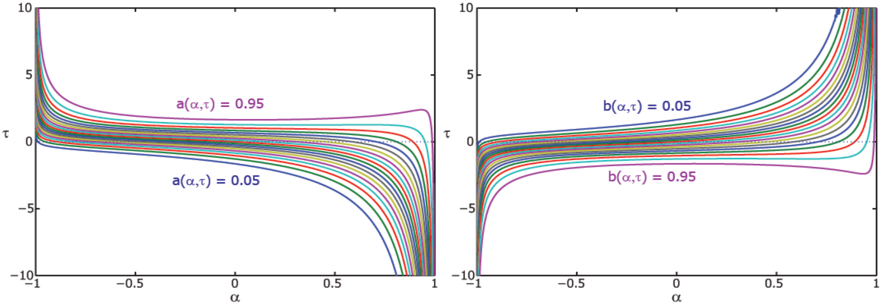
Transition of Stationary State Process

For $|\alpha| < 1$, transition probabilities of state process (s_t) from low state to low state $a(\alpha, \tau)$ and high state to high state $b(\alpha, \tau)$ are given by

$$a(\alpha, \tau) = \mathbb{P}\{s_t = 0 | s_{t-1} = 0\}$$
$$= \frac{\int_{-\infty}^{\tau\sqrt{1-\alpha^2}} \Phi\left(\tau - \frac{\alpha x}{\sqrt{1-\alpha^2}}\right) \varphi(x) dx}{\Phi\left(\tau\sqrt{1-\alpha^2}\right)}$$

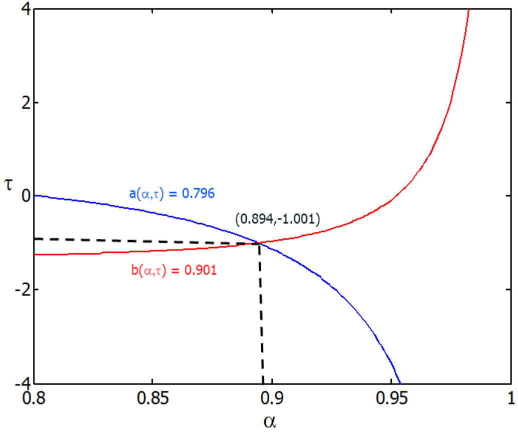
$$b(\alpha, \tau) = \mathbb{P}\{s_t = 1 | s_{t-1} = 1\}$$
$$= 1 - \frac{\int_{\tau\sqrt{1-\alpha^2}}^{\infty} \Phi\left(\tau - \frac{\alpha x}{\sqrt{1-\alpha^2}}\right) \varphi(x) dx}{1 - \Phi\left(\tau\sqrt{1-\alpha^2}\right)}.$$

Transition Probability Contours in (α, τ)-Plane



The contours of $a(\alpha, \tau)$ and $b(\alpha, \tau)$ are presented for the levels from 0.05 to 0.95 in the increment of 0.05, upward for $a(\alpha, \tau)$ and downward for $b(\alpha, \tau)$. Hence the bottom line in the left panel is the contour of $a(\alpha, \tau) = 0.05$, and the top line on the right panel represents the contour of $b(\alpha, \tau) = 0.05$.

One to One Correspondence Between (α, τ) and (a, b)



We set $a(\alpha, \tau) = 0.796$ and $b(\alpha, \tau) = 0.901$, the transition probabilities we obtain from our estimates from the Hamilton's model, and plot their contours in the (α, τ)-plane.

Equivalence

If

$$\rho = 0 \quad and \quad |\alpha| < 1,$$

the new endogenous regime switching model becomes identical to the conventional markov switching model.

- In this case, the transition probability in the new model depends only on α and τ
- For each combination of (α, τ), we can find the unique combination of transition probabilities $(a, b) = (\mathbb{P}\{s_t = 0|s_{t-1} = 0\}, \mathbb{P}\{s_t = 1|s_{t-1} = 1\})$, and vise versa. Thus, we may always find a regime switching model with an autoregressive latent factor that is observationally equivalent to any given conventional markov switching model.
- However, our approach produces an important by-product that is not available from the conventional approach: an extracted time series of the latent factor.

Maximum Likelihood Estimation

- The new endogenous model can be estimated by ML method. The log-likelihood function is given as

$$\ell(y_1, \ldots, y_n) = \log p(y_1) + \sum_{t=2}^{n} \log p(y_t | \mathcal{F}_{t-1})$$

  where $\mathcal{F}_t = \sigma\big(x_t, (y_s)_{s \leq t}\big)$, i.e., the information given by $x_t, y_1, \ldots, y_t$ for each $t = 1, \ldots, n$.
- Of course, the log-likelihood function includes a vector of unknown parameters $\theta \in \Theta$, say, which specifies conditional mean and volatility functions of the state dependent variable (y_t). It is, however, suppressed for the sake of brevity.
- For the mean and volatility models, θ consists of state dependent mean and volatility parameters, $(\underline{\mu}, \bar{\mu})$, $(\underline{\sigma}, \bar{\sigma})$, τ, α, ρ, and $(\gamma_1, \ldots, \gamma_k)$.

On to New Filter

To estimate the general switching model by the maximum likelihood method, we need to develop a filter. The conventional markov switching filter is no longer applicable, since the state process (s_t) for the new model is not a markov chain unless $\rho = 0$.

To develop the modified markov switching filter that can be used to estimate the new model, define

$$\Phi_\rho(x) = \Phi\big(x/\sqrt{1-\rho^2}\big),$$

We show in the next slide that the bivariate process (s_t, y_t) on $\{0,1\} \times \mathbb{R}$ is a $(k+1)$-st order markov process.

Markov Structure of Endogenous State Process

The bivariate process (s_t, y_t) on $\{0,1\} \times \mathbb{R}$ is a $(k+1)$-st order markov process, whose transition density with respect to the product of the counting and Lebesgue measure is given by

$$p(s_t, y_t | s_{t-1}, \ldots, s_{t-k-1}, y_{t-1}, \ldots, y_{t-k-1})$$
$$= p(y_t | s_t, \ldots, s_{t-k}, y_{t-1}, \ldots, y_{t-k}) p(s_t | s_{t-1}, \ldots, s_{t-k-1}, y_{t-1}, \ldots, y_{t-k-1}),$$

where

$$p(y_t | s_t, \ldots, s_{t-k}, y_{t-1}, \ldots, y_{t-k}) = \mathbb{N}\left(m_t, \sigma_t^2\right)$$

and

$$\begin{aligned} & p(s_t | s_{t-1}, \ldots, s_{t-k-1}, y_{t-1}, \ldots, y_{t-k-1}) \\ & = (1 - s_t)\omega_\rho(s_{t-1}, \ldots, s_{t-k-1}, y_{t-1}, \ldots, y_{t-k-1}) \\ & \quad + s_t\left[1 - \omega_\rho(s_{t-1}, \ldots, s_{t-k-1}, y_{t-1}, \ldots, y_{t-k-1})\right], \end{aligned}$$

The transition probability ω_ρ of the endogenous state process (s_t) to low state is given in the next slides, separately for the cases with $|\rho| < 1$ and $|\rho| = 1$

Transition Probability to Low State ω_ρ for $|\rho| < 1$

- If $|\alpha| < 1$,

$$\omega_\rho(s_{t-1}, \ldots, s_{t-k-1}, y_{t-1}, \ldots, y_{t-k-1})$$
$$= \frac{\left[(1-s_{t-1})\int_{-\infty}^{\tau\sqrt{1-\alpha^2}} + s_{t-1}\int_{\tau\sqrt{1-\alpha^2}}^{\infty}\right] \Phi_\rho\left(\tau - \rho\frac{y_{t-1}-m_{t-1}}{\sigma_{t-1}} - \frac{\alpha x}{\sqrt{1-\alpha^2}}\right)\varphi(x)dx}{(1-s_{t-1})\Phi(\tau\sqrt{1-\alpha^2}) + s_{t-1}\left[1-\Phi(\tau\sqrt{1-\alpha^2})\right]},$$

- If $\alpha = 1$,

for $t = 1$, $\omega_\rho(s_0) = \Phi(\tau)$ with $\mathbb{P}\{s_0 = 0\} = 1$ and $\mathbb{P}\{s_0 = 1\} = 1$ respectively when $\tau > 0$ and $\tau \leq 0$ and, for $t \geq 2$,

$$\omega_\rho(s_{t-1}, \ldots, s_{t-k-1}, y_{t-1}, \ldots, y_{t-k-1})$$
$$= \frac{\left[(1-s_{t-1})\int_{-\infty}^{\tau/\sqrt{t-1}} + s_{t-1}\int_{\tau/\sqrt{t-1}}^{\infty}\right] \Phi_\rho\left(\tau - \rho\frac{y_{t-1}-m_{t-1}}{\sigma_{t-1}} - x\sqrt{t-1}\right)\varphi(x)dx}{(1-s_{t-1})\Phi(\tau/\sqrt{t-1}) + s_{t-1}\left[1-\Phi(\tau/\sqrt{t-1})\right]},$$

with $\Phi_\rho(x) = \Phi\left(x/\sqrt{1-\rho^2}\right)$.

Important Remarks

- The transition probability of the state process (s_t) at time t from time $t-1$ depends upon y_{t-1} and other lags as well as s_{t-1} and other lags
- The state process (s_t) alone is therefore not Markov
- However, the state process augmented with the observed time series (s_t, y_t) becomes a Markov process
- If $\rho = 0$, (s_t) reduces to a Markov process independent of (y_t) as in the conventional Markov switching model, with the transition obtained earlier.

which highlights the differences between the new and conventional Markov switching models.

Modified Markov Switching Filter

To develop the modified Markov switching filter, we write

$$p(y_t|\mathcal{F}_{t-1}) = \sum_{s_t} \cdots \sum_{s_{t-k}} p(y_t|s_t, \ldots, s_{t-k}, \mathcal{F}_{t-1}) p(s_t, \ldots, s_{t-k}|\mathcal{F}_{t-1}).$$

Since

$$p(y_t|s_t, \ldots, s_{t-k}, y_{t-1}, \ldots, y_{t-k}) = \mathbb{N}\left(m_t, \sigma_t^2\right),$$

it suffices to have $p(s_t, \ldots, s_{t-k}|\mathcal{F}_{t-1})$ to compute the log-likelihood function. This is done by the repeated implementations of the prediction and updating steps, as in the usual Kalman filter.

Prediction Step

For the prediction step, we note that

$$
\begin{aligned}
&p(s_t, \ldots, s_{t-k} | \mathcal{F}_{t-1}) \\
&= \sum_{s_{t-k-1}} p(s_t | s_{t-1}, \ldots, s_{t-k-1}, \mathcal{F}_{t-1}) p(s_{t-1}, \ldots, s_{t-k-1} | \mathcal{F}_{t-1}),
\end{aligned}
$$

and

$$
\begin{aligned}
&p(s_t | s_{t-1}, \ldots, s_{t-k-1}, \mathcal{F}_{t-1}) \\
&= p(s_t | s_{t-1}, \ldots, s_{t-k-1}, y_{t-1}, \ldots, y_{t-k-1}).
\end{aligned}
$$

Prediction Step - Continued

The transition probability of state is given as

$$
\begin{aligned}
&p(s_t|s_{t-1},\dots,s_{t-k-1},y_{t-1},\dots,y_{t-k-1})\\
&\quad= (1-s_t)\omega_\rho(s_{t-1},\dots,s_{t-k-1},y_{t-1},\dots,y_{t-k-1})\\
&\qquad+ s_t\left[1-\omega_\rho(s_{t-1},\dots,s_{t-k-1},y_{t-1},\dots,y_{t-k-1})\right],
\end{aligned}
$$

where, in turn, if $|\alpha| < 1$,

$$
\begin{aligned}
&\omega_\rho(s_{t-1},\dots,s_{t-k-1},y_{t-1},\dots,y_{t-k-1})\\
&= \frac{\left[(1-s_{t-1})\int_{-\infty}^{\tau\sqrt{1-\alpha^2}} + s_{t-1}\int_{\tau\sqrt{1-\alpha^2}}^{\infty}\right]\Phi_\rho\left(\tau-\rho\frac{y_{t-1}-m_{t-1}}{\sigma_{t-1}}-\frac{\alpha x}{\sqrt{1-\alpha^2}}\right)\varphi(x)dx}{(1-s_{t-1})\Phi(\tau\sqrt{1-\alpha^2})+s_{t-1}\left[1-\Phi(\tau\sqrt{1-\alpha^2})\right]},
\end{aligned}
$$

Therefore, $p(s_t,\dots,s_{t-k}|\mathcal{F}_{t-1})$ can be readily computed, once $p(s_{t-1},\dots,s_{t-k-1}|\mathcal{F}_{t-1})$ obtained from the previous updating step.

Updating Step

Finally, for the updating step, we have

$$
\begin{aligned}
p(s_t, \ldots, s_{t-k} | \mathcal{F}_t) &= p(s_t, \ldots, s_{t-k} | y_t, \mathcal{F}_{t-1}) \\
&= \frac{p(y_t | s_t, \ldots, s_{t-k}, \mathcal{F}_{t-1}) p(s_t, \ldots, s_{t-k} | \mathcal{F}_{t-1})}{p(y_t | \mathcal{F}_{t-1})},
\end{aligned}
$$

where $p(y_t | s_t, \ldots, s_{t-k}, \mathcal{F}_{t-1}) = p(y_t | s_t, \ldots, s_{t-k}, y_{t-1}, \ldots, y_{t-k})$.

We may readily obtain $p(s_t, \ldots, s_{t-k} | \mathcal{F}_t)$ from $p(s_t, \ldots, s_{t-k} | \mathcal{F}_{t-1})$ and $p(y_t | \mathcal{F}_{t-1})$ from previous prediction step.

Extraction of Latent Factor

From prediction step, we have

$$p(s_t, ..., s_{t-k}|\mathcal{F}_{t-1}) = \sum_{s_{t-k-1}} p(s_t|s_{t-1}, ..., s_{t-k-1}, \mathcal{F}_{t-1})p(s_{t-1}, ..., s_{t-k-1}|\mathcal{F}_{t-1}),$$

$$p(w_t, \ldots, s_{t-k}|\mathcal{F}_{t-1}) = \sum_{s_{t-k-1}} p(w_t|s_{t-1}, ..., s_{t-k-1}, \mathcal{F}_{t-1})p(s_{t-1}, ..., s_{t-k-1}|\mathcal{F}_{t-1}).$$

From updating step, we get

$$p(s_t, ..., s_{t-k}|\mathcal{F}_t) = \frac{p(y_t|s_t, ..., s_{t-k}, \mathcal{F}_{t-1})p(s_t, ..., s_{t-k}|\mathcal{F}_{t-1})}{p(y_t|\mathcal{F}_{t-1})},$$

$$p(w_t, s_{t-1}, ..., s_{t-k}|\mathcal{F}_t) = \frac{p(y_t|w_t, s_{t-1}, ..., s_{t-k}, \mathcal{F}_{t-1})p(w_t, s_{t-1}, ..., s_{t-k}|\mathcal{F}_{t-1})}{p(y_t|\mathcal{F}_{t-1})}.$$

Extraction of Latent Factor

When $|\alpha| < 1$ and $|\rho| < 1$, we can show

$$p\left(w_t | s_{t-1} = 1, s_{t-2}, ..., s_{t-k-1}, \mathcal{F}_{t-1}\right)$$
$$= \frac{\left(1 - \Phi\left(\sqrt{\frac{1-\rho^2+\alpha^2\rho^2}{1-\rho^2}}\left(\tau - \frac{\alpha(w_t - \rho u_{t-1})}{1-\rho^2+\alpha^2\rho^2}\right)\right)\right)}{1 - \Phi\left(\tau\sqrt{1-\alpha^2}\right)} \mathbb{N}\left(\rho u_{t-1}, \frac{1-\rho^2+\alpha^2\rho^2}{1-\alpha^2}\right)$$

$$p\left(w_t | s_{t-1} = 0, s_{t-2}, ..., s_{t-k-1}, \mathcal{F}_{t-1}\right)$$
$$= \frac{\Phi\left(\sqrt{\frac{1-\rho^2+\alpha^2\rho^2}{1-\rho^2}}\left(\tau - \frac{\alpha(w_t - \rho u_{t-1})}{1-\rho^2+\alpha^2\rho^2}\right)\right)}{\Phi\left(\tau\sqrt{1-\alpha^2}\right)} \mathbb{N}\left(\rho u_{t-1}, \frac{1-\rho^2+\alpha^2\rho^2}{1-\alpha^2}\right).$$

Extraction of Latent Factor

By marginalizing, we can obtain

$$p\left(w_t|\mathcal{F}_t\right) = \sum_{s_{t-1}} \cdots \sum_{s_{t-k}} p\left(w_t, s_{t-1}, ..., s_{t-k}|\mathcal{F}_t\right).$$

which yields the inferred factor

$$\mathbb{E}\left(w_t|\mathcal{F}_t\right) = \int w_t p\left(w_t|\mathcal{F}_t\right) dw_t.$$

We may easily extract the inferred factor, once the maximum likelihood estimates of $p(w_t|\mathcal{F}_t), 1 \leq t \leq n$, are available.

GDP Growth Rates

We use

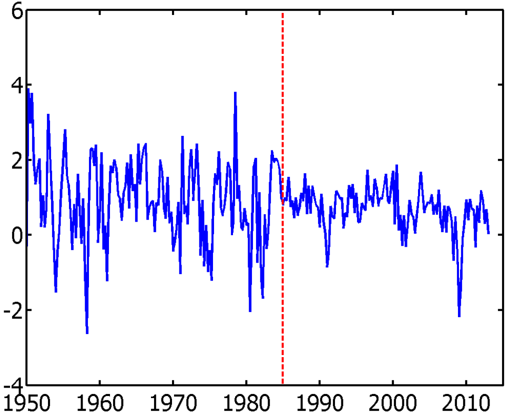
- Seasonally adjusted quarterly real US GDP for two sample periods: 1952-1984 and 1984-2012
- GDP growth rates are obtained as the first differences of their logs

to fit the mean model

$$\gamma(L)\,(y_t - \mu(s_t)) = \sigma u_t$$

where $\gamma(z) = 1 - \gamma_1 z - \gamma_2 z^2 - \gamma_3 z^3 - \gamma_4 z^4$.

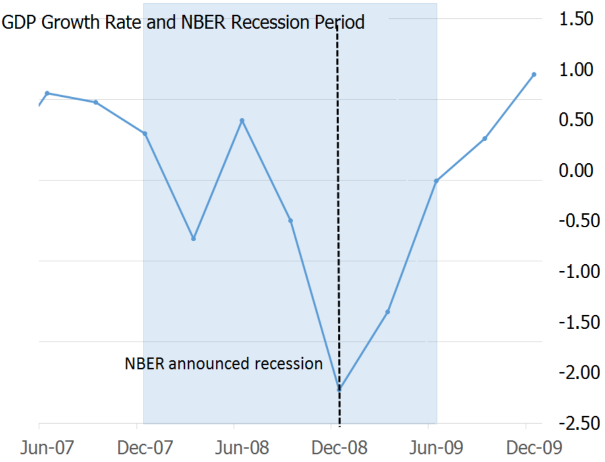
US Real GDP Growth Rates



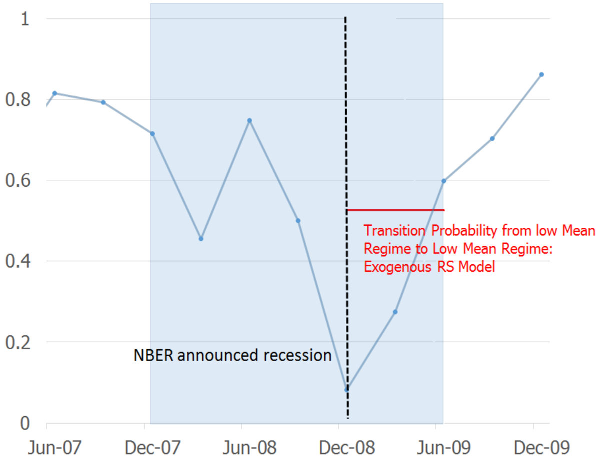
Estimation Result: GDP Growth Rate Model

Sample Periods	1952-1984		1984-2012	
Endogeneity	Ignored	Allowed	Ignored	Allowed
$\underline{\mu}$	-0.165	-0.083	-0.854	-0.756
	(0.219)	(0.161)	(0.298)	(0.318)
$\overline{\mu}$	1.144	1.212	0.710	0.705
	(0.113)	(0.095)	(0.092)	(0.085)
γ_1	0.068	0.147	0.154	0.169
	(0.123)	(0.104)	(0.105)	(0.106)
γ_2	-0.015	0.044	0.350	0.340
	(0.112)	(0.096)	(0.105)	(0.104)
γ_3	-0.175	-0.260	-0.077	0.133
	(0.108)	(0.090)	(0.106)	(0.104)
γ_4	-0.097	-0.067	0.043	0.049
	(0.104)	(0.095)	(0.103)	(0.115)
σ	0.794	0.784	0.455	0.453
	(0.065)	(0.057)	(0.034)	(0.032)
ρ		**-0.923**		**1.000**
		(0.151)		(0.001)
log-likelihood	-173.420	-169.824	-80.584	-76.443
p-value		0.007		0.004

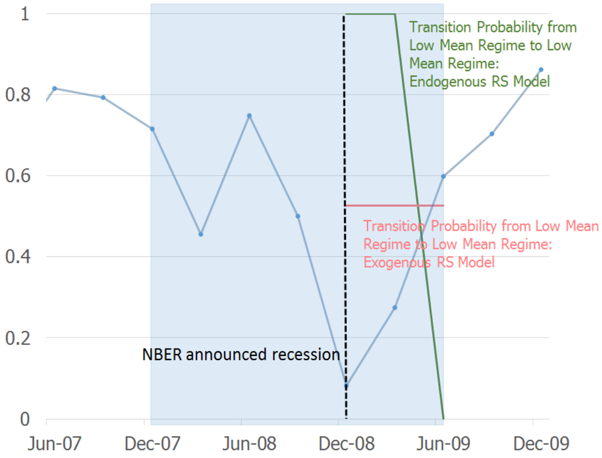
Transition Probability Comparison



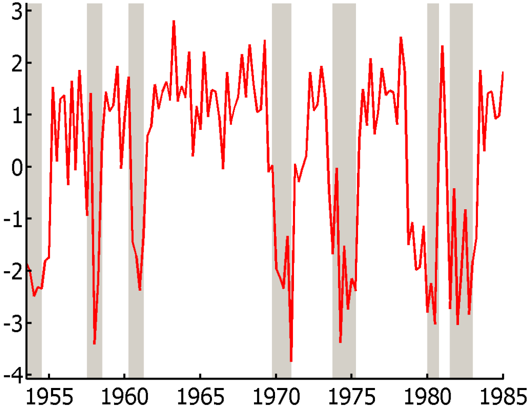
Transition Probability Comparison



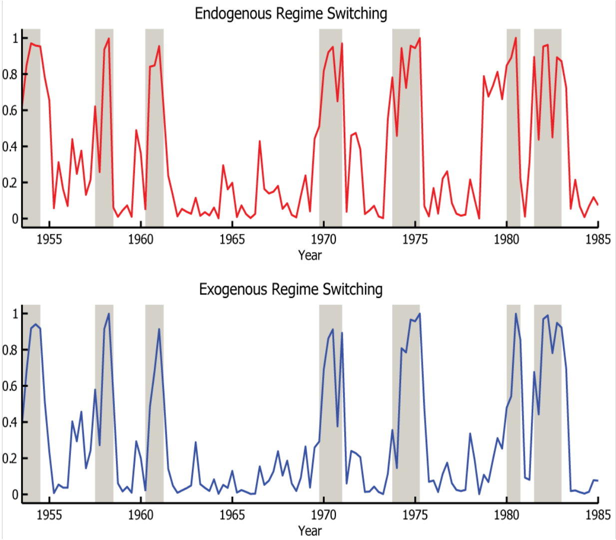
Transition Probability Comparison



NBER Recession Period and Latent Factor: 1952-1984



Recession Probabilities: 1952-1984



Stock Return Volatility

We use

- Monthly CRSP returns for 1926/01 - 2012/12 (1,044 obs.)
- One-month T-bill rates used to obtain excess returns
- Demeaned excess returns

to fit the volatility model

$$y_t = \sigma(s_t)u_t,$$

where

$$\sigma(s_t) = \underline{\sigma}(1 - s_t) + \bar{\sigma}s_t$$

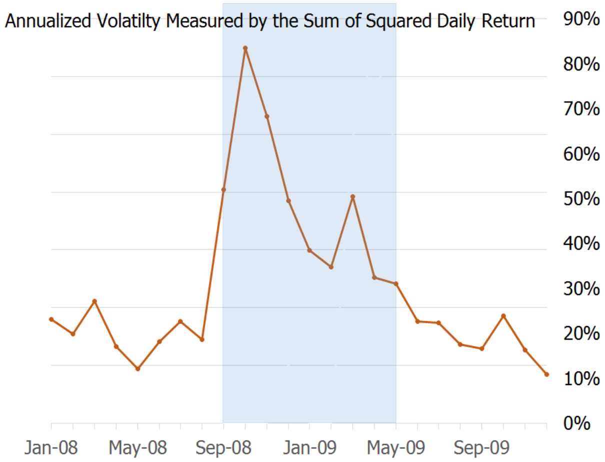
and

$$s_t = 1\{w_t \geq \tau\}.$$

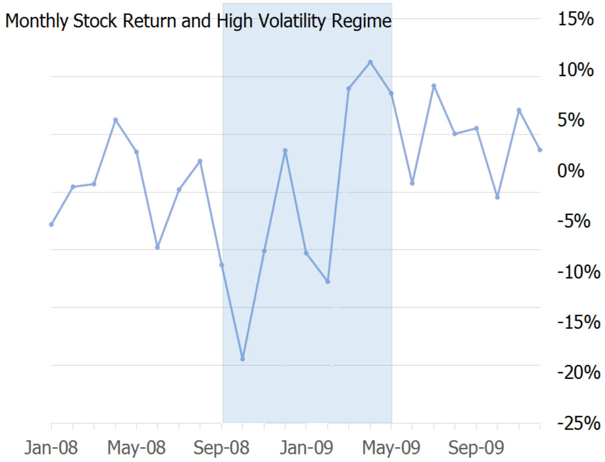
Estimation Result: Monthly Volatility Model

Sample Periods	1926-2012		1990-2012	
Endogeneity	Ignored	Allowed	Ignored	Allowed
$\underline{\sigma} = \sigma(s_t)$ when $s_t = 0$	0.0385	0.0380	0.0223	0.0251
	(0.0010)	(0.0011)	(0.0018)	(0.0041)
$\overline{\sigma} = \sigma(s_t)$ when $s_t = 1$	0.1154	0.1153	0.0505	0.0554
	(0.0087)	(0.0090)	(0.0030)	(0.0082)
ρ		**-0.9698**		**-1.0000**
		(0.0847)		(0.0059)
log-likelihood	1742.28	1747.98	507.70	511.28
p-value (LR test for $\rho = 0$)		**0.001**		**0.007**

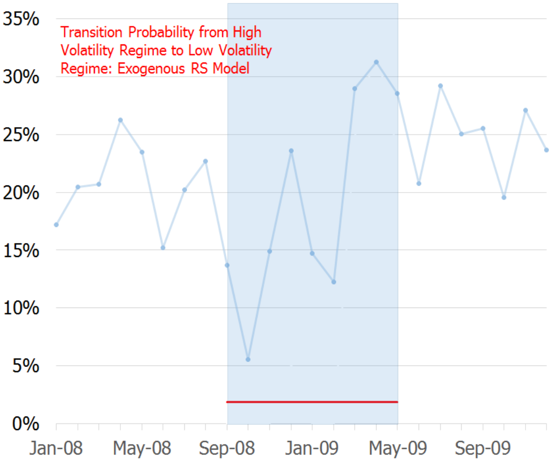
Transition Probability Comparison



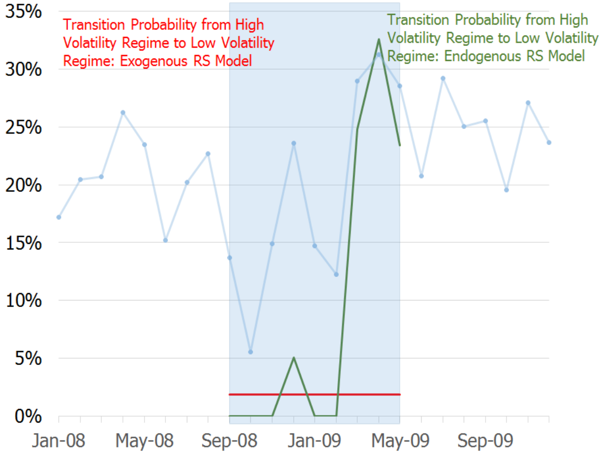
Transition Probability Comparison



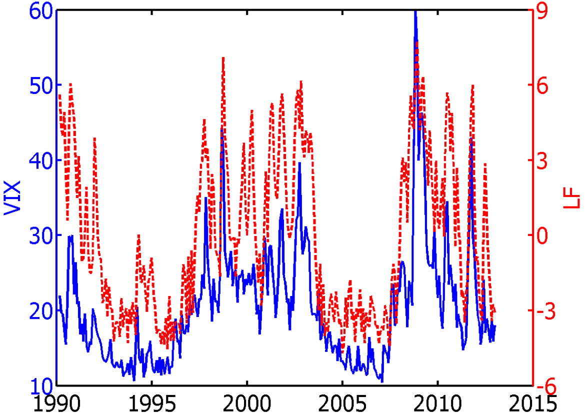
Transition Probability Comparison



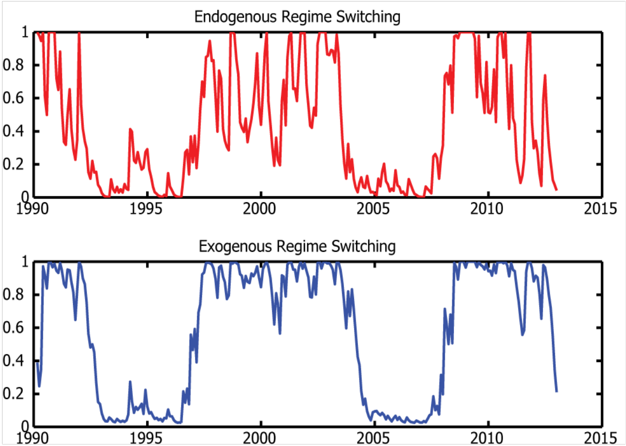
Transition Probability Comparison



Extracted Latent Factor from Volatility Model and VIX



High Volatility Probabilities: 1990-2012



Monetary and Fiscal Policy Interactions

Chang and Kwak (2016), Endogenous Monetary-Fiscal Regime Change in the United States.

- Policy interaction is crucial for the determination of price level. However, empirical research so far has focused on policy variables interactions.
- We use a new endogenous regime switching methodology to allow for the policy regime interactions.
- We characterize the monetary and fiscal policy regimes using the extracted latent factor from endogenous monetary and fiscal regime switching models.
- We analyze policy interactions using the extracted latent factors.

Policy Interactions

Monetary and fiscal regimes *jointly* accomplish the tasks of

- determining inflation/price level
- stabilizing government debt
- Leeper (1991), Woodford (1995), Sims (2013)

Conventional view (Regime M)

- MP aggressively adjusts the policy interest rate in response to inflation, and FP passively adjusts taxes and spending to ensure that debt returns to steady state
- Monetarist/ new Keynesian outcomes

Alternative view (Regime F)

- FP actively determines inflation by making primary surpluses insensitive to debt, and MP passively maintains real value of debt by permitting the necessary change in the current and future price level to occur
- Fiscal theory of the price level

Policy Interactions and Policy Parameters

Consider stylized monetary policy rule

$$\text{MP:} \;\; i_t = a_c + a_\pi \pi_t + u_t^m$$

where i_t is nominal interest rate, π_t inflation, and fiscal policy rule

$$\text{FP:} \;\; \tau_t = \beta_c + \beta_b b_{t-1} + \beta_g g_t + u_t^f$$

where τ_t is tax revenues, b_{t-1} real market value of outstanding government debt, and g_t government purchases. Let r denote net real interest rate.

- Regime M: $a_\pi > 1$ and $\beta_b > r$ (Active MP/Passive FP) MP actively controls inflation and FP passively stabilizes debt.
- Regime F: $0 \leq a_\pi < 1$ and $\beta_b < r$ (Passive MP/Active FP) FP actively determines the price level and MP passively stabilizes debt.

Empirical Approaches in Policy Interactions

- Dynamic patterns of correlation among policy variables
  - King and Plosser (1985), Melitz (2000), Muscatelli et al. (2002), Kliem et al. (2015)
  - Correlation among policy variables can tell us nothing about interaction between policy regimes
- Exogenous regime switching policy rules in DSGE models
  - Davig and Leeper (2006), Ballabriga and Martinez-Mongay (2002) Gonzalez-Astudillo (2013), Bianchi and Ilut (2014)
  - Exogenous regime change is silent about a mechanism which connects changes in macroeconomic environment to switches in policy regimes

New Approach to Regime Switching Policy Rules

Consider the policy rule equation

$$y_t = x_t' \beta_{s_t} + u_t,$$

where

- y_t: policy instrument, x_t: policy target variables, β_{s_t}: state dependent policy parameters
- u_t: policy disturbance representing multitude of all other factors such as policy shocks and other policy concerns that affect the policy making but are not measured by the policy target variables x_t. Not regarded as an exogenous shock.

$$\mathbb{E}\left[u_t \middle| s_t, x_t, \mathcal{G}_{t-1}\right] = 0$$

- $\mathcal{G}_{t-1}$ is the information available at time $t-1$ to the policy makers

Policy Regime and Latent Policy Factor

- s_t is a state variable at t driven by the latent factor w_t as

$$s_t = 1\{w_t \geq \tau\},$$

- w_t is a latent policy factor representing the internal information set used by a policy maker.

$$w_t = \alpha w_{t-1} + v_t,$$

- v_t and u_{t-1} are jointly i.i.d. normal with unit variance and $\text{cov}\,(u_{t-1}, v_t) = \rho$.
- Policy choice of β_{s_t} depends on previous u_{t-1} and on an exogenous component realized at t.

Regime Switching Policy Rules

- Endogenous regime switching policy rule

$$\beta_{s_t} = \arg\min_{\beta} \mathbb{E}\left[(y_t - x_t'\beta)^2 \middle| s_t, x_t, \mathcal{G}_{t-1} \right],$$

- $\mathcal{G}_{t-1}$ is the information available at time $t-1$ to the policy makers, which includes entire history of policy instrument y, policy target variables x, and state variable s up to time $t-1$.
- β_{s_t} minimizes MSE loss incurred by u_t conditionally on s_t and x_t and all other information in $\mathcal{G}_{t-1}$. State dependent policy choice β_{s_t} naturally entails the policy rule which is well formulated as a regression satisfying usual ortho. condition.
- Conventional regime switching policy rule

$$\beta_{s_t} = \arg\min_{\beta} \mathbb{E}\left[(y_t - x_t'\beta)^2 \middle| s_t, x_t, \mathcal{F}_{t-1} \right],$$

- $\mathcal{F}_{t-1}$ only includes policy instrument y and policy target x observed at time $t-1$, excluding all other past policy rules and disturbances.

Monetary-Fiscal Regime Changes in the US

Data: Quarterly, 1949:1 - 2014:2 (262 obs)

Monetary Policy (MP)

- Nominal interest rate (i)
- Inflation rate (π)

Fiscal Policy (FP)

- All fiscal variables are for the federal government only
- Tax-output ratio (τ)
- Debt-output ratio (b)
- Government spending-output ratio (g)

MP: Regime Switching in Policy Parameters

Consider a regime switching monetary policy (MP)

$$i_t = a_c(s_t^m) + a_\pi(s_t^m)\pi_t + \sigma^m u_t^m$$

- a_c, a_π are state dependent MP parameters

$$a_j(s_t^m) = a_{j,0}(1 - s_t^m) + a_{j,1}s_t^m, \quad \text{for} \quad j = c, \pi$$

- s_t^m is a state in MP at t determined by the latent factor w_t^m as

$$s_t^m = 1\{w_t^m \geq \psi_m\}$$

- The latent MP factor w_t^m is interpreted as internal information of monetary policy makers, and generated endogenously as

$$w_t^m = \alpha_m w_{t-1}^m + v_t^m$$

and

$$\begin{pmatrix} u_t^m \\ v_{t+1}^m \end{pmatrix} = \mathbb{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho_m \\ \rho_m & 1 \end{pmatrix} \right)$$

Endogeneity of Latent Policy Factor

- The latent MP factor is generated endogenously with cov $(u^m_{t-1}, v_t) = \rho$.

- The monetary policy shock u^m_t reflects the multitude of factors that influence policy's choices of the interest rate i_t that are not embedded in π_t. For example, if news contained in commodity prices portends higher future inflation, but not higher current inflation π_t, then u^m_t will be positive, and the interest rate i_t will be higher than the current inflation π_t alone would predict.

- The correlation ρ between the current policy disturbance u^m_t and the future associated latent policy regime s^m_{t+1} is estimated. The value of s^m_{t+1} in turn influences the likelihood that the policy rule parameters $a_c(s^m_{t+1})$ and $a_\pi(s^m_{t+1})$ will take on distinct values associated with Regime M or Regime F.

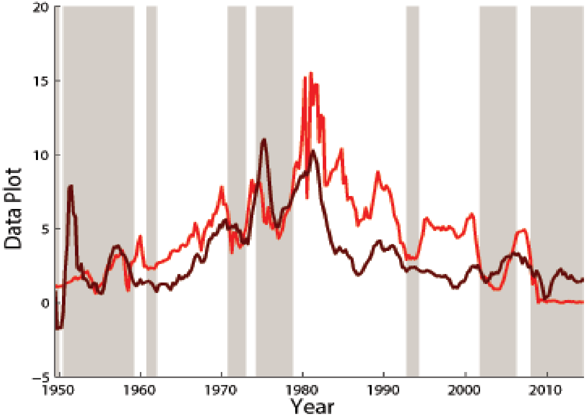
MP: Estimation Results

Parameter	Estimate	S.E
α_m	0.985	(0.009)
ψ_m	-0.964.	(2.320)
ρ_m	0.999	(0.025)
$a_c(s_t^m = 0)$	0.441	(0.245)
$a_c(s_t^m = 1)$	2.572	(0.257)
$a_\pi(s_t^m = 0)$	0.654	(0.059)
$a_\pi(s_t^m = 1)$	1.044	(0.060)
σ^m	1.302	(0.059)
log-likelihood	-456.309	
p-value(LR test for $\rho_m = 0$)	0.00000217	

$u_t^m > 0 \Rightarrow w_{t+1}^m \uparrow$: More likely to have active MP in future

Average interest rate by regime: 6.11 (Active), 2.43 (Passive)
Average inflation rate by regime: 3.39 (Active), 3.02 (Passive)

MP: Estimated State Distribution



Red: T-bill rate, Black: Inflation rate, Shaded: Passive MP

FP: Regime Switching in Policy Parameters

Consider a regime switching fiscal policy (FP)

$$\tau_t = \beta_c(s_t^f) + \beta_b(s_t^f)b_{t-1} + \beta_g(s_t^f)g_t + \sigma^f u_t^f$$

- $\beta_c, \beta_b, \beta_g$ are state dependent FP parameters

$$\beta_j(s_t^f) = \beta_{j,0}(1 - s_t^f) + \beta_{j,1}s_t^f, \quad \text{for} j = c, b, g$$

- s_t^f is a state in FP at t driven by the latent factor w_t^f as

$$s_t^f = 1\{w_t^f \geq \psi_f\}$$

- The latent FP factor w_t^f is interpreted as internal information of fiscal policy makers, and generated endogenously as

$$w_t^f = \alpha_f w_{t-1}^f + v_t^f$$

and

$$\begin{pmatrix} u_t^f \\ v_{t+1}^f \end{pmatrix} = \mathbb{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho_f \\ \rho_f & 1 \end{pmatrix} \right)$$

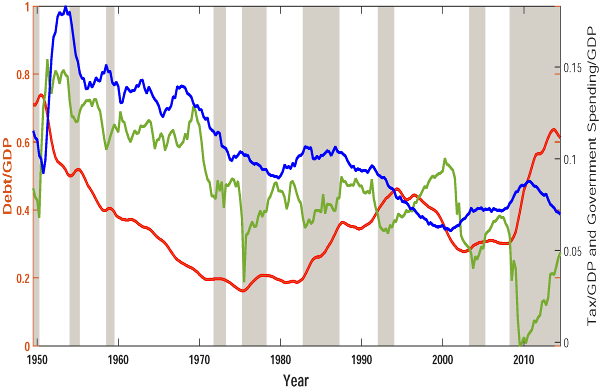
FP: Estimation Results

Parameter	Estimate	S.E
α_f	0.972	(0.020)
ψ_f	-0.546	(1.243)
ρ_f	0.999	(0.001)
$\beta_c(s_t^f = 0)$	-0.028	(0.010)
$\beta_c(s_t^f = 1)$	0.012	(0.006)
$\beta_b(s_t^f = 0)$	-0.033	(0.001)
$\beta_b(s_t^f = 1)$	0.056	(0.011)
$\beta_g(s_t^f = 0)$	1.025	(0.091)
$\beta_g(s_t^f = 1)$	0.599	(0.053)
σ^f	0.014	(0.001)
log-likelihood	727.71	
p-value(LR test for $\rho_f = 0$)	0.0002	

$u_t^f > 0 \Rightarrow w_{t+1}^f \uparrow$: More likely to have passive FP in future

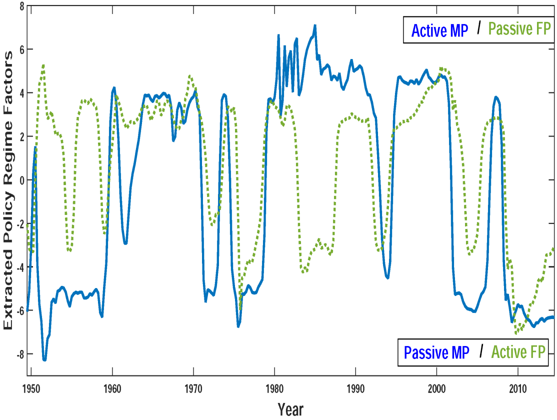
Average tax/GDP ratio by regime: 0.06 (Active), 0.1 (Passive)
Average debt/GDP ratio by regime: 0.38 (Active), 0.34 (Passive)

FP: Estimated State Distribution



Green: Tax/GDP, Blue: Govt. spending/GDP,
Orange: Debt/GDP, Shaded: Active FP

Extracted Monetary and Fiscal Policy Factors



Solid blue: MP Factor, Dashed green: FP Factor, Correlation=0.48
Policy factors usually move together to deliver one active and one passive policy with some exceptions

Policy Interactions

To investigate whether a change in MP regime induces a change in FP regime that delivers a unique equilibrium (and vice versa), we analyze policy interactions using extracted policy regime factors.

- **Extracted policy regime factors**: proxies of internal information of policy authorities in determination of policy regimes
- **Endogenous evolution of regime**: monetary policy's choice of its rule may influence fiscal policy's choice of its rule (and vice versa)

Analyses with Extracted Policy Factors

- TVC-VAR on the extracted monetary and fiscal policy factors to investigate the dynamic interactions of the policy regimes. Estimated by the classical kernel method in Giraitis et al. (2014)
- SVAR on two policy factors and key macro variables to analyze effects of macroeconomic shocks on regime factors and vice versa
- Adaptive LASSO to link policy factors to macroeconomy
- FAVAR à la Bernanke, Boivin and Eliasz (2005) to analyze effects of policy shocks to key macroeconomic variables based in the VAR on the five leading factors from PCA on 129 variables including our two extracted policy factors

TVC-VAR Estimation

We consider a TVC-VAR model given by

$$y_t = \Psi_t y_{t-1} + \eta_t$$

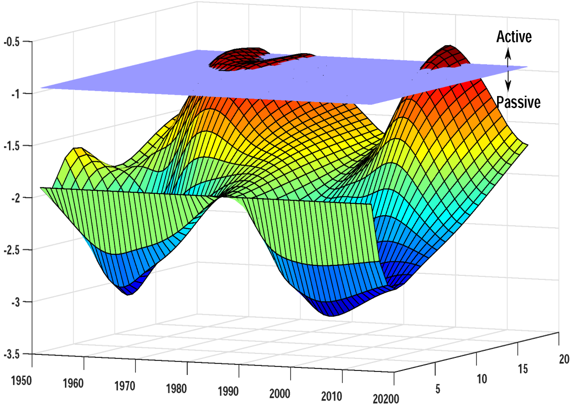
where $y_t = (w_t^m, w_t^f)'$, Ψ_t is 2×2 matrix of coefficient processes, and $\eta_t = (\eta_t^m, \eta_t^f)'$ is the noise with $E\eta_t\eta_s' = 0, t \neq s$, $t = 1, 2, \dots n$.

The TVC Ψ_t is estimated as

$$\widehat{\Psi}_t = \left(\sum_{s=1}^{n} k_{t,s} y_s y_{s-1}' \right) \left(\sum_{s=1}^{n} k_{t,s} y_{s-1} y_{s-1}' \right)^{-1}$$

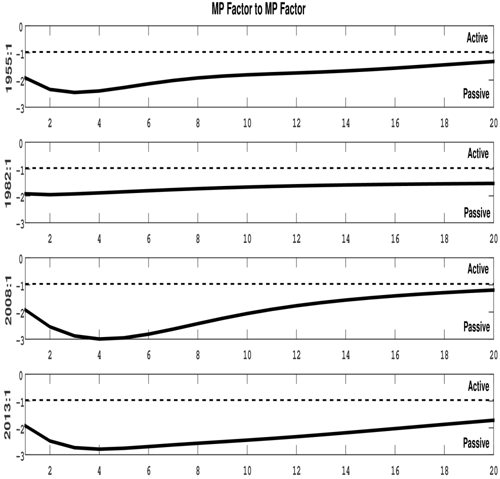
with the weights $k_{t,s} = K((t-s)/H_\Psi)$ are given by the kernel function K, and the bandwidth parameter H_Ψ.

Response of MP Regime to MP Regime



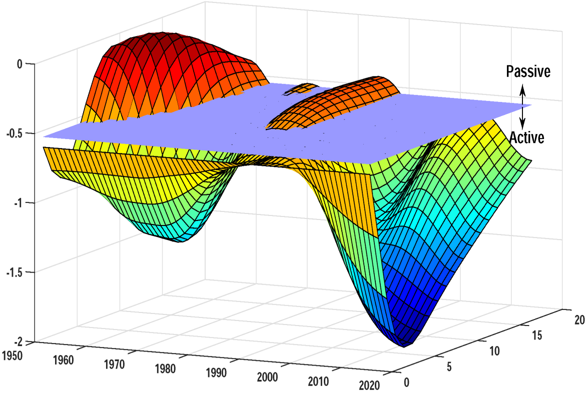
Passive MP below the surface, Active MP above the surface

MP Regime to MP Regime on Selected Years



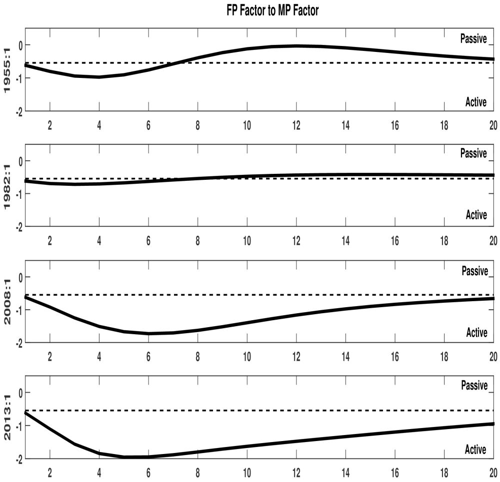
1955:1, 1982:1, 2008:1, 2013:1 from top to bottom

Response of FP Regime to MP Regime



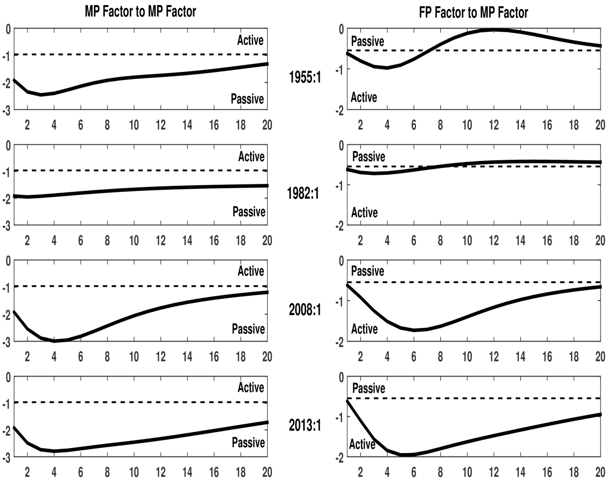
Active FP below the surface, Passive FP above the surface

FP Regime to MP Regime on Selected Years



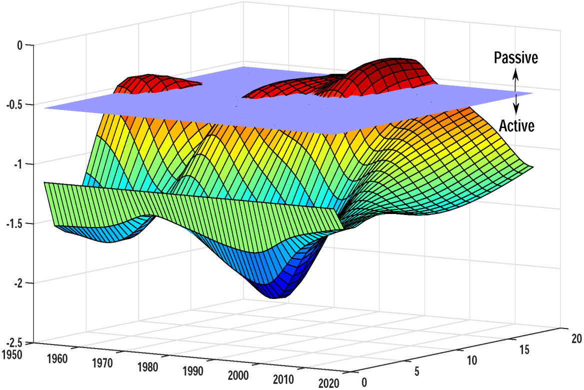
1955:1, 1982:1, 2008:1, 2013:1 from top to bottom

Policy Factors to MP Regime on Selected Years



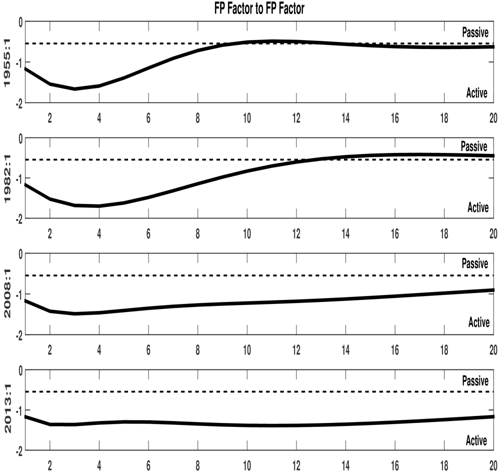
1955:1, 1982:1, 2008:1, 2013:1 from top to bottom rows

Response of FP Regime to FP Regime



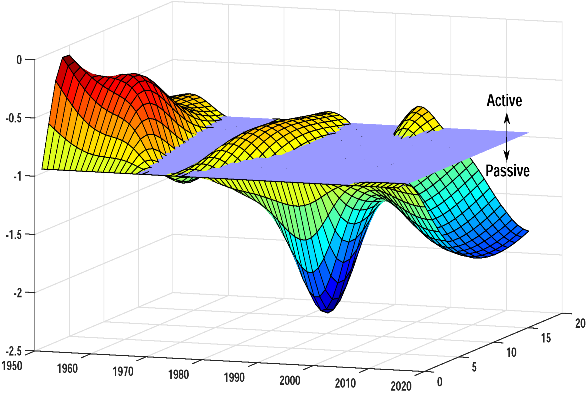
Active FP below the surface, Passive FP above the surface

FP Regime to FP Regime on Selected Years



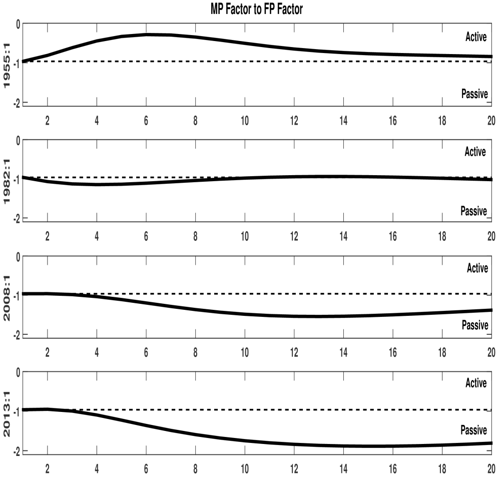
1955:1, 1982:1, 2008:1, 2013:1 from top to bottom row

Response of MP Regime to FP Regime



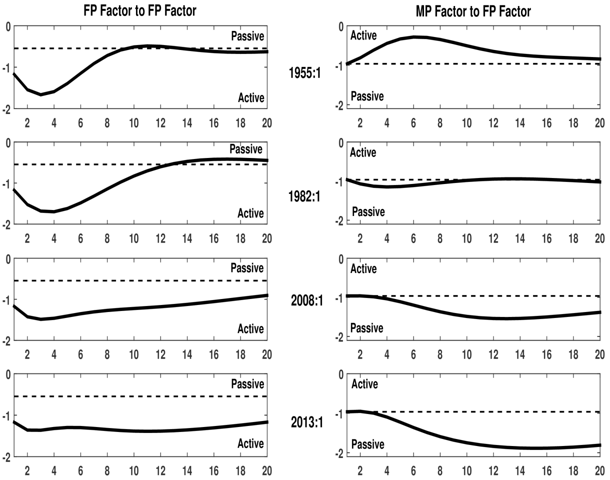
Active MP above the surface, Passive MP below the surface

MP Regime to FP Regime on Selected Years



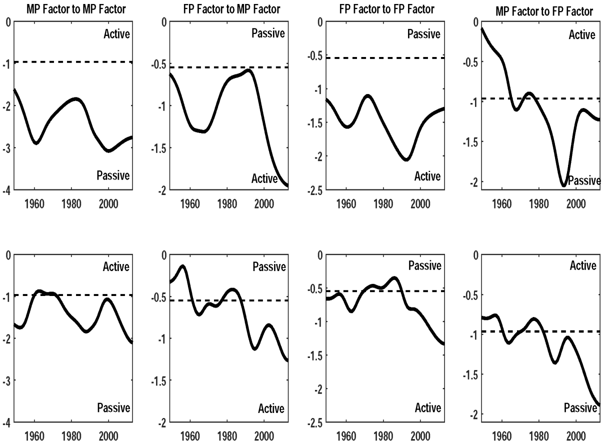
1955:1, 1982:1, 2008:1, 2013:1 from top to bottom row

Policy Factors to FP Regime on Selected Years



1955:1, 1982:1, 2008:1, 2013:1 from top to bottom rows

Response of ELFs on Selected Horizons



Top: 5 quarters from the initial shock
Bottom: 10 quarters from the initial shock

SVAR on Policy and Non-Policy Sector Variables

- To analyze effects of macroeconomic shocks on regime factors, we take a SVAR approach based on the structural form

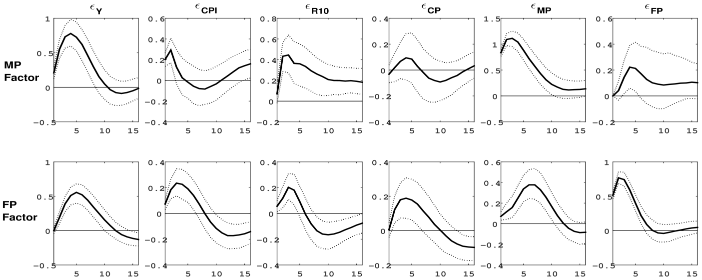
$$\sum_{s=0}^{p} A_s y_{t-s} = \epsilon_t$$

- y_t is a vector of observed time series and ϵ_t is a vector of i.i.d structural disturbances that are exogenous to the model.
- Non-policy sector: real GDP, CPI, long term interest rate, commodity price index
- Policy sector: MP and FP factors
- Non-policy and policy sector disturbances of the economy

$$\epsilon_t = \begin{bmatrix} \epsilon_{Nt} \\ \epsilon_{Pt} \end{bmatrix}$$

where ϵ_{Nt} and ϵ_{Pt} are the vectors of non-policy and policy sector disturbances.

Policy Factors to Macro Shocks



	Structural Shocks					
Variables	ϵ_Y	ϵ_{CPI}	ϵ_{R10}	ϵ_{CP}	ϵ_{MP}	ϵ_{FP}
MP	31.3	1.5	3.8	4.3	55.4	3.6
FP	31.1	1.7	8.6	11.1	12.9	34.7

IRFs of Policy factors to macro shocks (above) and contributions of structural shocks (%) to the variance of policy factors (below)

Linking Policy Factors to Macroeconomy

Which macro variables are correlated with the dynamics of policy regime factors?

Adaptive LASSO is used to select effective macro variables.

The adaptive LASSO is a solution of least squares problems with weighted l_1 penalty

$$\hat{\beta}_L = \underset{\beta}{\operatorname{argmin}} \parallel y - X\beta \parallel^2 + \lambda \sum_{i=1}^{N} |\beta_i| / |\hat{\beta}_i|,$$

where λ is a nonnegative regularization parameter, $N = dim(X)$, and $|\hat{\beta}_i|$ is the adaptive weight based on the OLS or ridge $\hat{\beta}_i$. BIC is used for model selection.

Selected Macroeconomic Variables for MP Factor

Series	Est.Coeff	s.e
Debt/GDP ratio	2.03	0.349
10YTCR	1.79	0.758
Bank prime loan rate	1.33	0.534
Index of consumer expectations	1.19	0.262
Total loans and leases	1.11	0.445
Average weekly hours:manufacturing	0.93	0.395
NAPM new orders index	-1.02	0.304
Privately owned housing starts	-2.05	0.465

Selected Macroeconomic Variables for FP Factor

Series	Est.Coeff	s.e
Debt/GDP ratio	2.53	0.387
Output gap	1.29	0.217
Bank prime loan rate	0.80	0.376
Average hourly earing:manufacurng	0.69	0.176
Total consumer credit outstanding	0.41	0.173
Privately owned housing starts	-1.41	0.334
Net interest payment/Govt.outlays	-1.46	0.302
10YTCR-FFR	-2.20	0.511

Effects of Policy Factors on Macroeconomy

We analyze effects of policy regimes on key macroeconomic variables based on FAVAR à la Bernanke, Boivin and Eliasz (2005).

Observation equation:

$$X_t = \Lambda C_t + e_t = \Lambda^f F_t + \Lambda^w W_t + e_t$$

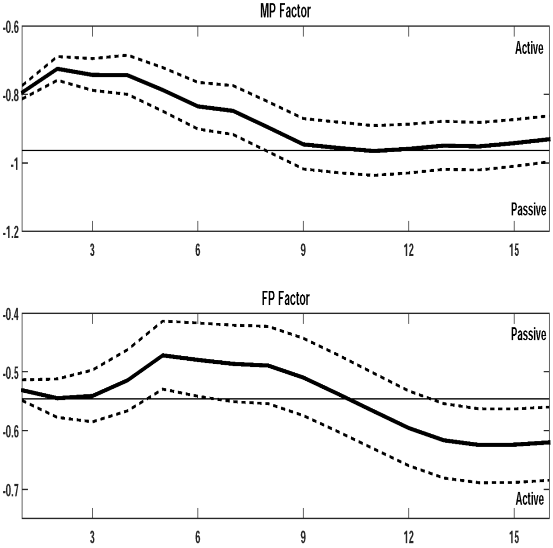
where W_t represents policy regime factors, F_t five leading principal components from 129 macro variables, and e_t an error term.

Transition equation:

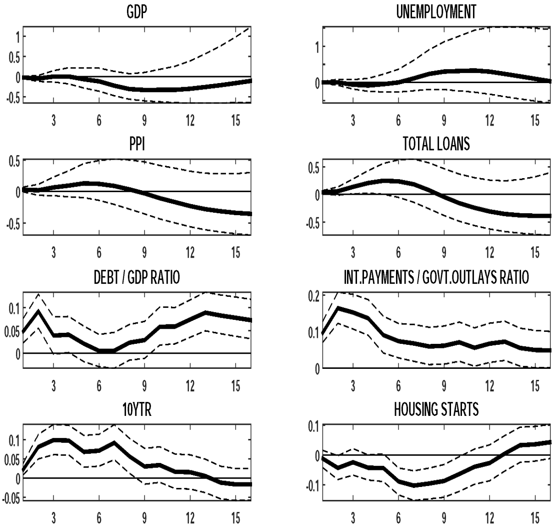
$$\begin{pmatrix} F_t \\ W_t \end{pmatrix} = \Phi_1 \begin{pmatrix} F_{t-1} \\ W_{t-1} \end{pmatrix} + \ldots + \Phi_p \begin{pmatrix} F_{t-5} \\ W_{t-5} \end{pmatrix} + v_t$$

where v_t an i.i.d error term.

Responses of Policy Regimes to MP Regime



Responses of Macrovariables to MP Regime

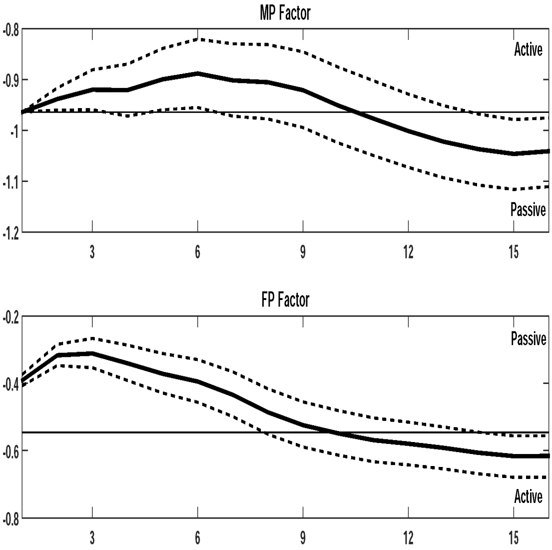


Explanatory Power of MP Regime

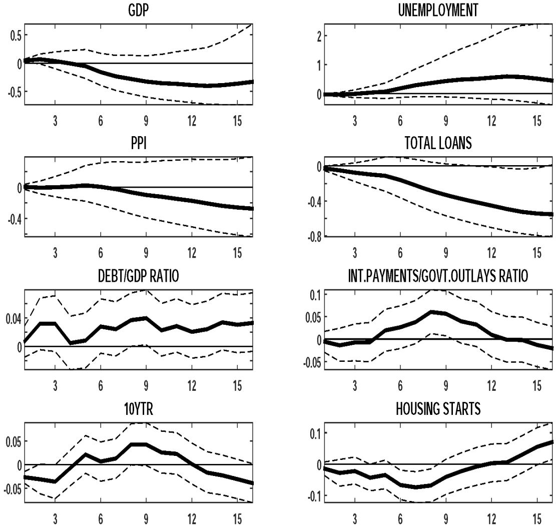
Table: Contributions of MP Regime Shocks (%) to the Variance of Variables

Variables	Variance Decomposition
MP Factor	48.9
FP Factor	5.2
GDP	5.4
Unemployment	10.1
PPI	12.2
Total loans	10.1
Debt/GDP ratio	20.8
Int.payment/Govt.outlays ratio	51.6
10YTR	19.3
Housing Starts	8.4

Responses of Policy Regimes to FP Regime



Responses of Macrovariables to FP Regime

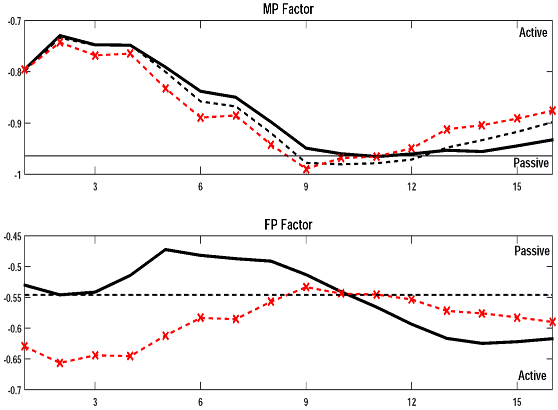


Explanatory Power of FP Regime

Table: Contributions of FP Regime Shocks (%) to the Variance of Variables

Variables	Variance Decomposition
FP Factor	64.3
MP Factor	4.6
GDP	4.2
Unemployment	6.0
PPI	8.5
Total loans	4.0
Debt/GDP ratio	8.6
Int.payment/Govt.outlays ratio	3.4
10YTR	7.9
Housing Starts	7.0

Counterfactual IRFs of ELFs to MP Regime

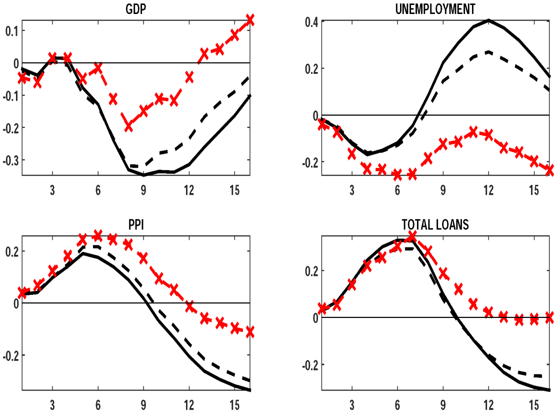


Black solid: Original responses (AM $\rightarrow$ PF)

Black dashed: No FP response (AM $\rightarrow$ No Response in FP)

Red star-dashed: Opposite FP regime response (AM $\rightarrow$ AF)

IRFs of Macrovariables to MP Regime

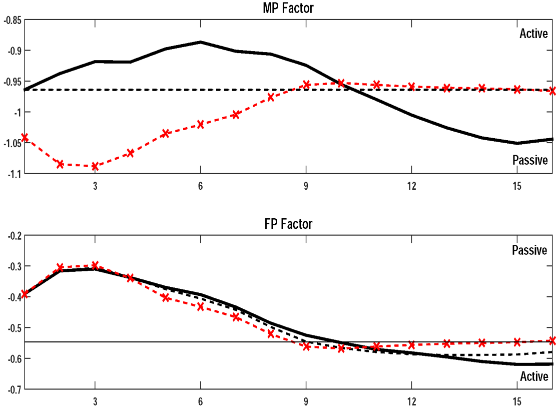


Black solid: Original responses (AM $\rightarrow$ PF)

Black dashed: No FP response (AM $\rightarrow$ No Response in FP)

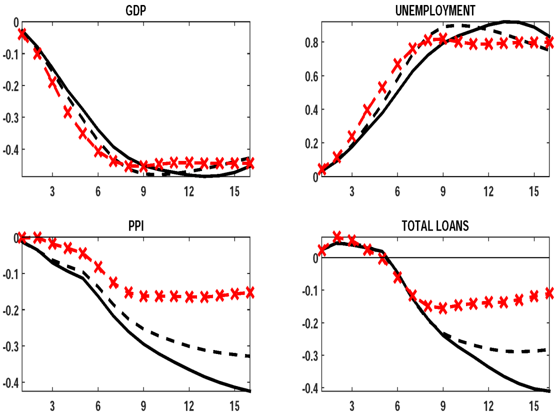
Red star-dashed: Opposite FP response (AM $\rightarrow$ AF)

Counterfactual IRFs of ELFs to FP Regime



Black solid: Original responses (PF $\rightarrow$ AM)
Black dashed: No MP response (PF $\rightarrow$ No Response in MP)
Red star-dashed: Opposite MP response (PF $\rightarrow$ PM)

IRFs of Macrovariables to FP Regime



Black solid: Original responses (PF $\rightarrow$ AM)

Black dashed: No MP response (PF $\rightarrow$ No Response in MP)

Red star-dashed: Opposite MP response (PF $\rightarrow$ PM)