Consumer Search and Price Competition*

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Abstract

We consider an oligopoly model in which consumers engage in sequential search based on partial product information and advertised prices. We derive a simple condition that fully summarizes consumers' shopping outcomes and use the condition to reformulate the pricing game among the sellers as a familiar discrete-choice problem. Exploiting the reformulation, we provide sufficient conditions that guarantee the existence and uniqueness of pure-strategy market equilibrium and obtain several novel insights about the effects of search frictions on market prices. Among others, we show that a reduction in search costs increases market prices, but providing more pre-search information raises market prices if and only if there are sufficiently many sellers.

JEL Classification Numbers: D43, D83, L13.

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1 Introduction

We consider an oligopoly model in which consumers sequentially search for the best product based on partial product information and advertised prices. A key distinguishing feature from

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traditional consumer search models is the observability of prices before consumer search. Consumers still face a non-trivial search problem, because they do not possess full information about their values for the products. In this environment, prices affect each seller's demand not only through their effects on consumers' final purchase decisions, but also through their effects on consumer search behavior. We study how the presence of the latter channel affects sellers' pricing incentives and what its economic consequences are. In particular, we investigate the effects of search frictions on market prices.

Consumer search models with observable prices have been drawing growing attention. The Internet has significantly lowered the cost of collecting price information. Now it is common to check prices online and visit stores only to get hands-on information and/or finalize a purchase. In the meantime, the model captures some salient features of online marketplaces and price comparison websites. A consumer typically begins with a summary webpage displaying multiple items. She clicks a certain set of items, collects more detailed information, and then makes a final purchase decision. Our model captures such consumer behavior particularly well. The analysis of our model can produce meaningful insights about the role of the Internet in traditional markets and the working of online marketplaces.

Similar models have been studied in three recent papers, Armstrong and Zhou (2011), Shen (2015), and Haan, Morage-González and Petrikaite (2015).[1] All three papers analyze a symmetric duopoly environment but consider different correlation structures for consumers' prior (known) and match (hidden) values. Both prior and match values are perfectly negatively correlated between the products in Armstrong and Zhou (2011), whereas both are independent in Haan, Morage-González and Petrikaite (2015). Shen (2015) examines an intermediate case where each consumer's prior values are perfectly negatively correlated, while her match values are independent, between the two products. Our model adopts the same independence structure as Haan, Morage-González and Petrikaite (2015) but allows for general market structure and asymmetric sellers.

It is well-recognized that such consumer search models do not admit tractable characterization. There are two main difficulties. First, consumer search behavior is complicated and hard to summarize. Each consumer undergoes sequential search, whose complexity grows

[1] An early precursor to these papers is Bakos (1997), who studies several versions of a (circular) location model. One of his extensions considers the case where quality (value) information is significantly costlier than price information. The limit version where price information can be obtained at zero cost is equivalent to the case where prices are publicly observable.

fast as the number of sellers increases or new features are introduced into the model. This is likely to be the reason why all previous studies have restricted attention to the duopoly case. Second, the sellers' best response functions do not behave well in general. There may not exist a pure-strategy equilibrium, and the model rarely produces sharp comparative statics results.

We overcome the first difficulty by identifying a necessary and sufficient condition that summarizes consumers' search outcomes.[2] We utilize an elegant solution by Weitzman (1979) for a class of sequential search problems and show that, although Weitzman's solution is necessary to fully describe optimal search *behavior*, the optimal search *outcome* (i.e., a consumer's eventual purchase decision) can be fully summarized by a simpler condition that is familiar in discrete-choice models. The condition pinpoints the extent to which search frictions distort consumers' purchase decisions (i.e., how a consumer's purchase decision under sequential search differs from that under perfect information) and allows us to reformulate the pricing game among the sellers as a discrete-choice problem.

For the second difficulty, we obtain sufficient conditions under which the seller's best response functions are well-behaved and, therefore, there exists a unique pure-strategy market equilibrium. We exploit the induced discrete-choice structure of our model and characterize sufficient conditions on the primitives of our model under which we can apply both general results in the literature on supermodular games and specific results in discrete-choice models. Despite certain limitations,[3] our characterization allows us to derive some sharp comparative statics results and, therefore, learn more about the working of the model, as we elaborate below. In addition, our analysis is likely to be informative for the environments that are not covered by our sufficient conditions.

We pay special attention to the relationship between search frictions and market prices. It was recognized early on that the Internet dramatically reduces market frictions and, therefore, should deliver more efficient market outcomes, by transforming traditional businesses as well as creating many new markets. This promise has been fulfilled in various ways by

[2]An effectively identical condition has been independently discovered by Armstrong (2016). See also Armstrong and Vickers (2015), who consider a more general problem of which demand systems have discrete-choice foundations and show that the demand system under consumer search belongs to the class.

[3]In particular, our sufficient conditions do not encompass a benchmark case where consumers are ex ante symmetric (i.e., do not possess any prior product information), for which it is known that there does not exist a pure-strategy equilibrium (see Armstrong and Zhou, 2011).

now, but several phenomena that are at odds with it still persist. In particular, it has been repeatedly reported that the Internet has neither significantly lowered markups nor reduced price dispersion (see, e.g., Ellison and Ellison, 2005; Baye, Morgan and Scholten, 2006) These suggest that search frictions are significant even in online markets and cast doubt on the conventional wisdom that a reduction in search frictions is necessarily beneficial to consumers. The following results provide some new insights for these important issues.

As a methodological contribution, we show that the effects of various changes in search frictions can be summarized by their effects on dispersion of the induced discrete-choice distributions. This is useful, because there is a systematic relationship between preference diversity and equilibrium prices in discrete-choice models: equilibrium prices increase as consumers' preferences become more diverse.[4] In other words, we derive several comparative statics results regarding search frictions, which are hard to obtain directly, by studying their effects on the induced distributions and utilizing a result that links between preference diversity and equilibrium prices in discrete-choice models.

We show that an increase in the value of search raises market prices. Specifically, we establish that, provided that the sellers are symmetric, the equilibrium price increases as search costs decrease or the distribution of match values becomes more dispersive (which increases the expected return of search).[5] Note that this is opposite to the standard result in the literature. As the value of search decreases, a consumer is less likely to leave for another seller and, therefore, more likely to purchase from the current seller. The sellers then have an incentive to extract more surplus from visiting consumers and, therefore, charge higher prices. This is the main mechanism behind the opposite result in the literature. However, it crucially depends on the assumption of unobservable prices (i.e., no price advertisement), which implies that the sellers cannot influence consumer search behavior. In our model, the sellers compete in prices to attract consumers. When the value of search falls, price competition becomes more severe, which induces the sellers to lower their prices.

In contrast, improving pre-search information quality has an ambiguous effect on market

[4]This is a classic idea in the literature on Bertrand competition under product differentiation. We contribute to the literature by providing an appropriate measure of preference diversity (product differentiation). See Section 5.1 for a more comprehensive discussion and our result.

[5]We note that, whereas the first result regarding search costs has also been established by Armstrong and Zhou (2011) and Haan, Morage-González and Petrikaite (2015), the second result regarding the distribution of match values is, to our knowledge, new to the literature.

prices. We show that providing more precise information for consumers before search increases market prices if and only if the number of sellers is above a certain threshold. There are two opposing effects. On the one hand, it reduces consumers' incentives to explore more products, which, as above, intensifies price competition among the sellers. On the other hand, consumers' preferences before search (prior values) become more dispersed, which relaxes price competition. We prove that the latter effect dominates the former, and thus providing more product information before search increases market prices if and only if there are sufficiently many sellers.

These results allow us to reinterpret various empirical findings in the literature, which, conversely, justifies the empirical relevance of our model. For instance, Lynch and Ariely (2000) run a field experiment with online wine sales and find that providing more product information lowers consumers' price sensitivity. Bailey (1998) and Ellison and Ellison (2014) report that online prices are often higher than off-line prices. This naturally arises in our model, given that search costs are significantly lower online than off-line. Ellison and Ellison (2009) report that markups are relatively higher for high-quality products than for low-quality products. Within our model, this can be understood as consumer preferences being more diverse, or the relative cost of search being lower, for high-quality products.

We also provide two novel insights for the case where the sellers are asymmetric. First, we study which sellers have a stronger incentive to post higher prices. We show that Weitzman index, which is the most natural candidate in the current sequential search context, does not provide enough guidance in general. We provide a sufficient condition under which the sellers' prices can be clearly ranked and also show that Weitzman index can be still useful to predict price rankings in some specific contexts. Second, we analyze the effects of search costs on asymmetric sellers. We show that when one seller has a higher marginal cost than the other, an identical increase in search costs raises demand for the low-cost seller but lowers demand for the high-cost seller. Intuitively, this is because consumers become more price-sensitive as search costs increase, and the low-cost seller posts a lower price. One noteworthy implication of this result is that the high-cost seller has a stronger incentive to lower his price than the low-cost seller as search costs increase. Since the former posts a higher price than the latter, this means that the price difference between the two sellers falls as search costs rise. In other words, an increase in search frictions may reduce price dispersion. This result contrasts well with a classical insight in search theory that price dispersion is a symptom of search frictions.

This paper joins a growing literature on ordered search, which investigates the effects of (both exogenous and endogenous) search order on market outcomes and various ways sellers influence consumer search behavior (order). See Armstrong (2016) for a comprehensive and organized introduction of the literature and several useful discussions. In light of this literature, we consider the case where each consumer's search order is fully endogenized and a seller influences search behavior through the choice of her price (which is arguably the most basic instrument).

One interpretation of our model is to introduce consumer search into a canonical model of Bertrand competition under product differentiation. Indeed, our model reduces to that of Perloff and Salop (1985) if consumers incur no search costs. We make it transparent how consumer search models with price advertisements are related to discrete-choice models (and what the former can learn from the latter). In addition, we show that dispersive order is an appropriate measure for preference diversity and explain how the result can be used to obtain several comparative statics results regarding search frictions.

As explained above, our model can be interpreted as a model of online marketplaces. In this regard, our paper is related to two strands of literature on electronic commerce. First, there are several theoretical studies that develop an equilibrium online shopping model. For example, Baye and Morgan (2001) analyze a model in which both the sellers and consumers decide whether to participate in an online marketplace, while Chen and He (2011) and Athey and Ellison (2011) present an equilibrium model that combines position auctions with consumer search. Our paper is unique in that the focus is on consumer search within an online marketplace. Second, a growing number of papers draw on search theory to study online markets. For example, Kim, Albuquerque and Bronnenberg (2010) develop a non-stationary search model to study the online market for camcoders. De los Santos, Hortaçsu and Wildenbeest (2012) test some classical search theories with online book sale data and argue that fixed sample size (i.e., simultaneous) search theory explains the data better than sequential search theory. Dinerstein, Einav, Levin and Sundaresan (2014) estimate online search costs and retail margins with a consumer search model based on the "consideration set" approach, and apply them to evaluate the effect of search redesign by eBay in 2011. Although empirical analysis is beyond the scope of this paper, we think that our equilibrium model is tractable and structured enough to be taken to data.

The rest of the paper is organized as follows. We introduce the environment in Section 2. We analyze consumers' optimal shopping problems in Section 3 and characterize the market equilibrium in Section 4. We study the effects of search frictions on market prices in Section 5 and provide two results, one about price rankings and the other about price dispersion, in Section 6. All omitted proofs are in the appendix.

2 Environment

The market consists of n sellers, each indexed by $i = \{1, ..., n\}$, and a unit mass of consumers. The sellers face no capacity constraint, while each consumer demands one unit among all products. The sellers simultaneously announce prices. Consumers observe those prices and search optimally.

Each seller i supplies a product at no fixed cost and a constant marginal cost c_i. We denote by $p_i \in \mathcal{R}_+$ seller i's price. In addition, we let $\mathbf{p}$ denote the price vector for all sellers (i.e., $\mathbf{p} = (p_1, ..., p_n)$) and $\mathbf{p}_{-i}$ denote the price vector except for seller i's price (i.e., $\mathbf{p}_{-i} = (p_1, ..., p_{i-1}, p_{i+1}, ..., p_n)$). Denote by $D_i(\mathbf{p})$ the measure of consumers who eventually purchase product i. Seller i's profit is then defined to be $\pi_i(\mathbf{p}) \equiv D_i(\mathbf{p})(p_i - c)$. Each seller maximizes his profit $\pi_i(\mathbf{p})$.

A (representative) consumer's random utility for seller i's product is given by $\tilde{V}_i = V_i + Z_i$. The first component V_i represents the consumer's prior value for product i, while the second component Z_i is the residual part that is revealed to the consumer only when she visits seller i and inspects his product. As for prices, we let $\mathbf{v} = (v_1, ..., v_n)$ and $\mathbf{z} = (z_1, ..., z_n)$ denote the realization of a consumer's value profile for each component.

The products are horizontally differentiated. We assume that V_i and Z_i are drawn from the distribution functions F_i and G_i, respectively, identically and independently across consumers and products (and independently each other), where both F_i and G_i have full support over the real line and continuously differentiable density f_i and g_i, respectively. Independence across products allows us to utilize the optimal search solution by Weitzman (1979), while independence between V_i and Z_i leads to a clean and easy-to-interpret characterization.[6]

[6]We note that independence between V_i and Z_i is restrictive not by itself, but because of a joint additive-utility specification ($\tilde{V}_i = V_i + Z_i$). It is always possible to reinterpret (redefine) Z_i, so that it is independent

Search is costly, but recall is costless. Specifically, each consumer must visit seller i and discover her match value z_i in order to be able to purchase product i. She needs to incur search cost $s_i(> 0)$ on her first visit. She can purchase the product immediately or recall it at any point during her search. Each consumer can leave the market at any point and take an outside option u_0.

A consumer's ex post utility depends on her value for the purchased product $\tilde{v}_i$, its price p_i, and her search history. Let N be the set of sellers a consumer visits. If she purchases product i (in N), then her ex post utility is equal to

$$U(v_i, z_i, p_i, N) = v_i + z_i - p_i - \sum_{j \in N} s_j.$$

If she does not purchase and takes an outside option, then her ex post utility is equal to

$$U(N) = u_0 - \sum_{j \in N} s_j.$$

Each consumer is risk neutral and maximizes her expected utility.

The market proceeds as follows. First, the sellers simultaneously announce prices $\mathbf{p}$. Then, each consumer shops (searches) based on available information $(\mathbf{p}, \mathbf{v})$. We study subgame perfect Nash equilibrium of this market game.[7] We first characterize consumers' optimal shopping behavior (given any price vector) and then analyze the pricing game among the sellers.

3 Consumer Behavior

In this section, we analyze consumers' optimal sequential search problems.

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of V_i (see, e.g., Eső and Szentes, 2007). In this case, a restriction is only due to the utility specification. On the other hand, Z_i can always be defined as $Z_i \equiv \tilde{V}_i - E[\tilde{V}_i|V_i]$ (see, e.g., Krähmer and Strausz, 2011). In this case, independence between V_i and Z_i imposes a restriction.

[7] For notational simplicity, we do not formally define consumers' search strategies. See Weitzman (1979) for a formal (recursive) definition of search strategy.

3.1 Optimal Shopping

Given prices $\mathbf{p}$ and prior values $\mathbf{v}$, each consumer faces a sequential search problem. She decides in which order to visit the sellers and, after each visit, whether to stop, in which case she chooses which product to purchase, if any, among those she has inspected so far, or visit another seller. Although this is, in general, a complex combinatorial problem, an elegant solution is known by Weitzman (1979). Independence between v_i and z_i leads to an even sharper characterization, as reported in the following proposition.[8]

Proposition 1 *Given $\mathbf{p} = (p_1, ..., p_n)$ and $\mathbf{v} = (v_1, ..., v_n)$, the consumer's optimal search strategy is as follows: for each i, let z_i^* be the value such that*

$$s_i = \int_{z_i^*}^{\infty} (1 - G_i(z_i))dz_i. \tag{1}$$

(i) Search order: the consumer visits the sellers in the decreasing order of $v_i + z_i^* - p_i$ (i.e., she visits seller i before seller j if $v_i + z_i^* - p_i > v_j + z_j^* - p_j$).

(ii) Stopping: let N be the set of sellers the consumer has visited so far. She stops, and takes the best available option by the point, if and only if

$$\max\{u_0, \max_{i \in N} v_i + z_i - p_i\} > \max_{j \notin N} v_j + z_j^* - p_j.$$

Weitzman's solution is based on a single index for each option (seller). Let r_i be the reservation value such that a consumer is indifferent between obtaining utility r_i immediately (which saves additional search costs s_i) and visiting seller i (which gives her an option to choose between r_i and $v_i + z_i - p_i$):

$$r_i = -s_i + \int \max\{r_i, v_i + z_i - p_i\} dG_i(z_i).$$

Weitzman (1979) shows that the optimal search strategy is to visit the sellers in the decreasing order of r_i and stop as soon as the best realized value by the point exceeds all remaining

[8] The measure of consumers who are indifferent over multiple choices is negligible, because F_i and G_i are assumed to be continuously increasing for all i. For notational convenience, we ignore those consumers throughout the paper.

r_i's. In our model, due to the additive-utility specification, Weitzman's index simplifies to $r_i = v_i + z_i^* - p_i$, where z_i^* is given by equation (1).

3.2 Shopping Outcomes

Despite its elegance, Weitzman's solution cannot be directly used to summarize consumers' shopping outcomes and derive the demand functions. Consider the simplest case where there are two sellers. Even in this case, there are three different paths through which a consumer eventually purchases product i. First, a consumer may visit seller i first and purchase immediately. Second, a consumer may visit seller i first, try seller j as well, but recall product i. Third, a consumer may visit seller j first but purchase product i. Total demand for seller i is the sum of all these demands. The number of paths grows exponentially fast as the number of sellers n increases.

One of our main breakthroughs is to identify a necessary and sufficient condition for consumers' eventual purchase decisions and, therefore, provide a simple way to summarize shopping outcomes. In order to motivate the result, consider the same duopoly case as above. The three paths through which a consumer purchases product i correspond to each of the following conditions:

(i) $v_i + z_i^* - p_i > v_j + z_j^* - p_j$ (visit i first) and $v_i + z_i - p_i > v_j + z_j^* - p_j$ (stop at i).

(ii) $v_i + z_i^* - p_i > v_j + z_j^* - p_j$ (visit i first), $v_i + z_i - p_i < v_j + z_j^* - p_j$ (not stop at i), and $v_i + z_i - p_i > v_j + z_j - p_j$ (prefer i to j).

(iii) $v_i + z_i^* - p_i < v_j + z_j^* - p_j$ (visit j first), $v_i + z_i^* - p_i > v_j + z_j - p_j$ (not stop at j), and $v_i + z_i - p_i > v_j + z_j - p_j$ (prefer i to j).

Notice that the first condition can be simplified to $v_i + \min\{z_i^*, z_i\} - p_i > v_j + z_j^* - p_j$, while the second and the third conditions together can be reduced to $v_i + \min\{z_i^*, z_i\} - p_i \leq v_j + z_j^* - p_j$ and $v_i + \min\{z_i^*, z_i\} - p_i > v_j + z_j - p_j$. Intuitively, a consumer purchases product i if she either does not visit seller j or finds a sufficiently low realized value of z_j. Combining these inequalities, we arrive at the following single inequality:

$$v_i + \min\{z_i, z_i^*\} - p_i > v_j + \min\{z_j, z_j^*\} - p_j.$$

This simple condition can be extended for the general case by considering each pair of sellers and accommodating the outside option, as formally reported in the following lemma.

Lemma 1 (Eventual Purchase) *Let $w_i \equiv v_i + \min\{z_i, z_i^*\}$ for each i. Given $\mathbf{p}$, $\mathbf{v}$, and $\mathbf{z}$, the consumer purchases product i if and only if $w_i - p_i > u_0$ and $w_i - p_i > w_j - p_j$ for all $j \neq i$.*

Lemma 1 suggests that consumer shopping behavior can be summarized as in canonical discrete-choice models.[9] The only difference is that consumers' purchase decisions are made based, neither on true values $\tilde{v}_i$ nor on prior values v_i, but on newly identified values w_i, which we call *effective values* from now on. Clearly, w_i is related to underlying values $\tilde{v}_i$ and v_i. In particular, w_i converges to $\tilde{v}_i$ as s_i tends to 0 (in which case z_i^* approaches ∞) and is determined only by v_i as s_i tends to infinity (in which case z_i^* approaches $-\infty$). Intuitively, search frictions prevent consumers from making fully informed decisions, and the problem becomes more severe, and consumers rely more on their prior information $\mathbf{v}$, as search frictions increase. The specific truncation structure is driven by a monotonicity property of Weitzman's solution. If a consumer visits seller i, Weitzman's indices for all remaining sellers are lower than $v_i + z_i^* - p_i$. Therefore, the consumer necessarily stops if z_i exceeds z_i^*, which implies that the probability that a consumer purchases product i stays constant above z_i^*.

In order to utilize Lemma 1, we let H_i denote the distribution function for the new random variable $W_i = V_i + \min\{Z_i, z_i^*\}$, that is,

$$H_i(w_i) \equiv \int_{-\infty}^{z_i^*} F_i(w_i - z_i)dG_i(z_i) + \int_{z_i^*}^{\infty} F_i(w_i - z_i^*)dG_i(z_i). \tag{2}$$

The distribution function H_i crucially depends on s_i. If s_i tends to 0, then z_i^* becomes arbitrarily large (see equation (1)) and, therefore, H_i becomes the convolution of F_i and G_i. If s_i explodes, then z_i^* approaches negative infinity, in which case H_i depends only on F_i.

[9]Lemma 1 holds even if prices are not observable to consumers before search, as long as consumers have correct beliefs about prices (i.e., in equilibrium). However, the result does not hold if a seller deviates, because consumers' search decisions are based on their expectations about prices, while their final purchase decisions depend on actual prices charged. That property makes Lemma 1 less useful in such a setting.

4 Market Equilibrium

In this section, we consider the pricing game among the sellers and provide sufficient conditions under which there exists a unique pure-strategy equilibrium.

Lemma 1 implies that the demand function for each seller can be derived as in standard discrete-choice models. A consumer purchases product i if and only if his effective utility for product i, $w_i - p_i$, exceeds the outside option u_0 and the corresponding utility for each other product, $w_j - p_j$. Therefore, the measure of consumers who purchase product i is given by

$$D_i(\mathbf{p}) = \int_{u_0+p_i}^{\infty} \left(\prod_{j \neq i} H_j(w_i - p_i + p_j) \right) dH_i(w_i).$$

This demand system exhibits standard properties for imperfect substitutes: demand for seller i decreases in own price p_i and increases in competitors' prices $\mathbf{p}_{-i}$. However, the demand system does not behave well in general: $D_i(p_i, \mathbf{p}_{-i})$ may not be quasi-concave in p_i, and a seller's best response does not necessarily increase in $\mathbf{p}_{-i}$.

The literature has found that log-concavity is an appropriate restriction for the distribution functions. It not only guarantees the existence and uniqueness of equilibrium, but also generates intuitive comparative statics results (such as declining market prices as the number of sellers increases). The following result by Quint (2014), translated into our environment, is well applicable to our model.[10]

Theorem 1 (Quint, 2014) *Suppose that for each i, both $H_i(w_i)$ and $1 - H_i(w_i)$ are log-concave. Then, $D_i(\mathbf{p})$ is log-concave in p_i, and $\log D_i(\mathbf{p})$ has strictly increasing differences in p_i and p_j. In addition, there exists a unique pure-strategy equilibrium in the pricing game among the sellers.*

Distributional log-concavity ensures log-concavity and log-supermodularity in demand. These two properties imply that the pricing game is a supermodular game and, therefore, has a pure-strategy equilibrium, as an application of more general existence theorems (see Vives, 2005). Uniqueness is not implied by general theory, but driven by a specific structure

[10] Quint (2014) provides further relevant discussions, including common distribution functions that satisfy the log-concavity condition and weaker conditions sufficient for each result. For more thorough technical treatments of log-concavity, see, e.g., Bagnoli and Bergstrom (2005).

of the model, namely that $D_i(\mathbf{p})$ is invariant when all prices, together with $-u_0$, increase by the same amount, which is not a general property in supermodular games.

As shown above, H_i in our model is not exogenously given but depends on F_i, G_i, and s_i in a specific way. Therefore, log-concavity cannot be directly imposed on H_i and $1 - H_i$. A natural assumption is that all primitive distribution functions F_i, $1 - F_i$, G_i, and $1 - G_i$ are log-concave. However, the assumption alone does not guarantee the log-concavity of H. In fact, even a stronger assumption that the density functions f_i and g_i are log-concave is not sufficient.[11]

In order to understand the origin of the problem, consider the case where F_i is degenerate at v_i. In this case, H_i(w) jumps up at $v_i + z_i^*$ (see the solid line, corresponding to $\alpha = 0$, in the left panel of Figure 1) and, therefore, cannot be globally log-concave. This is driven by the upper truncation structure of the random variable W_i, which is, in turn, due to the sequential search nature of consumers' problems, as explained in the previous section. When F_i is continuously distributed over the real line, the atom at $v_i + z_i^*$ is continuously scattered, which ensures the continuity of H_i. However, if F_i is sufficiently concentrated around v_i, then the slope of H_i at $v_i + z_i^*$ can be arbitrarily large (see the dashed line, corresponding to $\alpha = 0.1$, in the left panel of Figure 1). Therefore, H_i may still fail to be log-concave.

We provide sufficient conditions under which this problem is not binding and Theorem 1 applies. We begin by imposing sufficiently strong log-concavity on the primitive distributions.

Assumption 1 *For each i, both density functions f_i and g_i are log-concave.*

Although this assumption does not guarantee the log-concavity of H_i, it suffices for $1 - H_i$, as formally stated in the following lemma.

Lemma 2 *Under Assumption 1, $1 - H_i$ is log-concave.*

Proof. Integrating by parts the first term in equation (2) leads to

$$1 - H_i(w_i) = \int_{-\infty}^{z_i^*} f(w_i - z_i)(1 - G_i(z_i))dz_i \tag{3}$$

[11] If the density function f is log-concave, then both distribution function F and survival function $1 - F$ are log-concave. See Bagnoli and Bergstrom (2005) for more details.

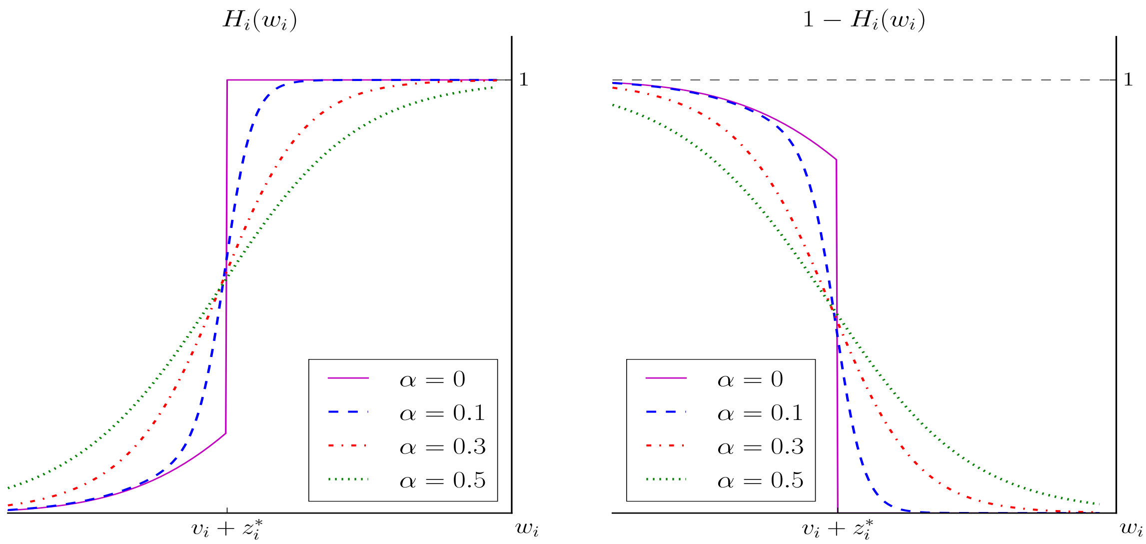

Figure 1: $H_i(w_i)$ and $1 - H_i(w_i)$ for different dispersion levels of F_i. For both panels, $F_i(v_i) = 1/(1 + e^{-v_i/\alpha})$ (logistic distribution), and $G_i = \mathcal{N}(0, 1)$ (standard normal distribution).

The log-concavity of g ensures the same property for $1 - G_i$. Since both f and $1 - G$ are log-concave, the integrand is log-concave in (w_i, z_i). The desired result then follows from Prekopa's theorem, which states that if the integrand is log-concave, then the integral is also log-concave.[12] ■

To understand the difference between H_i and $1 - H_i$, consider, again, the case where F_i is degenerate. Both H_i and $1 - H_i$ are discontinuous at $v_i + z_i^*$. However, $1 - H_i$ jumps down and, therefore, preserves log-concavity over the interval below $v_i + z_i^*$ (see the right panel of Figure 1). In addition, $1 - H_i(w_i)$ remains equal to 0 above $v_i + z_i^*$. These two properties ensure that $1 - H_i$ is log-concave when F_i is degenerate. When F_i is not degenerate, $1 - H_i(w_i)$ is continuous and stays positive. However, these properties do not disrupt log-concavity, and thus $1 - H_i$ is always log-concave under Assumption 1.

Our first result provides a sufficient condition under which H_i is globally log-concave. It

[12]See, e.g., Caplin and Nalebuff (1991) and Choi and Smith (2016) for a formal statement of the theorem and its uses in related contexts.

states that if F_i is sufficiently dispersed, then H_i is log-concave under Assumption 1.[13]

Proposition 2 *Fix random variables V_i and Z_i with density f_i and g_i, respectively. Define $V_i^\sigma \equiv \sigma V_i$ and $W_i^\sigma \equiv V_i^\sigma + \min\{Z_i, z_i^*\}$. Let H_i^σ denote the distribution function for W_i^σ. Then, there exists $\overline{\sigma} < \infty$ such that the distribution function H_i^σ is log-concave whenever $\sigma > \overline{\sigma}$.*

To understand this result, recall that the failure of log-concavity of H_i is due to the probability mass at z_i^*. Now notice that, since $W_i = V_i + \min\{Z_i, z_i^*\}$ (also see equation (2)), dispersion on F_i scatters this atom through the real line, which makes H_i increase more slowly and, therefore, mitigates the main problem. When F_i is sufficiently dispersed, the effect of the mass point would be small (i.e., H_i does not increase too fast at any point), and thus H_i can be log-concave (see the left panel of Figure 1).

Our second condition is based on the idea that global log-concavity is not necessary for Theorem 1. Specifically, it is clear that in equilibrium p_i exceeds c_i. Therefore, it suffices that H_i is log-concave only on the parameter region where $w_i \geq u_0 + c_i$.

Proposition 3 *Suppose Assumption 1 is satisfied. (i) Given $u_0 > -\infty$, there exists $\overline{s}_i < \infty$ such that if $s > \overline{s}_i$, then $H_i(w_i)$ is log-concave above $u_0 + c_i$. (ii) Given $s_i > 0$, there exists $\overline{u}_0$ such that if $u_0 > \overline{u}_0$, then $H_i(w_i)$ is log-concave above $u_0 + c_i$.*

Intuitively, if s_i is sufficiently large, then the value of visiting seller i is small. In this case, z_i^* lies in the irrelevant (sufficiently negative) region, while Assumption 1 ensures that H_i is log-concave in the relevant region. Similarly, if u_0 is sufficiently large, then consumers' effective values are relevant only when they are sufficiently large and, in particular, far exceed z_i^*. Again, Assumption 1 guarantees that H_i behaves well in the relevant region.

In the remaining sections, we restrict attention to the parameter space where H_i and $1 - H_i$ are log-concave at least over the relevant region and, therefore, there exists a unique pure-strategy equilibrium. Although restrictive, this allows us to go one step further and investigate sellers' pricing incentives in our model. In addition, we maintain Assumption 1. As in many existing studies, log-concavity allows us to derive clean and intuitive comparative statics results.

[13] Haan, Morage-González and Petrikaite (2015) conjecture this result and provide a set of confirming numerical examples. Our result formalizes their conjecture.

5 Symmetric Sellers: Search Frictions

In this section, we study how search frictions influence the sellers' pricing incentives.[14] For clear insights as well as tractability, we restrict attention to the case where the sellers are symmetric. Precisely, we assume that buyers' values for each product are drawn from identical distribution functions F and G, the sellers have an identical marginal cost c, and consumers face identical search costs for all sellers (i.e., for all i, $F_i = F$, $G_i = G$, $c_i = c$, and $s_i = s$). We let p^* and $\pi(p^*)$ denote the symmetric equilibrium price and profit, respectively.[15]

5.1 Preference Diversity

We begin by establishing a result that is useful through this section. The result is, in fact, of interest by itself, in regard to the literature on Bertrand competition under product differentiation. It is well-known that horizontal product differentiation provides a way to overcome the Bertrand paradox: each seller has some loyal consumers (who value the seller's product more than other products) and, therefore, can set a positive markup even under Bertrand competition. It is natural that the more differentiated consumers' preferences are, the higher prices the sellers charge. A challenge has been to identify an appropriate measure of preference diversity (product differentiation). In their seminal work, Perloff and Salop (1985) show that constant scaling of consumers' preferences necessarily increases the equilibrium price, but find that the result does not extend for mean-preserving spreads. Our result provides an answer to the long-standing open question.[16]

We utilize the following measure of stochastic orders, so called *dispersive order*.

Definition 1 *The distribution function H_2 is more dispersed than the distribution function H_1 if $H_2^{-1}(b) - H_2^{-1}(a) \geq H_1^{-1}(b) - H_1^{-1}(a)$ for any $0 < a \leq b < 1$.*

[14] We omit some standard comparative statics results. For example, it is easy to show that more intense competition, such as introducing an additional seller or increasing the outside option, lower market prices. See Quint (2014) for further results and illustrations.

[15] Recall that we focus on the environment in which there exists a unique pure-strategy equilibrium. In the symmetric environment, all sellers must charge the same price, as otherwise there are multiple equilibria.

[16] Zhou (forthcoming) studies the effects of bundling in the Perloff-Salop framework and independently discovers an almost identical result. Precisely, his Lemma 2 is equivalent to our Proposition 4, provided that there is no outside option (i.e., $u_0 = -\infty$). Our result is slightly more general than his, in that we account for the outside option. In addition, whereas his lemma is an isolated result in his paper, we fully utilize it for subsequent comparative statics.

Intuitively, a more dispersed distribution function increases more slowly (its inverse increases faster), as it density is more spread out. This order is location-free and, therefore, neither is implied by nor implies first-order or second-order stochastic dominance. Mean-preserving dispersive order, however, implies mean-preserving spread: if H_2 is more dispersed than H_1 with the same mean, then H_2 is a mean-preserving spread of H_1.[17]

The following result shows that there is a good sense in which dispersive order is an appropriate measure of product differentiation.

Proposition 4 *The equilibrium price p^* increases as H becomes more dispersive and $H(u_0 + c)$ weakly decreases.*

Proof of Proposition 4. The equilibrium condition for p^*, which stems from an individual seller's first-order condition and the symmetry requirement, can be rewritten as

$$\frac{1}{p^* - c} = \frac{\int h(\max\{u_0 + p^*, w\})dH(w)^{n-1}}{\frac{1}{n}(1 - H(u_0 + p^*)^n)}.$$

Letting $\phi \equiv H(u_0 + p^*)$ and changing the variable with $a = H(w)$, we get

$$\frac{1}{p^* - c} = \frac{h(H^{-1}(\phi))\phi^{n-1} + \int_\phi^1 h(H^{-1}(a))da^{n-1}}{\frac{1}{n}(1 - \phi^n)}. \tag{4}$$

If H becomes more dispersive, $dH^{-1}(a)/da = 1/h(H^{-1}(a))$ increases (i.e., $h(H^{-1}(a)$ decreases) for each a. If, in addition, $H(u_0 + c)$ decreases, then $\phi = H(u_0 + p^*)$ also decreases for any $p^* \geq c$, because a distribution function crosses a less dispersive one only once from above. Notice that both of these lower the right-hand side. The desired result now follows from the fact that the left-hand side is strictly decreasing in p^*, while the right-hand side is increasing in p^* (see the appendix for a proof of this last claim). ■

The relevance of dispersive order is particularly transparent when there is no outside option (i.e., $u_0 = -\infty$), which is the case considered by Perloff and Salop (1985) and many subsequent studies. In that case, the second condition about $H(u_0 + c)$ is vacuous, and thus

[17] See Shaked and Shanthikumar (2007) for further details. Dispersive order has been adopted and proved to be useful in other economic contexts. See, for example, Ganuza and Penalva (2010).

dispersive order alone dictates how market prices vary: market prices rise (fall) if H becomes more (less) dispersive.

5.2 Search Costs

The following result reports the effects of varying search costs s on the equilibrium price p^* and each seller's equilibrium profit $\pi(p^*)$.

Proposition 5 *Both equilibrium price p^* and equilibrium profit $\pi(p^*)$ decrease as s increases.*

Proof. We utilize Proposition 4 to prove the price result. Specifically, we show that the random variable $W = V + \min\{Z, z^*\}$ falls in the first-order stochastic dominance (which implies that $H(u_0 + c)$ increases) and becomes less dispersive as s increases. Notice that z^* decreases in s (see equation (1)). This immediately implies that W decreases in the sense of first-order stochastic dominance. For the dispersion result, let $\tilde{G}(z)$ denote the distribution function of the random variable $\min\{Z, z^*\}$. By its definition, $\tilde{G}(z) = G(z)$ if $z < z^*$ and $\tilde{G}(z) = 1$ if $z \geq z^*$, which implies that $\tilde{G}^{-1}(a) = \min\{G^{-1}(z), z^*\}$ for $a \in (0, 1)$. Clearly, the quantile function $\tilde{G}^{-1}(a)$ becomes weakly flatter at any $a \in (0, 1)$ as z^* decreases. This implies that $\min\{Z, z^*\}$ becomes less dispersive as s increases. The desired result follows once this result is combined with the fact that the density function f is log-concave.[18]

An increase in s affects each seller's profit $\pi_i(\mathbf{p}) = D_i(p_i, p_{-i})(p_i - c)$ through the following three channels:

$$\frac{d\pi_i(\mathbf{p})}{ds} = \frac{\partial p_i}{\partial s}\frac{\partial \pi_i(\mathbf{p})}{\partial p_i} + \frac{\partial p_{-i}}{\partial s}\frac{\partial \pi_i(\mathbf{p})}{\partial p_{-i}} + \frac{\partial z^*}{\partial s}\frac{\partial \pi_i(\mathbf{p})}{\partial z^*}.$$

Each term represents the marginal effect of own price, that of the other sellers' prices, and that of consumer search behavior, respectively. In equilibrium, the first term is equal to 0 by the envelope theorem ($\partial \pi_i(\mathbf{p})/\partial p_i = 0$). The second term is negative because $\partial p^*/\partial s \leq 0$, as shown above, and $\partial \pi_i(\mathbf{p})/\partial p_{-i} \geq 0$, as the products are imperfect substitutes one another. The last term is also negative because $\partial z^*/\partial s < 0$ and $\partial \pi_i(\mathbf{p})/\partial z^* \geq 0$: the latter inequality

[18] See Theorem 3.B.8 in Shaked and Shanthikumar (2007). If a random variable X is the convolution of two random variables X_1 and X_2 (i.e., $X = X_1 + X_2$) and X_1 has log-concave density, then X becomes more dispersive as X_2 becomes more dispersive.

stems from the fact that an increase in z^* increases the distribution function H in the sense of first-order stochastic dominance, induces less consumers to take the outside option and, therefore, increases each seller's demand $D_i(p^*)$. Overall, it is clear that $d\pi(p^*)/ds \leq 0$. ■

Both price and profit results are in stark contrast to those of most existing consumer search models where consumers discover prices through search. When prices are not observable before search, an increase in search costs decreases the value of additional search and, therefore, increases the probability that a consumer purchases from a given seller.[19] This induces the sellers to charge higher prices as search costs increase. When prices are observable before search, they directly influence consumer search (see Proposition 3): the lower price a seller offers, the more consumers visit him first. As search costs increase, consumers search less and are more likely to purchase from their first visit. This intensifies price competition among the sellers and leads to lower prices.

Proposition 5 raises an interesting possibility that consumer surplus may increase when search costs increase. An increase in search costs has a direct negative effect on consumer welfare. However, if the sellers lower their prices dramatically in response, overall consumer welfare may rise. Indeed, there is an example in which an increase in search costs is beneficial to consumers. It arises when consumers' outside option is sufficiently unfavorable and there are sufficiently few sellers. In this case, the sellers possess strong market power and, therefore, charge a high price. An increase in search costs induces them to drop their prices quickly, up to the point where the indirect effect outweighs the direct effect and, therefore, consumer welfare increases.

The following proposition addresses a closely related question of how an increase in returns to search affects the equilibrium price p^*. To obtain clean insights, we restrict attention to the case where consumers have no outside option.

Proposition 6 *Provided that consumers have no outside option, the equilibrium price p^* increases as G becomes more dispersive.*

[19] To be precise, the result in consumer search models with unobservable prices crucially depends on the log-concavity property of the relevant distributions. For example, in Anderson and Renault (1999) where prices are not observable and F is degenerate, the equilibrium price increases in s if $1-G$ is log-concave but decreases in s if $1-G$ is log-convex (and assuming that there exists a symmetric pure-strategy equilibrium). Our comparison applies only to the former case.

Proof. By the logic given for the price result in the proof of Proposition 5, it suffices to show that the random variable $\min\{Z, z^*\}$ becomes more dispersive as $G(z)$ becomes more dispersive. To this end, recall that the quantile function for the random variable $\min\{Z, z^*\}$ is given by $\tilde{G}^{-1}(a) = \min\{G^{-1}(a), z^*\}$ for $a \in (0,1)$. It suffices to show that the slope of $\tilde{G}^{-1}(a)$ increases for all $a \in (0,1)$. For $a < G(z^*)$, the result is immediate from $\tilde{G}^{-1}(a) = G^{-1}(a)$. For $a = G(z^*)$, the result follows from the fact that $G(z^*)$ rises as G becomes more dispersive: rewriting equation (1) with $b^* = G(z^*)$ and $b = G(z)$ yields $s = \int_{b^*}^{1}(1 - b)\partial G^{-1}(b)/\partial b db$. If G becomes more dispersive ($\partial G^{-1}(b)/\partial b$ rises), the integrand rises, and thus the lower support b^* must rise in order to maintain the equation.[20] ■

Notice that this is consistent with Proposition 5, as a decrease in search costs can be interpreted as a proportional increase in search returns. Proposition 6 demonstrates that the main insight in Proposition 5 extends beyond proportional changes and holds with any dispersive perturbations.

5.3 Pre-search Information Quality

In our model, consumers search because they have imprecise information about their values for the products. This means that search frictions can also be measured by the extent to which consumers are uncertain about their match values. We now examine the effects of improving pre-search information quality on the equilibrium price p^*.

For tractability, we specialize our model into a Gaussian learning environment, where both F and G are given by normal distributions with mean 0. In addition, we assume that F has variance α^2, while G has variance $1-\alpha^2$, for some $\alpha \in (0,1)$ (i.e., $V \sim \mathcal{N}(0, \alpha^2)$ and $Z \sim \mathcal{N}(0, 1-\alpha^2)$). Our choices of the variances are deliberate. Notice that $\tilde{V} = V + Z \sim \mathcal{N}(0,1)$ for any α. In other words, our variance specification ensures that the distribution for consumers' ex post values $\tilde{V}_i$ stays unchanged when α varies. The parameter α measures the quality of pre-search information: as α increases, consumers' ex post values $\tilde{V} = V + Z$ are influenced more by (known) V and less by (hidden) Z. We also assume that consumers have no outside option.

We find that, unlike Propositions 5 and 6, the equilibrium price may or may not increase

[20]This argument is due to Choi and Smith (2016).

as pre-search quality information improves. In particular, if the number of sellers is sufficiently small, then p^* decreases in α.

Proposition 7 *There exists an integer $n^*(\alpha)$ such that a marginal increase in α increases p^* if and only if $n \geq n^*(\alpha)$.*

Recall that the equilibrium price increases when H becomes more dispersive (Proposition 4). Importantly, the result depends only on H, not separately on F and G. This means that a decrease in α has two effects on p^*. On the one hand, it spreads out G, which, as shown in Proposition 6, tends to increase p^*. On the other hand, it reduces dispersion of F, which may translate into lower dispersion of H and, therefore, push down p^*.[21]

The following lemma, which establishes the relationship between α and dispersion of H, is useful to understand the specific pattern in Proposition 7.

Lemma 3 *There exists $w^*(< \infty)$ such that the slope of $H^{-1}(a)$ decreases in α if and only if $a > H(w^*)$.*

This lemma states that an increase in α has disproportionate effects on dispersion of H: the left portion of H becomes less dispersive, while the right portion grows more dispersive. Recall that $W = V + \min\{Z, z^*\}$. Since $\min\{Z, z^*\}$ is bounded above by z^*, if w is rather large, $H(w)$ is mostly determined by the behavior of F. Since F becomes more dispersive in α, $H(w)$ also does so for w large. If w is rather small, $H(w)$ is affected by all three V, Z, and z^*. The effects of the first two cancel each other out, because $V + Z \sim N(0, 1)$. The last effect through z^*, however, makes H less dispersive, because z^* decreases in α (see equation (1) and the proof of Proposition 5).

When there are many sellers, the effective value of a consumer's purchased product is likely to exceed w^*. This implies that the equilibrium price p^* mainly depends on the right side of H (i.e., the region above w^*). As shown in Lemma 3, H grows more dispersive in α over the region. The opposite reasoning holds if there are few sellers. In either case, Proposition 7 follows from Proposition 4.

[21]Unlike G, H may not become more dispersive when F becomes more dispersive. This is, of course, because of asymmetry between F and G. In particular, the upper truncation structure of Z generates a probability mass for each V. This does not interfere in dispersion of Z being translated into that of W, but may between V and W. The result still holds if the density function f is decreasing over the relevant region, but not in general.

6 Asymmetric Sellers: Prices and Price Dispersion

In this section, we return to the general setting with asymmetric sellers and study some questions that arise in the presence of seller asymmetry.

6.1 Who Post Higher Prices?

In the presence of seller asymmetry, the most natural question is which sellers post higher prices. This has not been addressed thoroughly in the literature, partly because most theoretical studies restrict attention to the symmetric-sellers case and partly because of its complexity. We take one step forward by providing a sufficient condition under which one seller posts a higher price than another. We demonstrate the usefulness of our condition with a series of corollaries.

Proposition 8 *If $W_i - c_i$ dominates $W_j - c_j$ in the hazard rate order and the reverse hazard rate order,*[22] *then $p_i - c_i \geq p_j - c_j$.*

For the intuition, consider the duopoly case with no outside option. In this case, seller i's profit function is given by

$$\pi_i(p_i, p_j) = \int H_j(w_i - p_i + p_j) dH_i(w_i) \cdot (p_i - c_i).$$

If H_j were degenerate at $\overline{w}_j$ (hypothetically), then the integral would be equal to $1 - H_i(\overline{w}_j + p_i - p_j)$, and thus seller i's profit maximization condition would reduce to

$$\frac{1}{p_i - c_i} = \frac{h_i(\overline{w}_j + p_i - p_j)}{1 - H_i(\overline{w}_j + p_i - p_j)}.$$

As H_i increases in the hazard rate order, the right-hand side decreases, which implies that the optimal price p_i increases. Similarly, if H_i were degenerate at $\overline{w}_i$, then seller i's profit

[22] A random variable X_1 with distribution function F_1 dominates another random variable X_2 with distribution function F_2 in the hazard rate order if $f_1(t)/(1 - F_1(t)) \leq f_2(t)/(1 - F_2(t))$ for all t. Similarly, X_1 dominates X_2 in the reverse hazard rate order if $f_1(t)/F_1(t) \geq f_2(t)/F_2(t)$ for all t.

maximization condition simplifies to

$$\frac{1}{p_i - c_i} = \frac{h_j(\overline{w}_j - p_i + p_j)}{H_j(\overline{w}_j - p_i + p_j)}.$$

Applying a similar argument, it follows that p_i increases as H_j decreases in the reverse hazard rate order. In the appendix, we prove that these two conditions suffice for the result in general (i.e., without hypothetical degeneracy assumptions).

Our first application of Proposition 8 concerns the relationship between marginal costs and markups. We show that otherwise symmetric sellers with higher marginal costs charge lower markups.

Corollary 1 *If $F_i = F_j$, $G_i = G_j$, $s_i = s_j$ and $c_i > c_j$ for some i and j, then $p_j - c_j \geq p_i - c_i$.*

Proof. Given Proposition 8, it suffices to show that $W_i - c_i$ rises in the hazard rate order and the reverse hazard rate order as c_i falls. The result follows from the log-concavity of H and $1 - H$: the former implies that $h(w_i + c_i)/H(w_i + c_i)$ increases, while the latter implies that $h(w_i + c_i)/(1 - H(w_i + c_i))$ deceases, as c decreases. ■

It is a tempting conjecture that Weitzman index based only on the value distributions and search costs (i.e., $v_i + z_i^*$) would be closely tied with prices. Specifically, if p_i is equal to 0 for all i, consumers visit the sellers in the decreasing order of $v_i + z_i^*$. Since a seller with a higher index would attract more consumers, it is plausible that the seller would post a higher price. Our second corollary of Proposition 8 shows that this conjecture does not hold in general.

Corollary 2 *Suppose $F_i = F_j$, $c_i = c_j$, and $z_i^* = z_j^*$ for some i and j. If z_j dominates z_i in the likelihood ratio order,[23] then $p_j \geq p_i$.*

Corollary 2 shows that even if two sellers have the same Weitzman index (based on the value distributions and search costs) and share other characteristics, one seller may post a higher price than the other. Intuitively, Weitzman index captures only the average behavior of a distribution above a certain point. However, a seller's optimal price depends on the

[23] Consider two random variables X_1 and X_2. We say X_2 dominates X_1 in the likelihood ratio order if $f_2(x)/f_1(x)$ rises in x. The likelihood ratio order is equivalent to the monotone likelihood ratio property. Even if z_j dominates z_i in the likelihood ratio, z_i^* can be equal to z_j^*, because z_k^* depends not only on G_k but also on s_k (see equation (1)).

entire behavior of the distribution, which cannot be summarized by a single index. To be more concrete, suppose $p_i = p_j$, so that consumers are equally divided between the two sellers. In this case, seller j has relatively fewer consumers on the margin and, therefore, faces a stronger incentive to increase her price than player i, which ultimately leads to the outcome $p_j > p_i$.

Our final application of Proposition 8 illustrates the relationship between associated search costs and prices. For the same reason as above, it is plausible that sellers with lower search costs would post higher prices. Unlike in the previous case, we present an affirmative result for this conjecture. Specifically, we provide a sufficient condition under which prices are inversely related to search costs (i.e., if $s_i < s_j$, then $p_i > p_j$). Notice that, since Weitzman index is decreasing in search costs, this result also shows that the index, despite Corollary 2, may still provide useful guidance for price rankings.

Corollary 3 *Suppose all sellers are identical except that $s_1 < ... < s_n$, and the common density function $f(v)$ is such that $-f'(v)$ is positive and log-concave in v for all $v > u_0 - z^*$, where $z^* \equiv \max_i z_i^*$. Then, $p_1 \geq ... \geq p_n$.*

Intuitively, when the sellers differ only in associated search costs, the difference is unidimensional and, therefore, can be fully captured by a single-valued Weintzman index. The result, although clearly limited, is useful because various common distributions in the exponential family, including Gaussian, Gumbel, and Laplace distributions, have the right tails that satisfy the necessary distributional properties.

6.2 Search Costs

We now study the effects of search costs in the presence of seller asymmetry. We focus on two questions, who benefits from a reduction in search costs and what is the relationship between price dispersion and search costs. For tractability, we restrict attention to the simplest duopoly environment where there is no outside option and the two sellers differ only in their marginal costs. We assume that seller 1's marginal cost is strictly lower than seller 2's ($c_1 < c_2$), which implies that in equilibrium seller 1 charges a lower price than seller 2 ($p_1 < p_2$).[24]

[24] See the proof of Proposition 9 in the appendix.

Our first result shows that a reduction in search costs is beneficial to the disadvantaged seller (with a higher marginal cost).

Proposition 9 *Demand for seller* 1 *(*$D_1(\mathbf{p})$*) increases, while demand for seller* 2 *(*$D_2(\mathbf{p})$*) decreases in* s*.*

Notice that this result counters a common belief that more efficient firms will flourish, while less efficient firms will eventually vanish, as search costs decrease. Consumers search more actively (visit more sellers) when search costs are lower. In particular, more consumers make a purchase decision after visiting both sellers. This is more beneficial to seller 2, who charges a higher price and, therefore, attracts less fresh visitors.[25]

Proposition 9 suggests that the disadvantaged seller has a stronger incentive to lower the price as search costs increase. This generates a unique implication for the relationship between price dispersion and search costs, as formally stated in the following proposition.

Proposition 10 *The relative markup ratio* $(p_2 - c_2)/(p_1 - c_1)$ *decreases in* s*. If* $c_2 - c_1$ *is sufficiently large, then the absolute price difference* $p_2 - p_1$ *also decreases in* s*.*

The result indicates that an increase in search costs may reduce price dispersion. This is contrary to a well-established insight in search theory that price dispersion is a symptom of search frictions and market prices are more dispersed when there are more search frictions (see, e.g., Burdett and Judd, 1983; Stahl, 1989). Again, the result is driven by the fact that prices are observable to consumers and the role of search is only to gather more information about their values.

We conclude this section with another consequence of Proposition 9. When the sellers are symmetric, market prices necessarily decrease as search costs s increase (Proposition 5). If the sellers are asymmetric, the result may not apply to some sellers. In particular, the advantaged seller (seller 1) may increase her price when s increases. This occurs when an increase in search costs discourages lots of consumers from visiting seller 2 after seller 1, and thus demand for seller 1 increases sufficiently fast. In this case, seller 2 has an even

[25] Armstrong (2016) finds a similar result in an environment where one seller is "prominent" and, therefore, visited by all consumers first. A reduction in search costs, which induces more consumers to visit both sellers, is beneficial to the non-prominent seller. Unlike our result, his result builds upon an asymmetric equilibrium in the symmetric environment, which exists because it is assumed that all consumers have an identical prior value (i.e., F_i is degenerate for each i) and prices are unobservable before search.

stronger incentive to lower her price, while seller 1 may find it more profitable to increase her price. This also means that, unlike the symmetric case where all firms' profits fall as search costs increase (see Proposition 5), some firms may benefit from an increase in search costs and obtain higher profits.

7 Conclusion

We study an oligopoly model in which the sellers advertise their prices and consumers conduct optimal sequential search. We derive a simple condition that fully summarizes consumers' search outcomes and allows us to reformulate the pricing game as a familiar discrete-choice problem. We also provide some sufficient conditions under which there exists a unique pure-strategy market equilibrium. Based on the characterization, we obtain a set of results that shed new light on the effects of search frictions on market prices. We show that a reduction in the value of search increases market prices, whereas providing more information before consumer search may or may not increase market prices. We also provide a sufficient condition under which one seller posts a higher price than another and demonstrate that a reduction in search costs may lead to more price dispersion in the presence of seller asymmetry.

Many interesting questions remain open. To name a few, we assume that all sellers are fully committed to their advertised prices. However, hidden fees, in various forms, are prevalent in reality. How does their potential presence affect consumer behavior and sellers' pricing incentives?[26] We consider the case where each seller sells only one product, but it is the exception rather than the rule. How should a multi-product seller price (or position) his products? Should the seller choose an identical price, or introduce difference prices, for ex ante symmetric products? If the products are asymmetric, which product should the seller make prominent and how?[27] We plan to address these and other related problems in the future.

[26] See Ellison (2005) and Dai (2016) for some developments along this line.

[27] See Gamp (2016) and Petrikaitė (2016) for some related problems.

Appendix: Omitted Proofs

Proof of Lemma 1. Sufficiency: $w_i - p_i > u_0$ implies that the consumer never takes an outside option u_0, because she is willing to visit at least one seller ($v_i + z_i^* - p_i > u_0$) and make a purchase ($v_i + z_i - p_i > u_0$). Given this, it suffices to show that if $w_i - p_i > w_j - p_j$, then the consumer never purchases product j.

- Suppose $z_j^* \leq z_j$, which implies that $w_j = v_j + z_j^*$. The consumer visits seller j only after seller i because $v_i + z_i^* - p_i \geq w_i - p_i > v_j + z_j^* - p_j$. Once she visits seller i, however, she has no incentive to visit seller j because $v_i + z_i - p_i > v_j + z_j^* - p_j$.

- Suppose $z_j^* > z_j$, which implies that $w_j = v_j + z_j$. In this case, even if she visits seller j, she either recalls a previous product ($v_i + z_i - p_i > v_j + z_j - p_j$) or continues to search ($v_i + z_i^* - p_i > v_j + z_j - p_j$) and finds a better product ($v_i + z_i - p_i > v_j + z_j - p_j$).

Necessity: if $w_i - p_i < u_0$, then the consumer does not visit seller i ($v_i + z_i^* - p_i < u_0$) or does not purchase product i even if she visits seller i ($v_i + z_i - p_i < u_0$). If $w_i - p_i < w_j - p_j$ for some $j \neq i$, then, for the same logic as above, the consumer never purchases product i. ■

Proof of Proposition 2. Since

$$(logH_i^\sigma(w_i^\sigma))'' = \frac{(h_i^\sigma)'(w_i^\sigma)H_i^\sigma(w_i^\sigma) - h_i^\sigma(w_i^\sigma)^2}{H_i^\sigma(w_i^\sigma)^2},$$

it suffices to show that $(h_i^\sigma)'(w_i^\sigma)H_i^\sigma(w_i^\sigma) - h_i^\sigma(w_i^\sigma)^2 \leq 0$ for all w, provided that σ is sufficiently large. Integrate equation (2) by parts, we have

$$H_i^\sigma(w_i^\sigma) = \int_{w_i^\sigma - z_i^*}^{\infty} G_i(w_i^\sigma - v_i^\sigma)dF_i^\sigma(v_i^\sigma) + F_i^\sigma(w_i^\sigma - z_i^*).$$

By straightforward calculus,

$$\frac{h_i^\sigma(w_i^\sigma)}{H_i^\sigma(w_i^\sigma)} = \frac{\int_{w_i^\sigma - z_i^*}^{\infty} g_i(w_i^\sigma - v_i^\sigma)dF_i^\sigma(v_i^\sigma) + (1 - G_i(z_i^*))f_i^\sigma(w_i^\sigma - z_i^*)}{\int_{w_i^\sigma - z_i^*}^{\infty} G_i(w_i^\sigma - v_i^\sigma)dF_i^\sigma(v_i^\sigma) + F_i^\sigma(w_i^\sigma - z_i^*)} \tag{5}$$

and

$$\frac{(h_i^\sigma)'(w_i^\sigma)}{h_i^\sigma(w_i^\sigma)} = \frac{\int_{w_i^\sigma - z_i^*}^{\infty} g_i'(w_i^\sigma - v_i^\sigma)dF_i^\sigma(v_i^\sigma) + (1 - G_i(z_i^*))(f_i^\sigma)'(w_i^\sigma - z_i^*) - g_i(z_i^*)f_i^\sigma(w_i^\sigma - z_i^*)}{\int_{w_i^\sigma - z_i^*}^{\infty} g_i(w_i^\sigma - v_i^\sigma)dF_i^\sigma(v_i^\sigma) + (1 - G_i(z_i^*))f_i^\sigma(w_i^\sigma - z_i^*)} \tag{6}$$

Changing the variables with $a = F_i^\sigma(v_i^\sigma)$ and $r = F_i^\sigma(w_i^\sigma - z_i^*)$, equation (5) becomes

$$\frac{h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)}{H_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} = \frac{\int_r^1 g_i((F_i^\sigma)^{-1}(r) - (F_i^\sigma)^{-1}(a) + z_i^*)da + (1 - G_i(z_i^*))f_i^\sigma((F_i^\sigma)^{-1}(r))}{\int_r^1 G_i((F_i^\sigma)^{-1}(r) - (F_i^\sigma)^{-1}(a) + z_i^*)da + r}.$$

Since $V_i^\sigma \equiv \sigma V_i$, we have $F_i^\sigma(v_i^\sigma) = F_i(v_i^\sigma/\sigma)$, $(F_i^\sigma)^{-1}(r) = \sigma F_i^{-1}(r)$, $f^\sigma((F_i^\sigma)^{-1}(r)) = f_i(F_i^{-1}(r))/\sigma$, and $(f_i^\sigma)'(F_i^{-1}(r)) = f_i(F_i^{-1}(r))/\sigma^2$. Using these facts and arranging the terms in the right-hand side above yield

$$\frac{\sigma h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)}{H_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} = \frac{\int_r^1 \sigma g_i(\sigma(F_i^{-1}(r) - F_i^{-1}(a)) + z_i^*)da + (1 - G_i(z_i^*))f_i(F_i^{-1}(r))}{\int_r^1 G_i(\sigma(F_i^{-1}(r) - F_i^{-1}(a)) + z_i^*)da + r}.$$

Since $F_i^{-1}(r) - F_i^{-1}(a) \le 0$, the denominator converges to r as σ explodes. Integrating $\int_r^1 \sigma g_i(\sigma(F_i^{-1}(r) - F_i^{-1}(a)) + z_i^*)da$ in the numerator by parts yields

$$G_i(z_i^*)f_i(F^{-1}(r)) + \int_r^1 G_i(\sigma(F_i^{-1}(r) - F_i^{-1}(a)) + z_i^*)df(F_i^{-1}(a)).$$

Again, since $F_i^{-1}(r) - F_i^{-1}(a) \le 0$, the second term vanishes as σ tends to infinity, and thus the numerator converges to $G_i(z_i^*)f_i(F_i^{-1}(r))$. Therefore,

$$\lim_{\sigma\to\infty} \frac{\sigma h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)}{H_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} = \frac{f_i(F_i^{-1}(r))}{r}.$$

Similarly, changing the variables with $a = F_i^\sigma(v_i^\sigma)$ and $r = F_i^\sigma(w_i^\sigma - z^*)$ in equation (6) and following a similar procedure, we have

$$\lim_{\sigma\to\infty} \frac{\sigma(h_i^\sigma)'(F_i^{-1}(r) + z_i^*)}{h_i(F_i^{-1}(r) + z_i^*)} = \frac{(1 - G_i(z_i^*))f_i'(F_i^{-1}(r))}{f_i(F_i^{-1}(r))}.$$

Altogether,

$$\begin{aligned}
&\lim_{\sigma\to\infty} \sigma\left[\frac{(h_i^\sigma)'((F_i^\sigma)^{-1}(r) + z_i^*)}{h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} - \frac{h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)}{H_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)}\right] \\
=\;& \frac{(1 - G_i(z_i^*))f_i'(F_i^{-1}(r))}{f_i(F_i^{-1}(r))} - \frac{f_i(F_i^{-1}(r))}{r} \\
=\;& (1 - G_i(z_i^*))\left[\frac{f_i'(F_i^{-1}(r))}{f_i(F_i^{-1}(r))} - \frac{f_i(F_i^{-1}(r))}{r}\right] - \frac{G_i(z_i^*)f_i(F_i^{-1}(r))}{r} < 0. \qquad (7)
\end{aligned}$$

For any $s_i \in (0, \infty)$, $G_i(z_i^*) \in (0, 1)$ by equation (1). The square bracket term is weakly negative because F is log-concave, thus the entire expression is weakly negative. Now we show the strict inequality (7) holds for all $r \in [0,1]$. For $r \in (0,1)$, the strict inequality (7) is true because $f_i(F_i^{-1}(r))/r > 0$ by the full support assumption. Since $f_i(F_i^{-1}(r))/r$ falls in r by the log-concavity of F_i, $f_i(F_i^{-1}(r))/r > 0$ at $r = 0$, and thus the strict inequality (7) also holds for $r = 0$. For $r = 1$, since f_i has full support, $f_i(F_i^{-1}(r))$ falls in r when r is large. Therefore $f_i'(F_i^{-1}(r))/f_i(F_i^{-1}(r)) < 0$ for some $r \in (0,1)$. Since $f_i'(F_i^{-1}(r))/f_i(F_i^{-1}(r))$ falls in r by the log-concavity of f_i, $f_i'(F_i^{-1}(r))/f_i(F_i^{-1}(r)) < 0$ when $r = 1$ and thus the strict inequality (7) holds when $r = 1$. Altogether, for each $r \in [0,1]$ there is a $\bar{\sigma}_r$ such that if $\sigma > \bar{\sigma}_r$, then $(h_i^\sigma)'(w)/h_i^\sigma(w) - h_i^\sigma(w)/H_i^\sigma(w) < 0$ where $w = F^{-1}(r)$. Since $[0,1]$ is a compact convex set, there exists $\bar{\sigma} = \max_{r\in[0,1]} \sigma_r \leq \infty$ such that if $\sigma > \bar{\sigma}$, then $(h_i^\sigma)'/h_i^\sigma - h_i^\sigma/H_i^\sigma < 0$ for all $r \in [0,1]$, or equivalently $H_i^\sigma(w)$ is log-concave for all w. ■

The following Lemma is useful for proving Proposition 3.

Lemma 4 *For any $a \in (0,1]$, there exists $s_a < \infty$ such that $h_i(F_i^{-1}(a))/H_i(F_i^{-1}(a))$ falls in s_i whenever $s_i \geq s_a$.*

Proof. Suppose $a \in (0,1)$, and let $w_i = F_i^{-1}(a)$. We show that $h_i(w_i)/H_i(w_i)$ falls in s if and only if s_i is large. By equation (1), $\partial z_i^*/\partial s_i = -[1 - G_i(z_i^*)]$. Then by equation (3), $\partial H_i(w_i)/\partial s_i = f_i(w_i - z_i^*)$. Therefore,

$$\frac{\partial}{\partial s_i} log \left[\frac{h_i(w_i)}{H_i(w_i)}\right] = \frac{f_i(w_i - z_i^*)}{h_i(w_i)} \left[\frac{f_i'(w_i - z_i^*)}{f_i(w_i - z_i^*)} - \frac{h_i(w_i)}{H_i(w_i)}\right]. \tag{8}$$

Suppose the square bracket term in the right-hand side is equal to 0 at some $s_i = s_a$. As s_i rises from s_a, $f_i'(w_i - z_i^*)/f_i(w_i - z_i^*)$ falls, because z_i^* falls in s_i and f_i is log-concave. The derivative of the second term in the square bracket with respect to s_i is equal to 0 at $s_i = s_a$. Thus $\partial[h_i(w_i)/H_i(w_i)]/\partial s_i \leq 0$ for all $s_i \geq s_a$. Equivalently, $\partial\left(h_i(w_i)/H_i(w_i)\right)/\partial s_i$ is reverse single-crossing in s_i.

To show $s_a < \infty$, it suffices to show $\partial(h_i(w_i)/H_i(w_i))/\partial s_i < 0$ as $s_i \to \infty$. If $s_i \to \infty$, then $z_i^* \to -\infty$ and the sign of $\partial(h_i(w_i)/H_i(w_i))/\partial s_i$ is the same as

$$\begin{aligned}
\lim_{z_i^* \to -\infty} \left[\frac{f_i'(w_i - z_i^*)}{f_i(w_i - z_i^*)} - \frac{h_i(w_i)}{H_i(w_i)}\right] &= \lim_{z_i^* \to -\infty} \left[\frac{f_i'(w_i - z_i^*)}{f_i(w_i - z_i^*)} - \frac{\int f_i(w_i - \min\{z_i, z_i^*\})g_i(z_i)dz_i}{\int F_i(w_i - \min\{z_i, z_i^*\})g_i(z_i)dz_i}\right] \\
&= \lim_{z_i^* \to -\infty} \left[\frac{f_i'(w_i - z_i^*)}{f_i(w_i - z_i^*)} - \frac{f_i(w_i - z_i^*)}{F_i(w_i - z_i^*)}\right] < 0.
\end{aligned}$$

The last inequality is true as $\lim_{z_i^* \to -\infty} f_i(w_i - z_i^*)/F_i(w_i - z_i^*) = 0$ and $\lim_{z_i^* \to -\infty} f_i'(w_i - z_i^*)/f_i(w_i - z_i^*) < 0$ by the log-concavity of f_i. Since $\partial(h_i(w_i)/H_i(w_i))/\partial s_i$ is reverse single-crossing in s_i and is strictly negative as s_i explodes, there exists $s_a < \infty$ such that $h_i(w_i)/H_i(w_i)$ falls in s_i for all $s_i > s_a$.

Finally, assume $a = 1$ and let $w_i = F_i^{-1}(a) = \infty$. In this case the right-hand side of equation (8) is strictly negative because $\lim_{w_i \to \infty} f_i'(w_i - z_i^*)/f_i(w_i - z_i^*) < 0$ by the log-concavity of f and $\lim_{w_i \to \infty} h_i(w_i)/H_i(w_i) = 0$. Therefore, $h_i(w_i)/H_i(w_i)$ falls in s_i when $a = 1$. ■

Proof of Proposition 3 . *Proof of (i):* To show that $H_i(w_i)$ is log-concave when s_i is large, it suffices to show the reverse hazard rate $h_i(w_i)/H_i(w_i)$ falls in w_i when s_i is large. Recall that $\partial z_i^*/\partial s_i = -1/[1 - G_i(z_i^*)]$ by equation (1) and $H_i(w_i) = \int_{w_i - z_i^*}^{\infty} G_i(w_i - v_i)dF_i(v_i) + F_i(w_i - z_i^*)$ by equation (2). Thus $\partial log(H_i(w_i))/\partial s_i = f_i(w_i - z_i^*)/H_i(w_i)$. Therefore $h_i(w_i)/H_i(w_i)$ can be written as

$$\frac{h_i(w_i)}{H_i(w_i)} = [1 - G_i(z_i^*)]\frac{\partial log(H_i(w_i))}{\partial s_i} + \frac{\int_{w_i - z_i^*}^{\infty} g_i(w_i - v_i)dF_i(v_i)}{\int_{w_i - z_i^*}^{\infty} G_i(w_i - v_i)dF_i(v_i) + F_i(w_i - z_i^*)}.$$

We argue that the right-hand side falls in w_i when s_i is sufficiently large. To see this, note that an immediate corollary of Lemma 4 is that $\partial log(H_i(w_i))/\partial w_i$ falls in s_i for all $w_i \geq u_0 > -\infty$ when s_i is sufficiently large. Equivalently, $\partial log(H_i(w_i))/\partial s_i$ falls in w_i for all $w_i \geq u_0$ when s_i is sufficiently large. Therefore, the first term in the displayed equation falls in w_i when s_i is large. It remains to show that the second term falls in w_i. To this end, consider the inverse of the second term

$$\begin{aligned}
& \left[\frac{\int_{w_i - z_i^*}^{\infty} g_i(w_i - v_i)dF_i(v_i)}{\int_{w_i - z_i^*}^{\infty} G_i(w_i - v_i)dF_i(v_i) + F_i(w_i - z_i^*)}\right]^{-1} \\
= & \frac{\int_{w_i - z_i^*}^{\infty} G_i(w_i - v_i)dF_i(v_i)}{\int_{w_i - z_i^*}^{\infty} g_i(w_i - v_i)dF_i(v_i)} + \frac{F_i(w_i - z_i^*)}{\int_{w_i - z_i^*}^{\infty} g_i(w_i - v_i)dF_i(v_i)} \\
= & \frac{\int_{-\infty}^{z_i^*} G_i(z_i)f_i(w_i - z_i)dz_i}{\int_{-\infty}^{z_i^*} g_i(z_i)f_i(w_i - z_i)dz_i} + \frac{F_i(w_i - z_i^*)}{f_i(w_i - z_i^*)}\frac{f_i(w_i - z_i^*)}{\int_{-\infty}^{z_i^*} g_i(z_i)f_i(w_i - z_i)dz_i}.
\end{aligned}$$

The second line applies a change of variable $z_i = w_i - v_i$. The first term rises in w_i by the log-concavity of f_i and G_i. The second term rises in w_i by the log-concavity of F_i and f_i. Altogether, the entire expression rises in w_i. Since all elements in the expression are positive, its inverse falls in w_i.

Proof of (ii): Since we assume the density function $f_i(v_i)$ is log-concave, it is single-peaked in v_i. Since f_i has full support and is a probability density function, it cannot be monotone and thus must rises and then falls as v_i rises. Thus there exists $\bar{u}_0$ such that $f_i'(w_i - z_i^*) \leq 0$ for all $w_i > \bar{u}_0$. It follows that for all $w_i > \bar{u}_0$, $h_i(w_i) = \int f_i(w_i - \min\{z_i, z_i^*\})g_i(z_i)dz_i$ falls in w_i. Thus $h_i(w_i)/H_i(w_i)$ falls in w_i for $w_i \geq \bar{u}_0$. ■

Proof of Proposition 4. We prove the last claim in the proof of Proposition 4 in the main text. Index $D_i(\mathbf{p}, u_0)$ by the outside option u_0 and let $\mathbf{p}^* = (p^*, \ldots, p^*)$. Then, the right-hand side of equation (4) can be rewritten as $-\partial Log[D_i(\mathbf{p}, u_0)]/\partial p_i|_{\mathbf{p}=\mathbf{p}^*}$. Due to the additive utility specification, $D_i(\mathbf{p}, u_0) = D_i(\mathbf{p}+u_0, 0)$, that is, demand for each seller stays constant if all prices and $-u_0$ increase by the same amount. This implies

$$\begin{aligned}&\frac{\partial}{\partial p^*}\left[\frac{-\partial Log[D_i(\mathbf{p}, u_0)]}{\partial p_i}|_{\mathbf{p}=\mathbf{p}^*}\right] = \frac{\partial}{\partial p^*}\left[\frac{-\partial Log[D_i(\mathbf{p}+u_0, 0)]}{\partial p_i}|_{\mathbf{p}=\mathbf{p}^*}\right]\\ =&\frac{\partial}{\partial u_0}\left[\frac{-\partial Log[D_i(\mathbf{p}+u_0, 0)]}{\partial p_i}|_{\mathbf{p}=\mathbf{p}^*}\right] = \frac{-\partial^2 Log[D_i(\mathbf{p}, u_0)]}{\partial p_i \partial u_0}|_{\mathbf{p}=\mathbf{p}^*}.\end{aligned}$$

Since $D_i(\mathbf{p}, u_0)$ is log-submodular in (p_i, u_0) by the proof of Theorem 1 in Quint (2014), the right-hand is positive and thus $-\partial Log[D_i(\mathbf{p}, u_0)]/\partial p_i|_{\mathbf{p}=\mathbf{p}^*}$ rises in p^*. ■

Recall from Section 5.3 that $V \sim \mathcal{N}(0, \alpha^2)$ and $Z \sim \mathcal{N}(0, 1-\alpha^2)$ in the accuracy model.

Proof of Lemma 3. It suffices to show there exists $a' \in (0, 1)$ such that $\partial h(H^{-1}(a))/\partial\alpha < 0$ if and only if $a > a'$. Let Φ denote the standard normal distribution function and ϕ denote its density function. Since $V \sim \mathcal{N}(0, \alpha^2)$ and $Z \sim \mathcal{N}(0, 1-\alpha^2)$, $F(v) = \Phi(v/\alpha)$ and $G(z) = \Phi(z/\sqrt{1-\alpha^2})$. Inserting these into equation (2) and differentiating $H(w)$ with respect to α yield

$$H_\alpha(w) \equiv \frac{\partial H(w)}{\partial\alpha} = -\left[1 - \Phi\left(\frac{z^*}{\sqrt{1-\alpha^2}}\right)\right]\left(\frac{w-z^*}{\alpha^2}\right)\phi\left(\frac{w-z^*}{\alpha}\right),$$

where $\partial z^*/\partial\alpha$ can be obtained from equation (1) by applying the implicit function theorem. Differentiating again with respect to w gives

$$h_\alpha(w) \equiv \frac{\partial h(w)}{\partial\alpha} = -\left[1 - \Phi\left(\frac{z^*}{\sqrt{1-\alpha^2}}\right)\right]\left[1 - \left(\frac{w-z^*}{\alpha}\right)^2\right]\frac{1}{\alpha^2}\phi\left(\frac{w-z^*}{\alpha}\right).$$

Now observe that

$$\frac{\partial h(H^{-1}(a))}{\partial\alpha} = h_\alpha(H^{-1}(a)) - H_\alpha(H^{-1}(a))\frac{h'(H^{-1}(a))}{h(H^{-1}(a))}.$$

Let $w = H^{-1}(a)$ and apply $H_\alpha(w)$ and $h_\alpha(w)$ to the equation. Then,

$$\frac{\partial h(H^{-1}(a))}{\partial\alpha} = \frac{-1}{\alpha^2}\left[1 - \Phi\left(\frac{z^*}{\sqrt{1-\alpha^2}}\right)\right]\phi\left(\frac{w-z^*}{\alpha}\right)\left[1 - \frac{(w-z^*)^2}{\alpha^2} - (w-z^*)\frac{h'(w)}{h(w)}\right].$$

Since $V \sim \mathcal{N}(0, \alpha^2)$ and $Z \sim \mathcal{N}(0, 1-\alpha^2)$, the density of $W = V + \min\{Z, z^*\}$ is

$$\begin{aligned} h(w) &= \frac{1}{\sqrt{1-\alpha^2}} \int_{-\infty}^{\infty} \phi\left(\frac{w - \min\{z, z^*\}}{\alpha}\right) \phi\left(\frac{z}{\sqrt{1-\alpha^2}}\right) dz \\ &= \frac{1}{\sqrt{1-\alpha^2}} \int_{-\infty}^{\infty} \phi\left(\frac{w - z^*}{\alpha} + \max\{r, 0\}\right) \phi\left(\frac{z^* - \alpha r}{\sqrt{1-\alpha^2}}\right) dr \end{aligned}$$

where the second line changes variable $r = (z^* - z)/\alpha$. Since $\partial\phi(x)/\partial x = -x\phi(x)$,

$$\frac{h'(w)}{h(w)} = -\frac{w - z^*}{\alpha^2} - \frac{\int_{-\infty}^{\infty} \max\{r, 0\}\phi\left(\frac{w-z^*}{\alpha} + \max\{r, 0\}\right) \phi\left(\frac{z^*-\alpha r}{\sqrt{1-\alpha^2}}\right) dr}{\alpha \int_{-\infty}^{\infty} \phi\left(\frac{w-z^*}{\alpha} + \max\{r, 0\}\right) \phi\left(\frac{z^*-\alpha r}{\sqrt{1-\alpha^2}}\right) dr}.$$

Applying this to the above equation leads to

$$\begin{aligned} \frac{\partial h(H^{-1}(a))}{\partial \alpha} &\propto -1 + \left(\frac{w - z^*}{\alpha}\right)^2 + (w - z^*)\frac{h'(w)}{h(w)} \\ &= -1 + \frac{(z^* - w)}{\alpha} \frac{\int_{-\infty}^{\infty} \mathbb{1}_{\{r \geq 0\}} r\phi\left(\frac{w-z^*}{\alpha} + \max\{r, 0\}\right) \phi\left(\frac{z^*-\alpha r}{\sqrt{1-\alpha^2}}\right) dr}{\int_{-\infty}^{\infty} \phi\left(\frac{w-z^*}{\alpha} + \max\{r, 0\}\right) \phi\left(\frac{z^*-\alpha r}{\sqrt{1-\alpha^2}}\right) dr}. \end{aligned}$$

The last expression is clearly negative if $w > z^*$. In addition, it converges to ∞ as w tends to $-\infty$. For $w \leq z^*$, it decreases in w because $(z^* - w)$ falls in w and the density $\phi((w - z^*)/\alpha + \max\{r, 0\})$ is log-submodular in (w, r). Therefore, there exists $w'(< z^*)$ such that the expression is positive if and only if $w < w'$. The desired result follows from the fact that $w = H^{-1}(a)$ is strictly increasing in a. ■

Proof of Proposition 7. Given that there is no outside option, the condition for the equilibrium price p^* is given by

$$\frac{1}{p^* - c} = n \int h(w) dH(w)^{n-1} = n \int_0^1 h(H^{-1}(a)) da^{n-1}.$$

By the implicit function theorem,

$$\frac{\partial p^*}{\partial \alpha} = -(p^* - c)^2 n \int_0^1 \frac{\partial h(H^{-1}(a))}{\partial \alpha} da^{n-1}.$$

The desired result follows by combining Lemma 3 above with the fact that for any real value function $\gamma : \mathcal{R} \to \mathcal{R}$, if $\int_0^1 \gamma(a) da^n \leq 0$ and there exists a' such that $\gamma(a) < 0$ if and only if

$a > a'$, then

$$\int_0^1 \gamma(a)da^{n+1} = \frac{n+1}{n}\int_0^1 \gamma(a)ada^n \leq 0.$$

The last inequality is due to the fact that a is positive and strictly increasing and, therefore, assigns more weight to the negative portion of $\gamma(a)$ in the integral. The result follows by letting $\gamma(a) = \partial h(H^{-1}(a))/\partial\alpha$. ■

Proof of Proposition 8. Let $\bar{p}_i = p_i - c_i$ be product i's markup. We prove the claim by contradiction — Assume $\bar{p}_i < \bar{p}_j$ and show that seller i would deviate and choose $\bar{p}_i \geq \bar{p}_j$. Let $X_i = -\max_{\ell\neq i}\{W_\ell - c_\ell - \bar{p}_\ell, u_0\}$. Seller i's demand is $D_i(\mathbf{p}) = P(W_i - c_i - \bar{p}_i > -X_i) = P(W_i - c_i + X_i > \bar{p}_i)$. Let R_i be the distribution function of the random variable $W_i - c_i + X_i$ and r_i be its density function. Then $D_i(\mathbf{p}) = 1 - R_i(\bar{p}_i)$ and thus seller i's FOC is

$$\frac{1}{\bar{p}_i} = \frac{-\partial D_i(\mathbf{p})/\partial\bar{p}_i}{D_i(\mathbf{p})} = \frac{r_i(\bar{p}_i)}{1 - R_i(\bar{p}_i)}.$$

To derive a contradiction, it suffices to show $r_i(\bar{p}_i)/[1 - R_i(\bar{p}_i)] \leq r_j(\bar{p}_j)/[1 - R_j(\bar{p}_j)]$ whenever $\bar{p}_i < \bar{p}_j$. Recall that $D_i(\mathbf{p})$ is log-concave in $\bar{p}_i$ and log-supermodular in $(\bar{p}_i, \bar{p}_j)$ by Theorem 1, thus $r_i(\bar{p}_i)/[1-R_i(\bar{p}_i)]$ rises in $\bar{p}_i$ and falls in $\bar{p}_j$. Similarly, $r_j(\bar{p}_j)/[1-R_j(\bar{p}_j)]$ rises in $\bar{p}_j$ and falls in $\bar{p}_i$. Therefore, it suffice to show $r_i/[1 - R_i] \leq r_j/[1 - R_j]$ whenever $\bar{p}_i = \bar{p}_j$. Fixing $\bar{p}_i = \bar{p}_j$ and all other markups, if $W_i - c_i$ and $W_j - c_j$ have the same distribution, then clearly $r_i/[1 - R_i] = r_j/[1 - R_j]$. To show $r_i/[1 - R_i] \leq r_j/[1 - R_j]$ when $W_i - c_i$ dominates $W_j - c_j$ in the hazard rate and reverse hazard rate order, it suffices to show (a) $r_i/[1 - R_i]$ falls as $W_i - c_i$ rises in the hazard rate order and (b) $r_j/[1 - R_j]$ rises as $W_i - c_i$ rises in the reverse hazard rate order.

Proof of (a): First, note that $r_i/[1 - R_i]$ falls if $W_i - c_i + X_i$ rises in the hazard rate order. By Lemma 1.B.3. in SS,[28] if the survivor function of X_i is log-concave, then $W_i - c_i + X_i$ rises in the hazard rate order when $W_i - c_i$ rises in the hazard rate order. To see why the survivor of X_i is log-concave, observe that

$$P(X_i > x) = P(\max_{j\neq i}\{W_j - c_j - \bar{p}_j, u_0\} < -x) = \prod_{j\neq i} H_j(c_j + \bar{p}_j - x)\mathbb{1}_{\{u_0<-x\}}$$

$$log(P(X_i > x)) = \sum_{j\neq i} log(H_j(c_j + \bar{p}_j - x)) + log(\mathbb{1}_{\{u_0<-x\}}) \tag{9}$$

where $\mathbb{1}_{\{u_0<-x\}}$ is an indicator function of the event $\{u_0 < -x\}$. The left-hand side of the second line is concave in x because each element in the right-hand side is. This proves (a).

[28]Lemma 1.B.3. in SS: Assume the random variables X and Y are such that X dominates Y in the hazard rate order. If W is a random variable independent of X and Y and has log-concave survivor function, then $X + Z$ dominates $Y + Z$ in the hazard rate order.

Proof of (b): Similar to (a), $W_j - c_j + X_j$ falls in the hazard rate order as X_j falls in the hazard rate order by Lemma 1.B.3. in SS because we have assumed the survivor of W_j is log-concave. It remains to show that X_j falls in the hazard rate order when $W_i - c_i$ rises in the reverse hazard rate order. As $W_i - c_i$ rises in the reverse hazard rate order, the ratio $h_i(c_i + \bar{p}_i - x)/H_i(c_i + \bar{p}_i - x)$ rises for all x and thus the slope of $log(H_i(c_i + \bar{p}_i - x))$ with respect to x falls at all x. Hence the slope of $log(P(X_j > x))$ with respect to x falls for all x by equation (9), which implies X_j falls in the hazard rate order. Altogether, $W_j - c_j + X_j$ falls in the hazard rate order as $W_i - c_i$ rises in the reverse hazard rate order. ■

Proof of Corollary 2. We show W_j dominates W_i in the hazard rate order and the reverse hazard rate order and then use Proposition 8 to prove the claim. Since the likelihood ratio order implies both the hazard rate order and the reverse hazard rate order by Theorem 1.C.1. in SS, it suffices to show W_j dominates W_i in the likelihood ratio order. Since $F_i = F_j = F$ and $z_i^* = z_j^* = z^*$, the likelihood ratio is

$$\frac{h_j(w)}{h_i(w)} = \frac{\int f(w - \min\{z, z^*\}) g_j(z) dz}{\int f(w - \min\{z, z^*\}) g_i(z) dz} = \int \frac{g_j(z)}{g_i(z)} \frac{f(w - \min\{z, z^*\}) g_i(z)}{\int f(w - \min\{z, z^*\}) g_i(z) dz} dz$$

The right-hand side can be interpreted as $E[g_j(X)/g_i(X)]$ where the random variable X has density $f(w - \min\{x, z^*\}) g_i(x)$. The random variable X rises in the first order stochastic dominance sense in w because f is log-concave. The function $g_j(X)/g_i(X)$ rises in X because Z_j dominates Z_i in the likelihood ratio order. Therefore, $h_j(w)/h_i(w)$ rises in w, or equivalently W_j dominates W_i in the likelihood ratio order. ■

Lemma 5 below shows that H_i falls in the likelihood ratio order as s_i rises under the premises of Corollary 3. Therefore, the conclusion of Corollary 3 follows from its premises by (i) Lemma 5, (ii) the fact that the likelihood ratio order implies both the hazard and reverse hazard rate order, and (iii) Proposition 8.

Lemma 5 *If $-f_i'(v)$ is positive and log-concave for all $v > u - z_i^*$, then $h_i(w_2)/h_i(w_1)$ falls in s_i for all $w_2 > w_1 \geq u_0$.*

Proof. Differentiating equation (3) yields

$$h_i(w) = \int_{-\infty}^{z_i^*} -f_i'(w - z)[1 - G_i(z)] dz.$$

To prove $h_i(w_2)/h_i(w_1)$ falls in s_i for all $w_2 > w_1 \geq u_0$, it suffices to show $(\partial h_i(w)/\partial s_i)/h_i(w)$ falls in w. Recall that $\partial z_i^*/\partial s_i = -1/[1 - G_i(z_i^*)]$. Thus

$$\frac{\partial h_i(w)/\partial s_i}{h_i(w)} = -\frac{f_i'(w - z_i^*)}{\int_{-\infty}^{z_i^*} f_i'(w - z)[1 - G_i(z)] dz}.$$

Since we assume $-f_i'(v)$ is positive and log-concave for all $v > u_0 - z^*$, $-f_i'(w-z)$ is log-supermodular in (w,z) for all $z \leq z_i^*$ and $w \geq u_0$. Therefore, the ratio $f_i'(w-z_i^*)/f_i'(w-z) > 0$ rises in w for all $z \leq z_i^*$ and $w \geq u_0$, and thus $[\partial h_i(w)/\partial s_i]/h_i(w)$ falls in w for $w \geq u_0$. Equivalently, $h_i(w_2)/h_i(w_1)$ falls in s for all $w_2 > w_1 \geq u_0$. ■

The following lemma is useful for proving Proposition 9.

Lemma 6 *Assume $F_1 = F_2 = F$, $G_1 = G_2 = G$ and $s_1 = s_2 = s$. The difference $W_2 - W_1$ grows less dispersive as the search cost s rises.*

Proof. Consider $W_2 - W_1 = V_2 - V_1 + \min\{Z_2, z^*\} - \min\{Z_1, z^*\}$. By Theorem 3.B.7 in SS, $W_2 - W_1$ grows more dispersive if (a) $V_2 - V_1$ has log-concave density and (b) the difference $\min\{Z_2, z^*\} - \min\{Z_1, z^*\}$ grows more dispersive. Since we have assume V_2 and V_1 have log-concave density, so does $V_2 - V_1$. Thus (a) is satisfied. To see (b), let T be the distribution function of the absolute difference $Y = |\min\{Z_2, z^*\} - \min\{Z_1, z^*\}|$:

$$T(y) = P(y \geq |\min\{Z_2, z^*\} - \min\{Z_1, z^*\}|) = 1 - 2\int G(\min\{z, z^*\} - y)dG(z).$$

By the definition of dispersive order, Y grows less dispersive in s if and only if its quantile function T^{-1} grows flatter as s rises, namely $\partial T^{-1}(a)/\partial a$ falls in s for all $a \in (0,1)$. Equivalently, $\partial T^{-1}(a)/\partial s = -[\partial T(y)/\partial s]/t(y)$ falls in $y = T^{-1}(a)$. Differentiating T with respect to y and s yields

$$\frac{\partial T^{-1}(a)}{\partial s} = -\frac{\partial T(y)/\partial s}{t(y)} = \frac{-g(z^* - y)}{\int g(\min\{z, z^*\} - y)dG(z)}.$$

The right-hand side falls in y by the log-concavity of g. Therefore, Y grows less dispersive as s rises. Since Z_1 and Z_2 have the same distribution, the distribution of the random variable $\min\{Z_2, z^*\} - \min\{Z_1, z^*\}$ is symmetric around 0. Therefore, $\min\{Z_2, z^*\} - \min\{Z_1, z^*\}$ also grows less dispersive as s rises.[29] ■

Proof of Proposition 9. When $n = 2$ and $u_0 = -\infty$, the demand function is $D_i(\mathbf{p}) = \int 1 - H_i(w - p_j + p_i)dH_j(w)$ for $i = 1, 2$. The first order condition for seller 1 and 2 are

$$p_1 - c_1 = \frac{\int(1 - H_1(w - p_2 + p_1))dH_2(w)}{\int h_1(w - p_2 + p_1)dH_2(w)} \text{ and } p_2 - c_2 = \frac{\int(1 - H_2(w - p_1 + p_2))dH_1(w)}{\int h_2(w - p_1 + p_2)dH_1(w)}.$$

[29] To see this, let $\tilde{T}$ be the distribution function of $\min\{Z_2, z^*\} - \min\{Z_1, z^*\}$. Since the distribution of $\min\{Z_2, z^*\} - \min\{Z_1, z^*\}$ is symmetric around 0, $\tilde{T}(y) = [1 - T(-y)]/2$ for $y < 0$ and $\tilde{T}(y) = [1 + T(y)]/2$ for $y \geq 0$. Thus $\tilde{T}^{-1}(a) = -T^{-1}(1 - 2a)$ for $a < 1/2$ and $\tilde{T}^{-1}(a) = T^{-1}(2a - 1)$ for $a \geq 1/2$. Recall that a random variable grows more dispersed if and only if its quantile function grows steeper at all quantile. Clearly, $\partial\tilde{T}^{-1}(a)/\partial a$ rises $\forall a \in (0,1)$ if $\partial T^{-1}(a)/\partial a$ rises $\forall a \in (0,1)$.

Define the price difference $\Delta \equiv p_2 - p_1$. Seller 1 and 2's first order conditions imply

$$c_2 - c_1 - \Delta = \frac{\int (1 - H_1(w - \Delta))dH_2(w)}{\int h_1(w - \Delta)dH_2(w)} - \frac{\int (1 - H_2(w + \Delta))dH_1(w)}{\int h_2(w + \Delta)dH_1(w)}. \tag{10}$$

Easily, the left-hand side falls in Δ. The right-hand side rises in Δ by the log-concavity of the demand functions. Therefore, equation (10) has a unique solution for Δ.

Let Q and q be the distribution function and the density function of the absolute difference $|W_2 - W_1|$. Since W_1 and W_2 have the same distribution, $H_1 = H_2 = H$. The probability $P(|W_2 - W_1| \geq \Delta) = 2 \int [1 - H(w + \Delta)]dH(w)$. Hence

$$Q(\Delta) = 1 - 2 \int (1 - H(w + \Delta))dH(w) \quad \text{and} \quad q(\Delta) = 2 \int h(w + \Delta)dH(w). \tag{11}$$

Since $D_1(\mathbf{p}) = 1 - D_2(\mathbf{p})$ and $Q(\Delta) = 1 - 2D_2(\mathbf{p})$, $Q(p_2 - p_1) = D_1(\mathbf{p}) - D_2(\mathbf{p})$. Therefore, $\partial D_1/\partial p_1 = \partial D_2/\partial p_2 = q(p_2 - p_1)/2$. Thus, equation (10) can be written as

$$c_2 - c_1 - \Delta = \frac{2Q(\Delta)}{q(\Delta)} \iff c_2 - c_1 - Q^{-1}(a) = \frac{2a}{q(Q^{-1}(a))} \tag{12}$$

where the second equation applies a change of variable $a = Q(\Delta)$. Since $c_2 - c_1 - p_2 + p_1 \geq 0$ by Corollary 1, $Q(\Delta) \geq 0$ in equilibrium. This implies $D_1(\mathbf{p}) \geq D_2(\mathbf{p})$ and thus $p_1 \leq p_2$.

Since $Q(\Delta)/q(\Delta)$ rises in Δ by the log-concavity of the demand functions, the fraction $a/q(Q^{-1}(a))$ in equation (12) rises in a. As H grows more dispersive, the absolute difference $|W_2 - W_1|$ rises in the first order stochastic dominance sense by Theorem 3.B.31 in SS and grows more dispersive by Lemma 6. Thus, $Q^{-1}(a)$ rises and $q(Q^{-1}(a))$ falls for all $a \in [0, 1]$. Since the left-hand side falls and the right-hand side rises in a, the solution of a falls as H grows more dispersive. Since $a \equiv Q(p_2 - p_1) = D_1(\mathbf{p}) - D_2(\mathbf{p})$ and $D_1(\mathbf{p}) + D_2(\mathbf{p}) = 1$, $D_2(\mathbf{p})$ rises and $D_1(\mathbf{p})$ falls as H grows more dispersive. ■

Proof of Proposition 10 . Since $p_i - c_i = D_i(\mathbf{p})/[\partial D_i(\mathbf{p})/\partial p_i]$ and $\partial D_1/\partial p = \partial D_1/\partial p$, we have $(p_2 - c_2)/(p_1 - c_1) = D_2(\mathbf{p})/D_1(\mathbf{p})$. Therefore, $(p_2 - c_2)/(p_1 - c_1)$ decreases in s by Proposition 9.

Next, recall from equation (12) that the price difference $\Delta = p_2 - p_1$ solves

$$c_2 - c_1 - \Delta = \frac{2Q(\Delta)}{q(\Delta)}$$

and $Q(\Delta)/2q(\Delta)$ rises in Δ by the log-concavity of the demand functions. Thus, there is a unique solution for Δ, call it Δ^*. Clearly Δ^* rises in $c_2 - c_1$ by the displayed equation. We have seen in the last part of the proof of Proposition 9 that $Q(\Delta)$ rises in s for all $\Delta \geq 0$. Therefore, if $\partial q(\Delta)/\partial s \leq 0$ at $\Delta = \Delta^*$, then $\partial \Delta^*/\partial s \leq 0$. Therefore, to prove $\partial \Delta^*/\partial s \leq 0$

when $c_2 - c_1$ is large, it suffices to show $\partial q(\Delta)/\partial s|_{\Delta=\Delta^*} \leq 0$ when $c_2 - c_1$ is large. Since $c_2 - c_1$ affects the derivative only through Δ^* and Δ^* rises in $c_2 - c_1$, it suffices to prove $\partial q(\Delta)/\partial s \leq 0$ when Δ is large.

To this end, let $\tilde{f}(\Delta) \equiv \int f(v)f(\Delta + v)dv$. Then by equation (11)

$$q(\Delta)/2 = \int\int \tilde{f}(\min\{z, z^*\} - \min\{\tilde{z}, z^*\} - \Delta)g(z)g(\tilde{z})d\tilde{z}dz.$$

Differentiate with respected to s and uses $\partial z^*/\partial s = -1/[1 - G(z^*)]$, then

$$\begin{aligned}\frac{\partial q(\Delta)}{\partial s} =& 2\int_{-\infty}^{z^*} [\tilde{f}'(z - z^* - \Delta) - \tilde{f}'(z^* - z - \Delta)]g(z)dz \\ =& 2\int_0^{\infty} [\tilde{f}'(-r - \Delta) - \tilde{f}'(r - \Delta)]g(-r + z^*)dr\end{aligned}$$

This expression is negative when Δ is large because

$$\begin{aligned}\lim_{\Delta\to\infty} [\tilde{f}'(-r - \Delta) - \tilde{f}'(r - \Delta)] =& \lim_{\Delta\to\infty} [\tilde{f}'(\Delta - r) - \tilde{f}'(\Delta + r)] \\ =& \lim_{\Delta\to\infty} \int f(v)[f'(\Delta - r + v) - f'(\Delta + r + v)]dv \\ =& \int f(v) \lim_{\Delta\to\infty} [f'(\Delta - r + v) - f'(\Delta + r + v)]dv \leq 0.\end{aligned}$$

The first equation is true because $\tilde{f}'(v) = -\tilde{f}'(-v)$ by the definition of $\tilde{f}$. The second equation uses $\tilde{f}(\Delta) \equiv \int f(v)f(\Delta + v)dv$. The third equation is by the Dominated Convergence Theorem, as the absolute value of the integrand is bounded. The inequality is true as $f'(\Delta - r + v) - f'(\Delta + r + v) \leq 0$ when Δ is large, because:

$$\lim_{\Delta\to\infty} \left[1 - \frac{f'(\Delta + r + v)}{f'(\Delta - r + v)}\right] f'(\Delta - r + v) = \lim_{\Delta\to\infty} \left[1 - \frac{f(\Delta + r + v)}{f(\Delta - r + v)}\right] f'(\Delta - r + v) \leq 0.$$

The equation is by L'Hospital's rule. Since $f(v)$ falls in v when v is big, the fraction $f(\Delta + r + v)/f(\Delta - r + v) \leq 1$ when Δ is big. Moreover $f'(\Delta - r + v) \leq 0$ if Δ is big. Altogether the inequality is true. ■

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