Central Bank Digital Currency: Welfare and Policy Implications

Stephen Williamson*
University of Western Ontario

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Abstract

A model of multiple means of payment is constructed to analyze the effects of the introduction of central bank digital currency (CBDC). The introduction of CBDC has three beneficial effects. It mitigates crime associated with physical currency, permits the payment of interest on a key central bank liability, and economizes on scarce safe collateral. CBDC admits another instrument of monetary policy, but raises issues of central bank independence and scarcity of assets to back central bank liabilities.

1 Introduction

As financial technology evolves, central banks need to re-evaluate their role, potentially introducing new central bank assets and liabilities, and altering their approach to monetary policy decision making and implementation. In recent years, financial markets have been flooded with privately-issued cryptocurrencies – Bitcoin, for example. While such cryptocurrencies have failed to gain wide acceptance as means of payment, it is possible that the associated distributed ledger technologies could have applications in central banking. In addition, old-fashioned physical currency is being replaced as a means of payment in conventional transactions by credit card, debit card, and other electronic means of payment. Yet, the demand for central-bank-issued currency is increasing in most countries. For example, U.S. currency outstanding relative to GDP rose from 5.5% in 2007 to 8.0% in 2018. How can more currency be held when most people are using it less? As pointed out, for example by Rogoff (2016), there is strong evidence, including the fact that more than 80% of U.S. currency outstanding is in the form of $100 bills, that the strong demand for currency is explained primarily by crime. The state of the central banking certainly appears suboptimal if a key function of central banking is to serve the needs of criminals.

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Central banks, including those in Sweden, Canada, and the U.K., have shown an increased interest in digital currencies (see Chapman and Wilkins 2019, Kumhoff and Noone 2018, and Bech et al. 2018), typically referred to as CBDC (central bank digital currency). Potentially, CBDC could take many forms. Ownership could be recorded and transferred on a decentralized ledger, as with cryptocurrencies, or recordkeeping could be done in a centralized fashion, as with conventional private bank liabilities and central bank reserves. Central banks could opt for monopoly issue by the central bank of CBDC, just as for physical currency, or there could be competition among private digital currencies and CBDC, or perhaps the central bank could issue CBDC and leave the mechanics of converting CBDC into other assets to the private sector. Physical currency could be eliminated completely with the issue of CBDC, or use of physical currency could be limited by abolishing large-denomination notes.

The goal of this paper is to study the factors which could result in a welfare improvement from the issue of CBDC, and to determine how CBDC issue might change the effects of monetary policy and the optimal behavior of the central bank. In the model, the use of physical currency as a means of payment is socially costly, because currency is subject to theft. This, at least in part, captures the idea that currency promotes illegal and socially costly activity. If CBDC is issued, then it potentially drives out physical currency in the model, and therefore eliminates the illegal activity associated with physical currency. As well, CBDC is an improvement over physical currency in that the central bank can pay interest on CBDC. It was recognized, at least as early as Friedman (1969), that an alternative to a zero-nominal-interest rate policy for implementing the Friedman rule is to pay interest on currency at the appropriate rate. There are practical obstacles to paying interest on physical currency, but if CBDC consists of centralized account balances, then it is straightforward to pay interest on those account balances. Finally, CBDC may play an important role in mitigating the incentive problems associated with private banking. Provided we can trust the central bank, and if the transactions costs associated with using CBDC and private bank deposits as means of payment are similar, then substituting CBDC for transactions deposits at private banks could increase welfare.

In the model, there are potentially three means of payment: physical currency, CBDC, and bank deposits. Physical currency and CBDC are issued by the central bank, while bank deposits are issued by private financial institutions. The fiscal authority can tax lump sum, and it issues one-period nominal bonds. There are also private assets in this economy. Private banks issue deposits, and hold private assets and government bonds as assets. As well, private banks are subject to limited commitment – they can abscond on their deposit liabilities, as in Williamson (2016, 2019a, 2019b) and Gertler and Kiyotaki (2011), for example. So, a bank's assets implicitly serve as collateral backing the bank's deposit liabilities. Collateral can be scarce in this economy if the government limits outstanding government debt sufficiently. Scarcity of collateral is reflected in binding collateral constraints for banks and low real interest rates.

In a world like the current one, in which the central bank issues physical currency and there is no digital currency, theft limits the quantity of currency in circulation, and produces a social loss in the model. In the model, consumers who make large-transaction purchases use bank deposits as means of payment, while small purchases are made with currency. Those making small transactions face a low probability of theft, but if those making large transactions used currency instead of bank deposits they would face a high probability of theft. Thus, we can observe little theft in equilibrium, but bank deposits are nevertheless preferred to currency in part because of safety concerns. With physical currency, inflation acts as a tax on theft. Higher inflation reduces the quantity currency held, which makes theft less profitable.

Given the use of negative interest rate policies by central banks in the world, including those in the Euro Area, Japan, Sweden, and Denmark, it has become clear that there exist practical limits to arbitrage that permit nominal interest rates on safe assets to fall below zero. In conventional monetary models, a negative nominal interest rate implies that economic agents can earn infinite profits by borrowing at a negative interest rate and purchasing zero-interest currency. In our model, arbitrage is limited by theft, but not by theft of physical currency from consumers. If banks' collateral constraints bind, then if the nominal interest rate is sufficiently low, banks would prefer to hold physical currency rather than government debt as implicit collateral backing bank deposits. But government debt is superior collateral to physical currency, as it is easier for bankers to abscond with currency than with government debt. This implies that the effective lower bound on the nominal interest rate is negative, and that the effective lower bound decreases as banks' collateral constraints tighten. That is, low real interest rates are associated with a lower effective lower bound.

In a regime with digital currency and no physical currency, monetary policy works differently, in part because the central bank has two policy instruments – the nominal interest rate on short-term government debt, and the interest rate on digital currency – rather than just one. An increase in the nominal interest rate on government debt, engineered through an open market sale of government debt, will result in substitution from digital currency to bank deposits, an increase in the real interest rate, and an increase in the inflation rate. An increase in the nominal interest rate on digital currency causes substitution from bank deposits to digital currency, a decrease in the real interest rate, and an increase in the inflation rate.

There are two kinds of substitution across means of payment that can result from monetary policy. One is substitution on the supply side in that, for example, if economic agents using currency as a means of payment hold more CBDC, then the central bank must acquire more government debt, and there is less government debt available to back bank deposits. The other is on the demand side in that, for example, a lower interest rate on government debt makes bank deposits less attractive, and people could choose to substitute digital currency for bank deposits as a means of payment. Substitution on the demand side has implications for how collateral is used in the aggregate, as substitution away from bank deposits mitigates incentive problems in the aggregate. That is, if the central bank can be trusted, then digital currency could be more efficient than private bank deposits, as it uses the aggregate stock of collateral more effectively.

An issue that arises with CBDC issue by the central bank is the available supply of government debt to back CBDC. If CBDC is less subject to theft than physical currency, as we assume, and bears interest, then the demand for CBDC could exceed the existing demand for physical currency. Under some conditions, the stock of government debt would be insufficient to support the demand for CBDC, except given low interest rates on CBDC. The central bank could purchase other assets than government debt, but there are good reasons to think that central bank purchases of private assets is a bad idea. So, barring that, it is possible to expand the issue of CBDC through central bank lending to private banks. Indeed, some central banks have been set up primarily as lending institutions – the Federal Reserve System before the 1920s, and the modern-day European Central Bank.

In the model, central bank loans can be a source of funding for private banks, just like retail bank deposits. But then, these bank liabilities need to be backed by safe collateral, given banks' limited commitment problem. There is then an inefficiency tradeoff. If government debt is exhausted as backing for CBDC, than an expansion of central bank lending to back additional CBDC issue will result in an efficiency loss, due to the incentive problem associated with private banking, and an efficiency gain, due to the increased use of CBDC.

Another issue that arises, is that the payment of interest on CBDC could threaten central bank independence. Typical central banking practice is for the central bank to pay its costs, make a stream of positive transfers to the fiscal authority, and be removed from the fiscal budgeting process. This typical arrangement is supported currently through the granting of a monopoly in zero-nominal-interest currency to the central bank. Then, so long as inflation and the nominal interest rate on government debt are sufficiently high, the central bank always faces a sufficiently large spread between the rates of return on its assets and its liabilities. This generates enough revenue to pay the central bank's costs and make a positive transfer to the fiscal authority.

With CBDC, the interest rate on CBDC is a choice variable, and the level of the CBDC interest rate will matter for central bank revenue. Maintaining a stream of positive transfers to the fiscal authority requires that the CBDC interest rate be sufficiently low relative to the interest rate on government debt. That will then imply a distortion, which is a cost of central bank independence. Without the distortion, the central bank would have to depend on capital injections from the fiscal authority.

The economics of cryptocurrencies has been studied by Abadi and Brunnermeir (2018) and Chiu and Koeppl (2018), among others. Davoodalhosseini (2018), Hendry and Zhu (2019) and Keister and Sanches (2018) analyze the role of CBDC in general equilibrium. Some papers that do a nice job of laying out the issues at stake in CBDC issue are Chapman and Wilkins (2019), Bech et al. (2018), and Kumhof and Noone (2018). Issues associated with theft of currency have been studied in He et al. (2008), Sanches and Williamson (2010), and Williamson (2012). Others have studied models with alternative means of payment, including Stokey (2019).

The paper is organized as follows. In the second section, the model is constructed. Then, in Sections 3 and 4, economies with physical currency and the threat of theft, and with CBDC, respectively, are analyzed. The final section is a conclusion.

2 Model

The model builds on a basic Rocheteau and Wright (2005) framework, with additional structure added to address the particulars of this problem. Periods are indexed by $t = 0, 1, 2, \ldots$, and in each period there are two sub-periods, the centralized market (CM), followed by the decentralized market (DM). There is a continuum of *buyers*, with unit mass, indexed by $i\epsilon$ $[0, 1]$, each of whom is infinite-lived with preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[-H_t^i + \theta_t^i u(x_t^i) \right],$$

where $0 < \beta < 1$, H_t^i denotes labor supply in the CM, θ_t^i is an i.i.d. (across buyers and time) preference shock, realized at the end of the CM, after consumption and production takes place, and x_t^i denotes consumption in the DM. Assume that $\Pr[\theta_t^i = \theta^L] = \rho$, and that $\Pr[\theta_t^i = \theta^H] = 1-\rho$, where $\theta^H > \theta^L > 0$, and $0 < \rho < 1$. Preference shocks are assumed to be public information.[1]

There also exists a continuum of bankers with unit mass, each of whom has preferences

$$E_0 \sum_{t=0}^{\infty} \left[-H_t + X_t \right],$$

where H_t and X_t are, respectively, labor supply and consumption for the banker in the CM. In addition, there is a continuum of sellers with unit mass, each with preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[X_t - h_t \right],$$

where X_t is consumption in the CM, and h_t is labor supply in the DM. In the CM and the DM, one unit of labor supply produces one unit of the perishable consumption good. Buyers cannot produce in the DM, and sellers cannot produce in the CM.

In the CM, all agents are together in one location. At the beginning of the CM, debts acquired in the previous period are settled, then production and exchange take place, buyers write contracts with bankers, and assets are traded.

[1] There are banking models of course, the Diamond-Dybvig (1983) banking model in particular, in which preference shocks are assumed to be private information. That's part of the structure in those models that can generate bank runs. It simplifies matters here, however, if we assume preference shocks are publicly observable, and bank runs are not germaine to the key issues we want to study here.

Finally, at the end of the period, buyers' preference shocks are realized, and each buyer can make contact with his or her bank.

In the DM, each buyer is matched at random with a seller. Each seller has access to technologies which permit him or her to process payments in currency, CBDC, or bank deposits. That is, the costs per transaction to accepting currency, CBDC, or bank deposits, respectively, are k^c, k^m, and k^d, in units of labor. As well, it is possible for the seller to steal the buyer's currency in a meeting in the DM. In the DM, a seller who meets a buyer holding currency can supply w units of labor and steal all of the buyer's currency with probability $\sigma(w)$, where $\sigma(\cdot)$ is strictly increasing, strictly concave, and twice differentiable, with $\sigma(0) = 0$, $\sigma'(0) = \infty$, and $\sigma(\infty) = 1$. For convenience, assume that the seller can observe the buyer's currency holdings. Assume that theft of CBDC and bank deposits is not feasible. Also, suppose there is no technology that permits the trade of government debt or private assets in the DM, although banks can hold these assets and issue tradeable bank deposits as liabilities. There is limited commitment, in that no economic agent can be forced to work, and there is no recordkeeping, so buyers cannot trade personal IOUs in the DM.

The basic assets (before being transformed by financial intermediaries) in this economy are physical currency, CBDC, one-period nominal government debt, and private assets. Physical currency bears a nominal interest rate of zero. One unit of CBDC held at the beginning of the CM of period t yields the digital-currency holder $R^m_{t-1} - 1$ units of digital currency, paid by the central bank. Assume that the CBDC technology permits this. The gross nominal interest rate on government debt is R^b_t. Private assets are perfectly divisible, with unit supply, and yield a dividend of y consumption goods at the beginning of the CM, per unit held.

2.1 Government

Confine attention to policies that are constant for all t, and stationary equilibria. Assume that there is no consolidated government debt outstanding at the beginning of period $t = 0$, so the period 0 consolidated government budget constraint is given by

$$\bar{c} + \bar{m} + \bar{b} = \tau_0, \tag{1}$$

where $\bar{c}$, $\bar{m}$, and $\bar{b}$ denote the quantities of physical currency, CBDC, and one-period government debt issued in period 0 (and in each succeeding period), all in units of the period 0 CM good. As well, τ_0 denotes the lump sum transfer to each buyer at $t = 0$. Then, in each subsequent period, again confining attention to stationary policies and stationary equilibria,

$$\bar{c} + \bar{m} + \bar{b} = \frac{\bar{c}}{\pi} + \frac{R^m \bar{m}}{\pi} + \frac{R^b \bar{b}}{\pi} + \tau \tag{2}$$

Here, π denotes the gross inflation rate, and the lump-sum transfer τ is constant for $t = 1, 2, 3, \ldots$. The left-hand side of (2) is the sum of total consolidated liabilities outstanding after new liabilities are issued, while the right-hand side is the sum of the total redemption value of consolidated government liabilities from the previous period, plus the transfer to buyers.

It is important for the analysis how we specify fiscal policy, as this will help determine the aggregate supply of collateral, which plays an important role in the analysis. We will assume, as in Andolfatto and Williamson (2015) and Williamson (2016, 2019a, 2019b) that the fiscal authority sets τ_0 and τ in response to monetary policy so that the real value of the consolidated government debt is a constant, v. That is,

$$v = \bar{c} + \bar{m} + \bar{b} \tag{3}$$

Given this fiscal policy rule, the fiscal authority determines the total value of the consolidated government debt, while the central bank determines its composition.

3 Equilibrium With Physical Currency

We will first consider a case resembling the status quo in most countries. The central bank issues physical currency, and there is no CBDC. As well, private banks will be in the business of issuing bank deposits subject to withdrawal in physical currency. This will serve to insure the bank's depositors, who will be buyers, against random needs for alternative means of payment. The role for banks is related to what exists in a Diamond-Dybvig (1983) model, though in that model the insurance role for banks is in part built into the production technology, which is not the case here.

3.1 Private Banks

Buyers write deposit contracts with banks before learning their preference shocks. As in Williamson (2012, 2016, 2019a, 2019b), the deposit contract will provide insurance against preference shocks, in that the preference shock outcome will determine what means of payment the depositor will use, at the optimum. We will assume (and later derive conditions that imply that this is equilibrium behavior), that buyers with $\theta_t^i = \theta^H$, denoted *large-transaction buyers*, prefer to use bank deposits as means of payment, and those with $\theta_t^i = \theta^L$, denoted *small-transaction buyers*, prefer to use physical currency. As we will show, this requires that k^d be sufficiently large relative to k^c (the cost to using bank deposits is sufficiently large relative to the cost of using physical currency). As well, it is important that the cost of theft will be relatively low in equilibrium for a small-transaction buyer, and relatively high for a large-transaction buyer. Rates of return on the assets backing bank deposits, relative to the rate of return on currency will also matter, as we will show.

A bank can insure depositors by providing each with an option either to withdraw currency at the end of the CM, or else trade a claim on the bank. Assume that it is impossible to trade private assets or government debt in the DM. The bank offers a deposit contract (z, c, d), where z is the quantity of CM

goods deposited with the bank by the depositor at the beginning of the CM, c is the quantity of currency that the depositor can choose to withdraw (in real terms) at the end of the CM, and d is the quantity of claims to CM goods in the next period that the depositor can trade in the DM if he or she does not withdraw currency. As well, in the CM the bank acquires a portfolio consisting of b government bonds and a private assets. Then, in equilibrium, the bank solves

$$\max_{z,c,d,b,a}\left[-z+\rho\left[1-\sigma(w^L)\right]\theta^L u\left(\frac{\beta c}{\pi}-k^c\right)+(1-\rho)\theta^H u\left(\beta d-k^d\right)\right] \tag{4}$$

subject to

$$z-b-\phi a-\rho c+\beta\left[\frac{R^b b}{\pi}+(\phi+y)\,a-(1-\rho)d\right]\geq 0, \tag{5}$$

and

$$\left[\frac{R^b b}{\pi}+(\phi+y)\,a\right](1-\gamma)-(1-\rho)d\geq 0. \tag{6}$$

The objective function (4) is the expected utility of the depositor, as a function of the deposit contract (z,c,d), given take-it-or-leave-it offers by the buyer in the DM meetings and a probability $\sigma(w^L)$ of theft in the DM if the buyer withdraws currency. That is, in the case in which the buyer receives preference shock θ^L in the CM, he or she withdraws currency, c, from the bank at the end of the CM. Then, the buyer meets a seller in the following DM. If the seller steals the buyer's currency, the buyer consumes nothing in the DM and the buyer receives utility $u(0)=0$. If, with probability $1-\sigma(w^L)$, the seller does not steal the buyer's currency, the buyer makes a take–it-or-leave it offer, and exchanges c units of currency with the seller for $\frac{\beta c}{\pi}-k^c$ units of goods. This compensates the seller for the costs of the transaction and producing goods, given that the seller will exchange the currency for goods in the next CM. If the buyer receives preference shock θ^H in the CM, then he or she will trade a claim on the bank in the DM. This is a claim to d consumption goods in the next DM. So, in the DM,the buyer makes a take-it-or-leave-it offer to the seller, and the buyer exchanges the d deposit claims for $\beta d-k^d$ goods, as that compensates the seller for the costs of producing those goods and carrying out the transaction.

Inequality (5) states that the present value of the net payoff for the bank is nonnegative, where ϕ denotes the price of private assets. That is, in the current CM, the bank receives deposits, and acquires assets (government bonds, private assets, and enough physical currency to provide for withdrawals). The bank can also work to acquire assets. This is essentially "sweat equity," i.e. internally generated capital. In the next CM, the bank pays off on its outstanding deposit liabilities, receives the payoffs on its assets, and consumes whatever is left.

Banks, just like the other individuals in this economy, are subject to limited commitment, and (6) is a collateral constraint, which states that the bank weakly prefers to pay off its deposit liabilities in the subsequent CM rather than absconding. The bank's assets – government bonds and private assets – are posted by the bank as collateral, but the bank can abscond with fraction γ of this collateral, should it default on its deposit liabilities. We will also assume that the bank can abscond with fraction γ^c of the value of any physical currency it holds in the subsequent CM, should it be holding any physical currency when it defaults. The bank's problem, (4) subject to (5) and (6), is set up assuming that all currency is withdrawn from the bank at the end of the current CM, but we will later consider the bank's incentive to hold currency as an asset when we determine the effective lower bond (ELB) on the nominal interest rate.

Assume that $\gamma^c > \gamma$, so that it is easier to abscond with currency than with other assets in the subsequent CM. This role for currency as a potential bank asset will only come into play when the nominal interest rate on government debt is sufficiently negative. Note that we have assumed that the bank cannot abscond with the currency it acquires to satisfy deposit withdrawals at the end of the CM when depositors learn their preference shocks. That is, we assume that there is no opportunity for the bank to abscond with cash in the CM when the bank acquires it initially.

Let x^{Lc} and x^{Hd}, denote, respectively, the quantities of consumption in the DM for small-transaction buyers (when their currency is not stolen) and for large-transaction buyers. So,

$$x^{Lc} = \frac{\beta c}{\pi} - k^c, \tag{7}$$

and

$$x^{Hd} = \beta d - k^d. \tag{8}$$

In solving the bank's problem, (4) subject to (5) and (6), first note that (5) holds with equality, i.e. each bank will earn zero profits, in present value terms. Then, given the optimal choice of c, we get

$$-1 + \frac{\beta}{\pi}\left[1 - \sigma(w^L)\right]\theta^L u'\left(x^{Lc}\right) = 0. \tag{9}$$

As well, from the bank's optimization problem, the following asset pricing relationships hold in equilibrium:

$$R^b = \frac{\pi}{\beta\left[\gamma + (1-\gamma)\theta^H u'\left(x^{Hd}\right)\right]}, \tag{10}$$

$$\phi = \frac{\beta y\left[\gamma + (1-\gamma)\theta^H u'\left(x^{Hd}\right)\right]}{1 - \beta\left[\gamma + (1-\gamma)\theta^H u'\left(x^{Hd}\right)\right]}, \tag{11}$$

where

$$\lambda = \beta\left[u'\left(x^{Hd}\right) - 1\right]$$

and λ is the multiplier associated with the collateral constraint (6). So, $u'\left(x^{Hd}\right) \geq 1$, i.e. $x^d \leq x^{H*}$, where x^{H*} is the surplus-maximizing quantity of consumption in a large-transaction meeting in the DM. Thus, when the collateral constraint binds, i.e. $\lambda > 0$, the assets used as collateral reflect liquidity premia. That is, in equation (10) the real interest rate on government debt is low, and in equation (11) the price of private assets is high when the bank's collateral constraint binds.

If the bank's collateral constraint (6) binds, then since (5) holds with equality, the quantity of bank capital – labor supply by the bank in the current CM – is

$$b + \phi a + \rho c - z = \frac{\gamma\beta\,(1-\rho)\,d}{(1-\gamma)},$$

so the binding collateral constraint is reflected in positive bank capital. That is, the bank demonstrates that it will not abscond on its liabilities by financing part of its portfolio through internally-generated capital. An alternative interpretation of the parameter γ is that this captures a regulatory capital constraint. We have not modeled explicitly the rationale for a bank capital requirement, but this alternative interpretation can be useful in understanding the results.

3.2 Equilibrium

In equilibrium, sellers choose effort in theft, w^L, optimally. This implies that

$$-1 + \sigma'(w^L)\frac{\beta c}{\pi} = 0,$$

or, given (7),

$$-1 + \sigma'(w^L)\left(x^{Lc} + k^c\right) = 0. \tag{12}$$

3.2.1 Banks' Collateral Constraints Do Not Bind

First, consider the case in which banks' collateral constraints do not bind, so $u'(x^{Hd}) = 1$, and $x^{Hd} = x^{H*}$. Then, from (10) and (11),

$$R^b = \frac{\pi}{\beta}, \tag{13}$$

$$\phi = \frac{\beta y}{1-\beta}. \tag{14}$$

Policy is given by (v, R^b), where v is determined by the fiscal authority and R^b is determined by the central bank (and supported by open market operations). In this equilibrium, changes in v are irrelevant, at the margin, as banks' collateral constraints are not binding. However, an increase in R^b implies an increase in the inflation rate, from (13). The real interest rate on government debt is $\frac{1}{\beta} - 1$, so an increase in the nominal interest rate increases the inflation rate one-for-one, which is a standard Fisher effect. Then, from (9) and (12), x^{Lc} and w^L both decrease. That is, the increase in the inflation rate implies that less currency is held in equilibrium, in real terms, there is less consumption in exchange using currency in the DM, and there is less theft as there is less currency to steal.

In this equilibrium, the collateral constraint (6) does not bind, so since $a = 1$ in equilibrium, and from (9), (12), (3), (13), and (14),

$$v + \frac{\beta y}{1-\beta} \geq \frac{\rho[1-\sigma(w^L)]\theta^L u'\left(\frac{1}{\sigma'(w)} - k^c\right)}{\sigma'(w^L)} + \frac{(1-\rho)\left[x^*\left(\theta^H\right) + k^d\right]}{1-\gamma} \tag{15}$$

That is, for this equilibrium to exist, the total value of assets – publicly supplied assets v and the value of private assets, respectively, on the left-hand side of (15) – must equal or exceed the demand for those assets. The first term on the right-hand side of (15) is the demand for currency, written as a function of effort in stealing currency, and the second term is the demand for assets induced by the use of bank deposits in transactions. The possibility of currency theft, along with other elements in this model, changes the conventional analysis of the lower bound on the nominal interest rate, in general. That is, currency theft and limited commitment will inhibit arbitrage, and could cause the effective lower bound on the nominal interest rate to be lower than zero, as we will show in what follows. However, in this equilibrium, so long as (15) holds when $R^b = 1$, the effective lower bound on the nominal interest rate is zero. That is, if $R^b < 1$ were to hold in equilibrium, then banks will prefer to hold currency rather than government bonds as an asset backing bank deposits. Therefore, since there is a positive supply of government bonds, the bond market does not clear, so $R^b < 1$ is not an equilibrium.

Restricting attention to stationary equilibrium allocations, what is the optimal setting for R^b given fiscal policy, v? Suppose that we simply add utility across agents – banks, buyers, and sellers. In equilibrium, each bank receives zero utility, as the nonnegative present value profit constraint for the bank, (5), holds with equality in equilibrium. As well, any output consumed in the CM must be produced by a buyer, and this will net out in utility terms. Further, if currency is stolen by the seller in a DM meeting with a buyer, this produces zero surplus in the DM (nothing is produced or consumed), and when the money is sold for goods in the next CM, production by buyers nets out with consumption by sellers. The remaining source of trades generating positive net welfare are meetings between low-transaction buyers and sellers in the DM in which currency is not stolen, and DM meetings in which bank deposit claims are traded between buyers and sellers. So, the measure of welfare in this equilibrium is

$$W = \rho\left[1-\sigma(w^L)\right]\left[\theta^L u(x^{Lc}) - x^{Lc} - k^c\right] + (1-\rho)\left[\theta^H u(x^{Hd}) - x^{Hd} - k^d\right]. \tag{16}$$

In this equilibrium, $x^{Hd} = x^{H*}$, so exchange using bank deposits is efficient and unaffected by the nominal interest rate. Therefore, at the margin, the change in welfare when the nominal interest rate increases is

$$\frac{dW}{dR^b} = \rho\left[1-\sigma(w^L)\right]\left[\theta^L u'(x^c) - 1\right]\frac{dx^{Lc}}{dR^b} - \sigma'(w^L)\left[\theta^L u(x^c) - x^c - k^c\right]\frac{dw^L}{dR^b} \tag{17}$$

On the right-hand side of (17), we can sign each of the two terms separately. The first term is the welfare effect due to the change in the quantity traded in each DM meeting where physical currency is exchanged for goods. We have already shown that $\frac{dx^{Lc}}{dR^b} < 0$, which is a conventional effect. A higher nominal interest rate implies a higher inflation rate (the effect is one for one), and this reduces real physical currency balances and the quantity of trade. So, as in conventional models in which the Friedman rule is optimal, the first term on the left-hand side of (17) is negative. But, we have determined that $\frac{dw^L}{dR^b} < 0$, so the second term is positive. That is, with higher nominal interest rates and higher inflation, less physical currency is held (in real terms), there is less to steal, theft goes down, and there are more positive-surplus-generating trades in the DM.

On net, then, it is not clear whether welfare goes up or down when the nominal interest rate increases, even at the zero lower bound. If we calculate the derivatives in equation (17), then we can write

$$\frac{dW}{dR^b} = \frac{\rho\left[1-\sigma(w)\right]\left[\theta^L u'(x^c)-1\right]\sigma''(w)(x^c+k^c)+\rho[\sigma'(w)]^2\left[\theta^L u(x^c)-x^c-k^{\ c}\right]}{\sigma''(w)(x^c+k^c)(1-\sigma(w))\theta^L u''(x^c)+\left[\sigma'(w)\right]^2\theta^L u'(w)}$$

Obtaining a characterization of optimal monetary policy appears difficult, even with simple examples. That is, signing the above derivative is not very productive, as there are no clear regularity conditions to give us insight into the problem. But, we can find examples in which theft is only a minor problem and $R^b = 1$ is optimal, or where theft is important and $R^b > 1$. That is, if theft is a serious problem, then the central bank may be willing to tolerate a high nominal interest rate and high inflation, because this has a deterrence effect on theft.

Finally, for this equilibrium, we need to check that it is not optimal for a bank to either offer small-transaction depositors the option to trade bank deposits rather than currency, or to offer large-transaction depositors the option to withdraw currency at the end of the CM rather than trade deposits in the DM. First, were a small-transaction depositor offered the option to trade bank deposits in the DM, he or she would be able to trade d^L bank deposits, and when meeting a seller in the DM would make a take-it-or-leave-it offer and consume

$$x^{Ld} = \beta d^L - k^d.$$

As the bank's collateral constraint does not bind, the best deposit contract the bank could offer a small transaction depositor satisfies

$$\theta^L u'(x^{Ld}) = 1,$$

that is $x^{Ld} = x^{L*}$, where $x^{L*}$is the surplus-maximizing quantity of consumption in a DM exchange involving a small-transaction buyer. The difference in the value of the bank's objective function if it offers small-transaction buyers the ability to trade deposits in the DM rather than withdrawing physical currency to spend, using (7) and (9), is

$$\psi^L = \rho\left\{\theta^L u\left(x^{L*}\right) - x^{L*} - k^d - \theta^L\left[1-\sigma(w^L)\right]\left[u(x^{Lc}) - u'(x^{Lc})(x^{Lc}+k^c)\right]\right\} \tag{18}$$

For existence of the equilibrium we have constructed, we require $\psi^L \leq 0$.

Second, for this equilibrium to exist, banks will not have the incentive to offer withdrawal of physical currency as an option to large-transaction depositors. Let c^H denote the quantity of currency that a large transaction depositor can withdraw from the bank, off equilibrium. Then, sellers who meet these buyers in the DM will choose theft effort w^H, with w^H satisfying

$$-1 + \sigma'(w^H)(x^{Hc} + k^c) = 0, \tag{19}$$

which is similar to (12), with x^{Hc} denoting the quantity of goods the large-transaction buyer would consume in the DM. As well, similar to (9), the following is implied by optimal choice of the out-of-equilibrium currency-withdrawal contract by the bank:

$$-1 + \frac{\beta}{\pi}\left[1 - \sigma(w^H)\right]\theta^H u'\left(x^{Hc}\right) = 0. \tag{20}$$

Then, the difference in the value of the bank's objective function from offering large-transaction buyers the ability to trade bank deposits rather than withdrawing currency is

$$\psi^H = (1-\rho)\left\{\theta^H u\left(x^{H*}\right) - x^{H*} - k^d - \theta^H\left[1 - \sigma(w^H)\right]\left[u(x^{Hc}) - u'(x^{Hc})(x^{Hc} + k^c)\right]\right\} \tag{21}$$

And, to support the equilibrium we have constructed, $\psi^H \geq 0$. Using (19), (20), and (21), we can show that $x^{Hc} > x^{Lc}$, and $w^H > w^L$. So, if large-transaction buyers used physical currency in transactions in the DM, they would consume more than small-transaction buyers in the DM, in states in which their currency is not stolen. However, large-transaction buyers would face a higher probability of theft were they to carry currency than do small-transaction buyers, as they are more lucrative targets for thieves.

In (18), since $\sigma(w^L) > 0$ and exchange would be efficient for small-transaction buyers were they to use bank deposits in transactions, therefore $k^c < k^d$ is a necessary condition for $\psi^L \leq 0$. That is, to support an equilibrium in which small transaction buyers use physical currency in transactions, the cost of accepting currency must be smaller than the cost of accepting deposits for a seller. Otherwise, physical currency would be too costly to use, due to the theft problem. But, we require that $\psi^H \geq 0$ in equilibrium, so bank deposits cannot be too costly to use, relative to currency. What helps to make $\psi^H \geq 0$ consistent with $\psi^L \leq 0$, is that $w^H > w^L$, so if large-transaction buyers were to use physical currency they would face a higher probability of theft than small-transaction buyers. Thus, large-transaction buyers are willing to compensate sellers for the high cost of accepting bank deposits, because large-transaction buyers value safety more than do small-transaction buyers.

Therefore, what is needed to support an equilibrium in which small-transaction buyers use currency, and large-transaction buyers use bank deposits in transactions, is consistent with what we observe. In practice, currency transactions mainly involve small purchases, and debit card and credit card purchases tend to be large. Observed theft of currency is low in Canada and the United States, but the possibility of theft can be important for the choice of means of payment. Small transactions using currency are low cost, and the higher costs associated with debit card and credit card transactions are mitigated by the benefit of safety if the transactions are large.

3.2.2 Banks' Collateral Constraints Bind

Next, consider an equilibrium in which banks' collateral constraints (6) bind. In this case, from (6), (3), (7), (8), (9), (10), (11), and (12), we obtain

$$v + \frac{\beta y \left[\gamma + (1-\gamma)\theta^H u'(x^{Hd})\right]}{1 - \beta\left[\gamma + (1-\gamma)\theta^H u'(x^{Hd})\right]} = \frac{\rho[1-\sigma(w^L)]\theta^L u'\left(\frac{1}{\sigma'(w^L)} - k^c\right)}{\sigma'(w^L)} + \frac{(1-\rho)\left(x^{Hd} + k^d\right)\left[\gamma + (1-\gamma)\theta^H u'(x^{Hd})\right]}{1-\gamma}. \tag{22}$$

Also, from (10), (9), and (12), we get

$$R^b = \frac{\left[1-\sigma(w^L)\right]\theta^L u'\left(\frac{1}{\sigma'(w^L)} - k^c\right)}{\beta\left[\gamma + (1-\gamma)\theta^H u'(x^{Hd})\right]} \tag{23}$$

Then, equations (22) and (23) solve for x^{Hd} and w^L, and we can work backward to solve for x^{Lc} from (12), and for π from (9). Equation (22) states that the total value of the supply of assets, including public assets, v, and private assets, respectively, on the left-hand side of (22), is equal to the total demand for those assets, i.e. the demand for currency, as a function of w^L, and the demand for assets that back bank deposits, as a function of x^{Hd}, the quantity of consumption in DM exchange involving bank deposits. Equation (23) states that, in equilibrium, the gross nominal interest rate is equal to the gross inflation rate, $\left[1-\sigma(w^L)\right]\theta^L u'\left(\frac{1}{\sigma'(w^L)} - k^c\right)$, multiplied by the gross real interest rate, $\frac{1}{\beta[\gamma+(1-\gamma)\theta^H u'(x^{Hd})]}$.

The left-hand side of (22) is strictly decreasing in x^{Hd}, and sufficient conditions for the right-hand side to be strictly increasing in x^{Hd} are that

$$-\frac{xu''(x)}{u'(x)} < 1 \tag{24}$$

(i.e., roughly, asset demands are assumed to be strictly increasing in the asset's rate of return) and γ sufficiently large. In equation (23), the right-hand side is strictly decreasing in w^L and strictly increasing in x^{Hd}. There are then two cases. In case 1, shown in Figure 1, the right-hand side of equation (22) is strictly increasing in w^L. That is,

$$\frac{\left\{\begin{array}{c} -[\sigma'(w)]^2 u'\left(\frac{1}{\sigma'(w)}-k^c\right) - \frac{[1-\sigma(w)]u''\left(\frac{1}{\sigma'(w)}-k^c\right)\sigma''(w)}{[\sigma'(w)]^2} \\ -\left[1-\sigma(w)\right] u'\left(\frac{1}{\sigma'(w)}-k^c\right)\sigma''(w) \end{array}\right\}}{[\sigma'(w)]^2} > 0 \tag{25}$$

If (25) holds, then we can depict the solution to (22) and (23) as the intersection of the two curves A and B, respectively, in Figure 1. A unique equilibrium exists, and an increase in R^b will have the same qualitative effects as in the case in which the banks' collateral constraints do not bind. That is, w^L falls, x^{Hd} rises, x^{Lc} falls, inflation rises, and ϕ falls. So, to support an increase in the nominal interest rate, the central bank conducts an open market sale, which reduces the quantity of currency in real terms, and increases the real quantity of bonds. An additional effect is that the increase in the real quantity of bonds relaxes banks' collateral constraints. The real interest rate then rises, and the price of private assets falls, reflecting a fall in liquidity premia.

In the second case, the sign goes the other way in (25), so in Figure 1 the A curve will have a positive slope. We then cannot rule out multiple equilibria, and counterintuitive effects of an increase in the nominal interest rate can arise. For our purposes, it is not clear that this second case is interesting.

As in the equilibrium in which banks' collateral constraints do not bind, for the equilibrium to exist requires that a bank not have the incentive to either offer large-transaction depositors the option to withdraw physical currency or the incentive to offer small-transaction depositors the option to trade bank deposits. So first, similar to (18),

$$\psi^L = \rho\theta^L \left\{\begin{array}{c} u\left(x^{Ld}\right) - u'(x^{Ld})(x^{Ld}+k^d) \\ -\left[1-\sigma(w^L)\right]\left[u(x^{Lc}) - u'(x^{Lc})(x^{Lc}+k^c)\right] \end{array}\right\},$$

and next, similar to (21),

$$\psi^H = (1-\rho)\theta^H \left\{\begin{array}{c} u\left(x^{Hd}\right) - u'(x^{Hd})(x^{Hd}+k^d) \\ -\left[1-\sigma(w^H)\right]\left[u(x^{Hc}) - u'(x^{Hc})(x^{Hc}+k^c)\right] \end{array}\right\}.$$

Then, in equilibrium, $\psi^L \leq 0$, and $\psi^H \geq 0$. In this case, it is possible to support an equilibrium of this type even if $k^d < k^c$, so that currency is relatively high-cost from a retailer's point of view. That is, if collateral is very scarce, then banks' collateral constraints are very tight, to the point where we could even have $x^{Ld} < x^{Lc}$. But theft is low-probability for small-transaction buyers, and potentially high-probability for large-transaction buyers. This then implies that small-transaction buyers prefer currency, even though it is high-cost for sellers, because deposit banking is very inefficient. But this inefficiency in deposit banking is not enough to outweigh the incentive that large-transaction buyers have to use deposits in exchange, due to safety concerns.

3.2.3 Effective Lower Bound

In most conventional monetary models, arbitrage dictates that the lower bound on the nominal interest rate is zero, typically because a negative nominal interest rate implies that economic agents can borrow at a negative rate, and make infinite profits by holding currency. In this model, arbitrage is potentially more difficult, because physical currency can be stolen, and due to limited commitment. However, in the case in which banks' collateral constraints do not bind, the lower bound on the nominal interest rate is nevertheless zero, as we showed.

But things change when banks' collateral constraints bind. Arbitrage can be accomplished by a bank, which can choose to hold physical currency rather than government debt as an asset backing bank deposit liabilities. But, recall that the bank is able to abscond with a larger fraction of physical currency than of government bonds, so government debt is a superior form of collateral for the bank to hold, from the point of view of the bank's depositors. The ELB threshold for R^b is achieved when banks are indifferent between holding physical currency and government debt as collateral. That is, if we let $\bar{R}^b$ denote the ELB, then

$$\bar{R}^b = \frac{\gamma^c + (1-\gamma^c)\theta^H u'(x^{Hd})}{\gamma + (1-\gamma)\theta^H u'(x^{Hd})}, \tag{26}$$

or, from (23) and (26), the ELB is achieved when

$$\gamma^c + (1-\gamma^c)\theta^H u'(x^{Hd}) = \left[1-\sigma(w^L)\right]\theta^L u'\left(\frac{1}{\sigma'\left(w^L\right)} - k^c\right), \tag{27}$$

and from (23) and (27), in equilibrium (w^L, x^{Hd}) must satisfy

$$\gamma^c + (1-\gamma^c)\theta^H u'(x^{Hd}) \leq \left[1-\sigma(w^L)\right]\theta^L u'\left(\frac{1}{\sigma'\left(w^L\right)} - k^c\right).$$

Note in (26) that the ELB is endogenous, and depends on the tightness of the bank's collateral constraint. In particular, the tighter is the bank's collateral constraint, the larger is $\theta^H u'(x^d)$, and the lower is $\bar{R}^b$. Further, when the collateral constraint does not bind, and $\theta^H u'(x^d) = 1$, then $\bar{R}^b = 1$, and the ELB is zero, as was previously shown. Effectively, the ELB can be less than zero because of a theft problem, but not the theft problem that the buyer faces when using currency as a means of payment. The key theft problem related to the ELB is that the bank can more easily abscond with currency than with other assets that might serve as collateral.

Another cost that we could model that would limit arbitrage is the storage cost associated with physical currency – roughly the cost of preventing theft of physical currency held in the bank vault. In the case of a binding collateral constraint, this would also contribute to an effective lower bound below zero, but would be operational when banks' collateral constraints do not bind, as well as when they do.

3.2.4 Welfare Effects of Monetary Policy When Collateral Constraints Bind

Ideally, we would like to characterize optimal monetary policy in a world with theft. In the case in which collateral constraints bind, what is the optimal nominal interest rate on government debt? Whether collateral constraints bind or not, our welfare measure is the same, given by (16). But, when collateral constraints bind, exchange is inefficient in DM exchanges involving bank deposits. Therefore, instead of (17) we get

$$\frac{dW}{dR^b} = \rho\left[1-\sigma(w^L)\right]\left[\theta^L u'(x^{Lc})-1\right]\frac{dx^{Lc}}{dR^b}$$
$$-\sigma'(w^L)\left[\theta^L u(x^{Lc}) - x^{Lc} - k^c\right]\frac{dw^L}{dR^b} + (1-\rho)\left[\theta^H u'(x^{Lc})-1\right]\frac{dx^{Hd}}{dR^b}$$

In general, the sign of this derivative is ambiguous. Even in the case where (25) holds, we run into the same issue as in the case in which collateral constraints do not bind, and encounter a second issue. That is, if (25) holds, then $\frac{dx^{Hd}}{dR^b} > 0$, $\frac{dx^{Lc}}{dR^b} < 0$, and $\frac{dw^L}{dR^b} < 0$. The first effect increases welfare, the second reduces it, and the third increases it, when the nominal interest rate goes up. The first and third effects are due to the fact that an increase in the nominal interest rate is supported by an open market sale that increases the real quantity of government bonds outstanding and reduces the real quantity of physical currency. This relaxes banks' collateral constraints, increasing welfare for buyers trading bank deposits, and reducing exchange using physical currency, which reduces welfare. In other related models that exhibit low real interest rates with binding collateral constraints, for example Andolfatto and Williamson (2015) and Williamson (2019c), it can be optimal to depart from the Friedman rule ($R^b = 1$) in the face of binding collateral constraints, as the welfare benefits of relaxing collateral constraints exceed the additional welfare costs of added inefficiency in transactions involving physical currency. The second effect, which is that theft falls as inflation rises, gives an additional welfare benefit from a higher nominal interest rate. But, because theft causes a greater inefficiency in DM exchange using physical currency than in exchange involving bank deposits when $R^b = 1$, or $R^b = \bar{R}^b$, this will tend to increase the marginal inefficiency loss from a higher nominal interest rate. So, it is hard to draw clear conclusions about the effects of theft on optimality in this context.

4 Central Bank Digital Currency

In modeling CBDC, we need to make some assumptions concerning the properties of CBDC relative to physical currency, and make choices about the amount of detail we want to model in a monetary system with CBDC. We will assume that CBDC, like physical currency, is a central bank liability. But, like a private bank deposit, CBDC is assumed to be an electronic account balance, and this account balance is transferable, at a cost, to a seller in the DM, and at no cost in the CM, among sellers and buyers. We will assume that CBDC balances cannot be stolen so, in this respect CBDC is assumed to be identical to bank deposits. In reality, cybersecurity is an issue, but a complicated one. In terms of how theft affects the users of means of payment, it seems reasonable to account for the theft of physical currency, and not for the potential theft of CBDC.

In addition to safety, CBDC will have the advantage, relative to physical currency, of bearing interest. We will assume that interest is paid to the holder of CBDC at the beginning of the CM. A third advantage of CBDC over physical currency is that central bankers are assumed not subject to limited commitment. The rules governing the central bank require it to back CBDC with government debt but, in contrast to private bankers, central bankers are assumed incapable of absconding with assets on the central bank's balance sheet.

There are some important details left out of the model that will have to be solved in the implementation of CBDC in practice. For example, CBDC will require a mechanism for clearing and settlement, just as there is a system for clearing and settlement of payments using central bank reserve accounts. But in practice the number of CBDC accounts will be very large relative to the number of reserve accounts at the central bank, and the number of daily transactions using CBDC will be similarly large, so clearing and settlement of CBDC payments will, in principle, require a different payments mechanism from what exists now. For example, to provide the full benefits of CBDC issue, interest should be paid on CBDC balances, and paid continuously, requiring 24/7 fast clearing and settlement. Reserve account payments are made only on business days and in daylight hours.

In this section, we will examine two cases. In the first, CBDC does not compete with bank deposits. That is, private banks offer deposit contracts under which CBDC is withdrawn when the depositor wants to make a small transaction in the DM, and large-transaction buyers in the DM trade bank deposits. In the second case, either large transaction buyers in the DM trade both CBDC and claims on private banks, or small-transaction buyers use bank deposits and CBDC in transactions in the DM. Thus, in the second case CBDC competes directly, though possibly in different ways, with private bank deposits. We will assume at the outset that physical currency is withdrawn from circulation and replaced by CBDC. But, we will ultimately determine conditions that imply that CBDC will dominate physical currency, so that physical currency is not held in equilibrium, even when available.

4.1 Case 1: CBDC Does Not Compete with Bank Deposits

Assume for convenience that physical currency is withdrawn from circulation by the central bank, and replaced with CBDC, which bears interest, and cannot be stolen. Let R^m denote the gross nominal interest rate on CBDC and, as in the previous section, we confine attention to stationary equilibria. We will assume that CBDC is integrated with private banking in a similar way to physical currency, in that private banks offer deposit contracts which permit small-transaction buyers to withdraw CBDC instead of physical currency. Of course, since CBDC is digital, it does not have to be physically "withdrawn." Withdrawal of CBDC means having the right to trade away a specified quantity of claims on the central bank in the DM. Here, a bank chooses the deposit contract (z, m, d), where m is the quantity of CBDC that a small-transaction buyer can withdraw from the bank at the end of the CM. As with physical currency, the bank acquires a portfolio (b, a) of government bonds and private assets in the CM. In equilibrium a bank solves

$$\max_{z,m,d,b,a} \left[-z + \rho\theta^L u\left(\frac{\beta R^m m}{\pi} - k^m\right) + (1-\rho)u\left(\beta d - k^d\right)\right] \tag{28}$$

subject to

$$z - \rho m - b - \phi a + \beta \left[\frac{R^b b}{\pi} + (\phi + y)\, a - (1-\rho)\, d\right] \geq 0, \tag{29}$$

and (6). From the bank's problem, the optimal choice for m implies

$$-1 + \frac{\beta R^m}{\pi}\theta^L u'(x^{Lm}) = 0,$$

where consumption in the DM for a small-transaction buyer, denoted x^{Lm}, is given by

$$x^{Lm} = \frac{\beta R^m m}{\pi} - k^m \tag{30}$$

As in the previous section, (6), (8), (10), and (11) hold.

First, consider the case in which the bank's collateral constraint does not bind. Then, $x^{Hd} = x^{H*}$, and from (10) and (**??**), we get

$$\theta^L u'(x^{Lm}) = \frac{R^b}{R^m}, \tag{31}$$

which determines x^{Lm}.

For existence of this equilibrium, it is necessary that the bank's collateral constraint (6) hold. So, from (6), (13), (14), (3), (31), and market-clearing in asset markets, we obtain

$$v + \frac{\beta y}{1-\beta} \geq \rho(x^{Lm} + k^m)\theta^L u'(x^{Lm}) + \frac{(1-\rho)(x^{H*} + k^d)}{1-\gamma}. \tag{32}$$

Existence of this equilibrium also requires that the bank not have an incentive to offer a large-transaction buyer the opportunity to withdraw CBDC at the end of the CM, and that the bank not have an incentive to offer a small-transaction buyer the option to trade bank deposits in the DM. Similar to our approach in the previous section, let ψ^H denote the difference in the value of the bank's objective function between offering a large-transaction buyer the option to trade bank deposits in the DM, vs. the option to withdraw CBDC in the CM. Then, similar to (21), we have

$$\psi^H = (1-\rho)\left\{\theta^H u(x^{H*}) - x^{H*} - k^d - \theta^H\left[u(x^{Hm}) - u'(x^{Hm})(x^{Hm}+k^m)\right]\right\}, \tag{33}$$

where x^{Hm} is the optimal choice of DM consumption for the large-transaction buyer, were he or she to use CBDC for transactions in the DM. Similar to (31), x^{Hm} is determined by

$$\theta^H u'(x^{Hm}) = \frac{R^b}{R^m}. \tag{34}$$

Then, the difference in the value of the bank's objective function between resulting from offering small-transaction buyers the option of trading bank deposits, similar to (18), is

$$\psi^L = \rho\left\{\theta^L u(x^{L*}) - x^{L*} - k^d - \theta^L\left[u(x^{Lm}) - u'(x^{Lm})(x^{Lm}+k^m)\right]\right\}. \tag{35}$$

For existence of this equilibrium, it is necessary that $\psi^H \geq 0$ and $\psi^L \leq 0$. If $R^b = R^m$, so that CBDC pays interest at the same rate as government bonds, and $k^m = k^d > 0$, then $\psi^H = \psi^L = 0$. Then, if we lower the gross nominal interest rate on CBDC, R^m, to make CBDC less attractive, this increases both $\psi^H(\theta^H)$ and $\psi^L(\theta^L)$, but then $\psi^H(\theta^H) > \psi^L(\theta^L) > 0$. Then, if we increase k^d or reduce k^m, $\psi^H(\theta^H)$ and $\psi^L(\theta^L)$ fall by the same quantity. Therefore, we can find policy and parameter values such that $\psi^H(\theta^H) > 0$ and $\psi^L(\theta^L) < 0$. Given the construction above, this involves $R^b > R^m$ and $k^m < k^d$. That is, CBDC must be less costly to use in transactions than bank deposits, and CBDC must pay a lower interest rate than government debt. This makes CBDC more attractive than bank deposits for small transactions, and bank deposits more attractive for large transactions.

In this equilibrium, note that there are two underlying central bank instruments, open market operations and the gross nominal interest rate on $CBDC$, denoted R^m. The central bank sets R^m, and then open market operations determine R^b. From equation (31), $\frac{R^b}{R^m}$ determines x^m, and a change in nominal interest rates that leaves $\frac{R^b}{R^m}$ unchanged is neutral. There are no real effects, but from (**??**) and (11), inflation increases if nominal interest rates increase – there is a one-for-one Fisher effect. In this sense, inflation does not matter. If R^b increases holding constant R^m, then from (11) inflation increases, and from (31) x^m falls. This is essentially a standard effect of inflation. With the nominal return on CBDC held constant, more inflation causes low-transaction buyers to economize on CBDC balances.

Next, suppose that banks' collateral constraints bind. Then, using (3), (28), (29), (6), (10), (11), (**??**), and (30), x^{Hd} and x^{Lm} are determined by

$$v + \frac{\beta y[\gamma + (1-\gamma)\theta^H u'(x^{Hd})]}{1-\beta\left[\gamma + (1-\gamma)\theta^H u'(x^{Hd})\right]} =$$
$$\frac{(1-\rho)\left(x^{Hd}+k^d\right)\left[\gamma + (1-\gamma)\theta^H u'(x^{Hd})\right]}{1-\gamma} + \rho\theta^L u'(x^{Lm})(x^{Lm}+k^m) \tag{36}$$

and

$$R^b = \frac{R^m \theta^L u'(x^{Lm})}{[\gamma + (1-\gamma)\theta^H u'(x^{Hd})]}. \tag{37}$$

Assuming that (24) holds and that k^d and k^m are sufficiently small, there exists a unique equilibrium, with (36) defining a downward-sloping locus, A, in Figure 2, and (37) defining the upward-sloping locus B in the figure. As in the case with nonbinding bank collateral constraints, changes in monetary policy that leave $\frac{R^b}{R^m}$ unchanged are neutral, but can change the inflation rate. If nominal interest rates go up and leave $\frac{R^b}{R^m}$ unchanged, then there is a one-for-one Fisher effect on the inflation rate. However, if R^b increases with R^m held constant, then from (36) and (37), this increases x^{Hd} and reduces x^{Lm}, so from (**??**) inflation increases. If R^m increases with R^b held constant, then from (36) and (37) x^{Lm} rises and x^{Hd} falls, so from (10) the inflation rate must rise. From (10) and (11), if x^{Hd} rises (falls), then the real interest rate on government debt rises (falls), because this reflects a relaxation (tightening) of bank collateral constraints. Thus, if R^b increases with R^m held constant, then the real interest rate rises because of the open market sale of government debt that is required to support the increase in the nominal interest rate on government debt. If R^m increases with R^b held constant, then the real interest rate falls, because of the open market purchase of government debt that is required, as the increase in R^m generates an increase in the demand for CBDC that needs to be accommodated.

There are no bounds on nominal interest rates, other than $R^m > 0$ and $R^b > 0$, in the following sense. If an equilibrium exists given a policy (v, R^b, R^m), in which large-transaction buyers trade bank deposits and small-transaction buyers trade CBDC in the DM, then, whether banks' collateral constraints bind or not, increasing or decreasing nominal interest rates, holding constant v and $\frac{R^b}{R^m}$, leaves all real variables unchanged in equilibrium. The inflation rate just increases one-for-one with nominal interest rates. So, this economy with CBDC might seem to work something like a Woodford (2003) cashless economy, in which there are no distortions from changes in anticipated inflation. But, that is not correct, as the two nominal interest rates controlled by the central bank, if they move independently, have real effects – whether banks' collateral constraints bind or not.

What is the effective lower bound in this economy with CBDC? Assume that a bank can abscond with CBDC in the same fashion as with government debt, by walking off with fraction γ of any CBDC held in its asset portfolio. Then, the lower bound on the nominal interest rate on government debt is the nominal interest rate on CBDC, that is $R^b \geq R^m$, otherwise CBDC dominates government debt in rate of return and no one would hold government debt in equilibrium.

For this equilibrium to exist, we require bounds on the ratio $\frac{R^b}{R^m}$. First, this ratio cannot be so high that all buyers want to conduct transactions using bank deposits. Second, $\frac{R^b}{R^m}$ cannot be so low that all buyers want to conduct transactions using only CBDC. To be more specific, as in our previous analysis, define ψ^j as the difference between the maximized value of a bank's objective function if a depositor with $\theta = \theta^j$, $j = L, H$, uses deposits in DM transactions or uses CBDC. In general, for this equilibrium to exist, $\psi^H \geq 0$ and $\psi^L \leq 0$.

First, let x^{Ld} and x^{Hm} denote, respectively, the quantities of the consumption for small transaction buyers were they to use deposits in transactions, and of large transaction buyers were they to use CBDC in transactions. Then, using the structure of the bank's problem (28) subject to (29), (6), and (34), in equilibrium x^{Ld} and x^{Hm} must satisfy

$$\theta^L u'(x^{Ld}) = \theta^H u'(x^{Hd}), \tag{38}$$

and

$$\theta^H u'(x^{Hm}) = \theta^L u'(x^{Lm}), \tag{39}$$

respectively.

Then, from (28) subject to (29), (6), as well as (38) and (39), we can write the two conditions we require as

$$\psi^H = (1-\rho)\theta^H \begin{bmatrix} u(x^{Hd}) - (x^{Hd} + k^d)u'(x^{Hd}) \\ -u(x^{Hm}) + (x^{Hm} + k^m)u'(x^{Hm}) \end{bmatrix} \geq 0, \tag{40}$$

$$\psi^L = \rho\theta^L \begin{bmatrix} u(x^{Ld}) - (x^{Ld} + k^d)u'(x^{Ld}) \\ -u(x^{Lm}) + (x^{Lm} + k^m)u'(x^{Lm}) \end{bmatrix} \leq 0, \tag{41}$$

respectively.

Given $(v, \frac{R^b}{R^m})$, an equilibrium of this type consists of $(x^{Hd}, x^{Hm}, x^{Ld}, x^{Lm})$ satisfying (36)-(41).

Proposition 1 *In an equilibrium with CBDC and binding collateral constraints, in which large-transaction depositors trade bank deposits in the DM, and small-transaction buyers trade CBDC: (i) a necessary condition for existence of equilibrium is $k^d > k^m$; (ii) in equilibrium $x^{Hd} > x^{Hm}$ and $x^{Ld} > x^{Lm}$.*

Proof. *(i)First, suppose that $k^d = k^m$. Then, (38), (39), (40) and (41) can be satisfied only if $x^{Hd} = x^{Hm}$ and $x^{Ld} = x^{Lm}$. But then, (39) and (37) imply that $R^b < R^m$, which cannot hold in equilibrium. Next, suppose that $k^d < k^m$. Then, if (41) holds, then $x^{Lm} > x^{Ld}$, which implies, from (38) and (39), that $x^{Hm} > x^{Hd}$. But then, from (39) and (37), $R^b < R^m$,which cannot hold in equilibrium. Therefore, $k^d > k^m$ in equilibrium. (ii) If $k^d > k^m$, then (40) implies that $x^{Hd} > x^{Hm}$. Then, from (38) and (39), $x^{Ld} > x^{Lm}$.* ■

Therefore, to support an equilibrium in which CBDC is used by small-transaction buyers and not by large-transation buyers (who continue to use bank deposits in exchange), the CBDC technology must involve lower transaction costs for sellers than is the case for bank deposits. As well, the interest rate on CBDC needs to be sufficiently low relative to the interest rate on government debt. This distorts decisions sufficiently that large-transaction buyers will prefer bank deposits to CBDC. But the interest rate on CBDC cannot be too low, else small-transaction buyers will prefer bank deposits to CBDC.

4.2 CBDC Competes with Bank Deposits

There are two ways that CBDC could compete directly with bank deposits. First, CBDC could be sufficiently unattractive that small-transaction buyers substitute bank deposits for CBDC. Second, CBDC could be sufficiently attractive that large-transaction buyers substitute CBDC for bank deposits. We will explore each case in turn.

4.2.1 Small-Transaction Buyers Use Deposits and CBDC in Transactions

In this equilibrium, in which the ratio $\frac{R^b}{R^m}$ is large, large transaction buyers trade bank deposits in the DM, and small transaction buyers either trade bank deposits or use CBDC. First, consider the case in which banks' collateral constraints do not bind. In this case $x^{jd} = x^{j*}$ for $j = L, H$, that is transactions in the DM using bank deposits are efficient for small-transaction buyers and large-transaction buyers. As well (31) and (34) hold and, in (35), $\psi^L = 0$ (banks are indifferent between offering CBDC or bank deposits to small transaction buyers). Given that $k^m < k^d$, there is some critical value for $\frac{R^b}{R^m}$, denoted R^L, such that, if $\frac{R^b}{R^m} < R^L$ then small transaction buyers will conduct transactions in the DM using CBDC, and if $\frac{R^b}{R^m} = R^L$ then some small-transaction depositors will use CBDC and some will use bank deposits in DM transactions. However, if $\frac{R^b}{R^m} > R^L$ then there is no equilibrium in which CBDC is held. Further, $R^L > 1$, since $k^m < k^d$.

Next consider the case in which banks' collateral constraints bind. Since it must be optimal for a bank not to offer payment in CBDC as an option for large-transaction buyers, inequality (40) must hold, just as in the case in the previous subsection. However, for small-transaction depositors to trade using both bank deposits and CBDC, the bank must be indifferent between offering small-transaction buyers either of these two options, which implies that

$$\psi^L = \rho\theta^L \left[u(x^{Ld}) - (x^{Ld} + k^d)u'(x^{Ld}) - u(x^{Lm}) + (x^{Lm} + k^m)u'(x^{Lm})\right] = 0. \tag{42}$$

Then, (38), (39), (37), and (42) determine x^{Lm}, x^{Ld}, x^{Hm}, and x^{Hd}. It remains to determine the fractions of small-transaction buyers who use bank deposits and CBDC. Letting δ^L denote the fraction of small-transaction buyers using CBDC, and $1 - \delta^L$ the fraction using banking deposits in exchange, then analogous to equation (36), given x^{Lm}, x^{Ld}, x^{Hm}, and x^{Hd}, the fraction of small-transaction CBDC users δ^L is determined by

$$\begin{aligned} v + \tfrac{\beta y[\gamma+(1-\gamma)\theta^H u'(x^{Hd})]}{1-\beta[\gamma+(1-\gamma)\theta^H u'(x^{Hd})]} &= \tfrac{(1-\rho)\left(x^{Hd}+k^d\right)[\gamma+(1-\gamma)\theta^H u'(x^{Hd})]}{1-\gamma} \\ +\delta^L\rho\theta^L u'(x^m)(x^m + k^m) &+ \tfrac{(1-\delta^L)\rho\left(x^{Ld}+k^d\right)[\gamma+(1-\gamma)\theta^L u'(x^{Ld})]}{1-\gamma} \end{aligned} \tag{43}$$

To solve for an equilibrium in this case, equations (37) and (38) give

$$R = \frac{\theta^L u'(x^{Lm})}{[\gamma + (1-\gamma)\theta^L u'(x^{Ld})]}, \tag{44}$$

where $R \equiv \frac{R^b}{R^m}$. Then, equations (42) and (44), solve for x^{Lm} and x^{Ld}, and we can then work backward to solve for x^{Hd} and x^{Hm} from (38) and (39), and then for δ^L from (43). For this case, it helps to restrict preferences to constant relative risk aversion, maintaining assumption (24).

Proposition 2 *Assume that* $u(x) = \frac{x^{1-\alpha}}{1-\alpha}$*, with* $0 < \alpha < 1$*. Suppose* γ *is sufficiently small, and* k^d *and* k^m *are sufficiently small. Then, in an equilibrium in which small-transaction depositors trade bank deposits and CBDC in the DM, and large-transaction depositors trade bank deposits in the DM, an increase in* R *results in decreases in* x^{Lm}*,* x^{Ld}*,* x^{Hm}*, and* x^{Hd}*, and a decrease in* δ^L*.*

Proof. *If* $u(x) = \frac{x^{1-\alpha}}{1-\alpha}$*, and* $\gamma = 0$*, from (42), (44), (38) and (39), we get closed-form solutions*

$$x^{Lm} = \left(\frac{1-\alpha}{\alpha}\right)\frac{\left(k^d R^{-1} - k^m\right)}{\left(R^{\frac{1}{\alpha}-1} - 1\right)} \tag{45}$$

$$x^{Ld} = R^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\frac{\left(k^d R^{-1} - k^m\right)}{\left(R^{\frac{1}{\alpha}-1} - 1\right)} \tag{46}$$

$$x^{Hm} = \left(\frac{\theta^H}{\theta^L}\right)^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\frac{\left(k^d R^{-1} - k^m\right)}{\left(R^{\frac{1}{\alpha}-1} - 1\right)} \tag{47}$$

$$x^{Hd} = \left(\frac{R\theta^H}{\theta^L}\right)^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\frac{\left(k^d R^{-1} - k^m\right)}{\left(R^{\frac{1}{\alpha}-1} - 1\right)} \tag{48}$$

Then, straightforward differentiation shows that, if R *increases, then* x^{Lm}*,* x^{Ld}*,* x^{Hm}*, and* x^{Hd} *all decrease. Then, note that, in (43), the left-hand side is strictly decreasing in* x^{Hd}*, and the right-hand side is strictly increasing in* x^{Hd}*,* x^{Lm}*, and* x^{Ld}*, given* $\alpha < 1$ *and* k^d *and* k^m *sufficiently small. As well,* $R > 1$ *implies, from (44), that* $x^{Lm} < x^{Ld}$*. Therefore,* δ^L *must decrease. Then, by continuity, these results hold for* $\gamma > 0$*, as long as* γ *is sufficiently small.* ■

The assumptions in the Proposition give intuitive results. That is, an increase in R, the ratio of the gross nominal interest rate on government debt to the gross nominal interest rate on CBDC, makes deposits more attractive relative to CBDC, and causes substitution on the part of small-transaction buyers, from CBDC to bank deposits. But, the activity of trading bank deposits needs to be supported by more assets, in the aggregate, than is the case for CBDC, because in equilibrium a buyer using bank deposits consumes more in the DM, and a higher volume of trade must be backed by more assets. This then implies that the quantity of consumption in each type of transaction in the DM falls, though the total volume of transactions accounted for by small-transaction buyers rises.

4.2.2 Large-Transaction Buyers Use Deposits and CBDC in Transactions

If R is sufficiently low, this makes CBDC attractive enough for large-transaction buyers that banks will be indifferent in equilibrium between offering a large-transaction buyer the option of using CBDC or bank deposits in DM exchange. First, if banks' collateral constraints do not bind, then $x^{jD} = x^{j*}$, for $j = L, H$, that is exchange is efficient for large-transaction buyers in the DM, and exchange would be efficient were small-transaction buyers to use bank deposits in DM exchange. As well, (31) and (34) hold, and in (33), $\psi^H = 0$. As in an equilibrium in which banks are indifferent concerning what means of payment small-transaction buyers will use in the DM, in this case there is a critical value for the ratio $\frac{R^b}{R^m}$, denoted R^H, such that this equilibrium exists if and only if $\frac{R^b}{R^m} = R^H$. Further, $R^L > R^H > 1$. So, if banks' collateral constraints do not bind, then for high R an equilibrium does not exist in which CBDC is held, for medium-range R small transaction buyers use CBDC and large-transaction buyers use deposits in exchange in the DM, and for low values of R all buyers use CBDC in exchange.

Next, suppose that banks' collateral constraints bind. In such an equilibrium, (40) holds as an equality, that is

$$\psi^H = (1-\rho)\theta^H \left[\begin{array}{c} u(x^{Hd}) - (x^{Hd} + k^d)u'(x^{Hd}) \\ -u(x^{Hm}) + (x^{Hm} + k^m)u'(x^{Hm}) \end{array} \right] = 0, \qquad (49)$$

In equilibrium, (37), (38), (39), and (41) hold, and then (37), (38), (39), and (49) determine x^{Lm}, x^{Ld}, x^{Hm}, and x^{Hd}. Finally, the fraction of large-transaction buyers who trade CBDC, δ^H, is determined, analogous to (43), by

$$\begin{aligned} v + \frac{\beta y[\gamma + (1-\gamma)\theta^H u'(x^{Hd})]}{1 - \beta\left[\gamma + (1-\gamma)\theta^H u'(x^{Hd})\right]} &= \\ \frac{(1-\rho)(1-\delta^H)\left(x^{Hd} + k^d\right)\left[\gamma + (1-\gamma)\theta^H u'(x^{Hd})\right]}{1-\gamma} \\ + \delta^H(1-\rho)\theta^H u'(x^{Hm})(x^{Hm} + k^m) + \rho\theta^L u'(x^{Lm})(x^{Lm} + k^m) \qquad (50) \end{aligned}$$

Then, qualitatively, our results for this equilibrium are identical to those we worked out in the previous case, in which low-transaction buyers use both bank deposits and CBDC in exchange. That is, we can apply Proposition 2 in exactly the same way, and determine that, under the assumptions in Proposition 2, an increase in R will reduce x^{Lm}, x^{Ld}, x^{Hm}, and x^{Hd}, and will reduce δ^H. That is, an increase in the ratio of the gross nominal interest rate on government debt to the gross nominal interest rate on CBDC reduces the quantity traded in each type of DM exchange, but increases the fraction of large-transaction buyers using deposits in exchange.

4.3 Collateral Shortage and CBDC

Under our assumptions from the previous section, when banks' collateral constraints bind, there are four critical values for R, which we will denote, R^{Lu}, R^{Ll}, R^{Hu}, and R^{Hl}, where $1 < R^{Hl} < R^{Hu} < R^{Ll} < R^{Lu}$. If $R\epsilon(R^{Hl}, R^{Hu})$, then large-transaction buyers use both bank deposits and CBDC in exchange in the DM; if $R\epsilon[R^{Hu}, R^{Ll}]$, then large-transaction buyers use bank deposits in exchange in the DM, and small-transaction buyers use CBDC; if $R\epsilon(R^{Ll}, R^{Lu})$, then large-transaction depositors use only bank deposits in transactions in the DM, and small transaction buyers use both bank deposits and CBDC.

But, there may exist a lower bound on R, denoted R^c, such that, when $R = R^c$, the central bank has purchased the entire stock of government debt, and there are no assets left to back further increases in the stock of CBDC outstanding. In such a case $R > R^c$ would not be feasible. For example, in an equilibrium in which large-transaction buyers use bank deposits in DM exchange, and small-transaction buyers use CBDC, an equilibrium consists of x^{Hd} and x^{Lm} solving (36) and (37) given R and v. However, if the central bank is restricted to purchasing government debt to back CBDC issues, then in equilibrium

$$v \geq \rho\left(x^{Lm} + k^m\right)\theta^L u'(x^{Lm}), \tag{51}$$

where the right-hand side of (51) is the demand for CBDC as a function of consumption of small-transaction buyers in the DM. Thus, (51) states that the consolidated government debt must exceed the demand for CBDC in equilibrium. We have shown that x^{Lm} rises as R falls, and given our assumptions, the right-hand side of (51) is strictly increasing in x^{Lm}. So, if v is sufficiently small, there exists a critical value R^c, such that when $R = R^c$, (51) binds, and if $R > R^c$ an equilibrium does not exist as there is insufficient government debt to satisfy the demand for CBDC at market interest rates.

If the fiscal authority keeps v low so that, in the example, (51) binds in circumstances in which the central bank would like to expand the stock of CBDC outstanding, then there are other options. For example, the central bank could purchase private assets outright. This might be viewed as undesirable, for reasons that are not made explicit in this model. For example, there may be private information issues associated with private assets – unobserved quality or a moral hazard problem – that private sector financial institutions are more adept at mitigating than is the central bank. Another possibility is that the central bank backs CBDC, in part, with loans to private sector banks, secured with private assets. We will explore this latter avenue for expanding CBDC in instances in which government debt is in short supply.

Let l denote the quantity of one-period loans, in real terms, to private banks from the central bank. Central bank loans to private banks are one-period loans made at a gross nominal interest rate R^l. In this context, we will modify the fiscal authority's policy rule to take the form

$$v = m + b - l, \tag{52}$$

that is the consolidated government debt is now $m + b - l$ at the beginning of any period, where $b \leq v$, i.e. the central bank's CBDC issues are always backed one-for-one with government debt and loans to private banks.

We need to modify the private bank's problem, in the example with small-transaction buyers using CBDC, and large-transaction buyers using bank deposits in exchange, to accommodate central bank lending. The bank's objective function (28) is the same, but constraints (29) and (6) are replaced by

$$z + l - \rho m - b - \phi a + \beta \left[\frac{R^b b}{\pi} + (\phi + y) a - (1 - \rho) d - \frac{R^l l}{\pi} \right] \geq 0, \tag{53}$$

and

$$\left[\frac{R^b b}{\pi} + (\phi + y) a \right] (1 - \gamma) - (1 - \rho) d - \frac{R^l l}{\pi} \geq 0, \tag{54}$$

respectively. So, in (53) and (54), central bank loans l are a source of funding for the bank, in addition to deposits, and the collateral constraint (54) states that the bank can abscond on central bank loans in the same fashion as for retail deposits.

In an equilibrium in which banks' collateral constraints bind, from the bank's problem, (28) subject to (53) and (54), the gross nominal interest rate on central bank loans satisfies

$$R^l = \frac{\pi}{\beta \theta^H u'(x^{Hd})} \tag{55}$$

Then, using (52), (54) with equality, and (55) to amend equation (36), we get

$$\begin{aligned} v + \frac{\beta y [\gamma + (1-\gamma)\theta^H u'(x^{Hd})]}{1 - \beta [\gamma + (1-\gamma)\theta^H u'(x^{Hd})]} &= \frac{(1-\rho)\left(x^{Hd} + k^d\right)[\gamma + (1-\gamma)\theta^H u'(x^{Hd})]}{1-\gamma} \\ &+ \rho \theta^L u'(x^{Lm})(x^{Lm} + k^m) + \frac{l\gamma}{(1-\gamma)\,\theta^H u'\left(x^{Hd}\right)} \end{aligned} \tag{56}$$

Then, in equilibrium, (56) and (37) solve for x^{LM} and x^{Hd} given policy (v, R, l), and the central bank must satisfy the constraint

$$v + l \geq \rho \theta^L u'(x^{Lm})(x^{Lm} + k^m), \tag{57}$$

that is the quantity of consolidated government debt plus central bank lending must be at least as large as the demand for CBDC at market interest rates. In other words, there must be sufficient central bank assets to back the quantity of CBDC demanded in equilibrium.

From (56) and (37), central bank policy with $l > 0$ for which (57) does not bind is inefficient. That is, if $l > 0$ and (57) does not bind in equilibrium, then the central bank could hold R constant, reduce l, and increase both x^{Hd} and x^{Lm}, which will increase welfare, as this increases surplus in all DM meetings. The welfare increase occurs because it is more efficient for the central bank to back CBDC with government bonds than with loans, if feasible, as loans by the central bank to private banks necessarily increase bank capital. That is, from

(53) and (54), the quantity of bank capital, i.e. the amount of production for the banker which is funding asset purchases is given by

$$cap = \frac{\gamma\beta}{(1-\gamma)}\left[(1-\rho)d + \frac{R^l l}{\pi}\right]$$

But, if (57) binds in equilibrium, then the central bank cannot increase R without increasing l. That is, if all government debt is held by the central bank, the central bank can increase the quantity of CBDC outstanding by increasing R and l, with (57) a binding constraint.

Thus, the central bank can circumvent a shortage of government debt in order to issue more CBDC, by lending to private banks. But it is more efficient if the fiscal authority accommodates the CBDC issue with more government debt.

4.4 Would Buyers Hold Physical Currency If It Were Available?

Thus far, we have assumed that the introduction of CBDC coincides with a withdrawal of physical currency from circulation. But, would economic agents have any incentive to hold physical currency if it were not eliminated? In any equilibrium we have examined, in which CBDC coexists with private bank deposits, we can ask whether a bank would offer a deposit contract that would permit either small-transaction buyers or large-transaction buyers to withdraw physical currency in the CM, if physical currency were available. If the bank strictly prefers not to offer such a deposit contract, then physical currency would not be valued in equilibrium, once CBDC is introduced.

From the bank's problem, (28) subject to (29) and (6), as well as (**??**), (30), (9), and (12), we can write the change in the value of the bank's objective function, were it to offer type j buyers a contract permitting withdrawal of physical currency rather than CBDC, for $j = L$, H, as

$$\chi^j = \theta^j \left\{ \begin{array}{c} \left[u(x^{jc}) - u'(x^{jc})(x^{jc} + k^c)\right] \\ -u(x^{jm}) + u'(x^{jm})(x^{jm} + k^m) \end{array} \left[1 - \sigma(w^j)\right] \right\}, \tag{58}$$

for $j = L, H$. So, note that χ^j is strictly decreasing in w^j, the effort of sellers in stealing physical currency from type j, strictly increasing in x^{jc}, the quantity of goods purchased with physical currency by type j, strictly decreasing in x^{jm}, the quantity of goods purchased with CBDC by type j, strictly decreasing in k^c, the cost for a seller of accepting physical currency, and strictly increasing in k^m, the cost to a seller of accepting CBDC. Then, from (12),$w^J > 0$ and $\sigma(w^j) > 0$. So if $R^m \geq 1$, then $x^{jc} < x^{jm}$. Therefore, if $R^m \geq 1$ and $k^m \leq k^c$, then from (58), $\chi^j < 0$, and a type j agent will not use physical currency in transactions. That is, it is sufficient for CBDC to pay a nonnegative nominal interest rate and for the CBDC technology to be such that sellers bear a lower cost to accepting CBDC than physical currency, to guarantee that physical currency will not be used in equilibrium. These positive features make CBDC dominate physical currency, given that physical currency can be stolen and CBDC cannot.

If we contemplate other negative effects, in addition to theft, of exchange using physical currency, such as tax evasion and trade in street drugs, CBDC permits the mitigation of such activities. That is, the payment of interest on CBDC protects CBDC-holders from inflation, so inflation then becomes a tax on illegal activity conducted with physical currency. So, CBDC gives policymakers some options, if one of the goals of offering CBDC is to mitigate illegal activity. Physical currency could be banned outright but, since CBDC can eliminate the effects of the inflation tax on non-criminal activity, it permits the use of the inflation tax to extract revenue from criminals.

4.5 CBDC and Central Bank Independence

While central bank arrangements can differ across countries, the typical central bank has three key properties. First, the central bank has a monopoly on the issue of physical currency; second, it has some degree of independence from the central government; and third, it does not have the power to tax, but is confined to intermediating assets, consisting primarily of government debt and loans to the financial sector. These properties are linked, in that central bank independence is maintained in part because the central bank remains outside the budgetary process of the central government. As long as the government provides a ready supply of government debt, the central bank can issue enough physical currency to satisfy the demand for it in a low-inflation environment. And the central bank's monopoly on zero-nominal-interest physical currency allows it to generate enough revenue to pay its costs and supply a stream of positive transfers to the fiscal authority.

But, if all central bank liabilities pay interest, there are conditions under which the central bank cannot generate a large enough profit to avoid depending on the fiscal authority for support, and this is a threat to central bank independence. To see what is going on, we need to separate the consolidated government budget constraints into separate constraints for the fiscal authority and the central bank, in a way that is consistent with (1), (2), and (3). In a stationary equilibrium, the fiscal authority issues v government bonds, in real terms, each period, and the central bank purchases b^{cb} bonds each period, in real terms, and issues $\bar{m}$ units of CBDC. So if τ^b is the central bank's transfer to the fiscal authority each period, in real terms, then the central bank's budget constraint in periods $t = 1, 2, 3, ...,$ is

$$m + \frac{R^b b^{cb}}{\pi} = \frac{R^m \bar{m}}{\pi} + b^{cb} + \tau^b. \tag{59}$$

That is, on the right-hand side of the equation, in order, the central bank pays off the interest and principal on CBDC held at the beginning of the period, purchases newly issued government debt, and makes a transfer to the fiscal authority. Outlays on the right-hand side of the equation are financed by the items on the left-hand side which are, respectively, the quantity of CBDC outstanding during the current period, and interest and principal on the government debt held by the central bank from the previous period. We can write the fiscal authority's budget constraint, in a stationary equilibrium, as

$$v + \tau^b = \frac{R^b v}{\pi} + \tau. \tag{60}$$

That is, in (60), the outlays on the right-hand side are the interest and principal on the government debt issued in the previous period, including what is held by the central bank, plus the transfer to the private sector. Outlays are financed by the quantities on the left-hand side of (60), which consist, respectively, of the total new debt issue, plus the transfer received from the central bank.

Assume that the central bank returns all profits to the fiscal authority, period-by-period, and does not accumulate capital. Then, $\bar{m} = b^{cb}$, i.e. liabilities (CBDC) always match assets (government debt) on the central bank's balance sheet. This implies, from (59), that

$$\tau^b = \frac{\bar{m}}{\pi}\left(R^b - R^m\right)$$

Then, from (30) and its analogous counterpart for large-transaction buyers, along with market clearing, we get

$$\tau^b = \frac{[\rho\delta^L(x^{Lm} + k^m) + (1-\rho)\delta^H(x^{Hm} + k^m)](R^b - R^m)}{\beta}. \tag{61}$$

So, in (61), central bank profits τ^b, which are transferred to the fiscal authority each period, depend on the quantity of consumption of buyers using CBDC in transactions, which in turn determines the demand for CBDC, and on the interest rate margin $R^b - R^m$, which is the difference between the gross nominal interest rates on the central bank's assets and its liabilities. Implicit in (61) is a standard Laffer curve relationship. The interest rate margin $R^b - R^m$ is effectively a tax on CBDC. If the tax increases, this will increase tax revenue (central bank profits) given the quantity of CBDC outstanding, but given our analysis, the real quantity of CBDC outstanding falls as $R^b - R^m$ rises. If the tax rate is zero, i.e. $R^b - R^m = 0$, then central bank profits are zero, and if $R^b - R^m$ is sufficiently high, then $\delta^L = \delta^H = 0$ and CBDC is not held in equilibrium and central bank profits are zero.

So, as long as the Laffer curve is well-behaved, central bank profits are increasing in a stationary equilibrium in $R^b - R^m$, the tax rate on CBDC. In general, some distortion is required, i.e. $R^b - R^m > 0$, to support an equilibrium with positive central bank profits. If we account for, say, a fixed cost of running the central bank, then for the central bank to be self-financing requires that $R^b - R^m$ be sufficiently large. That is, if central bank independence depends on keeping the central bank out of the central government's budgeting process, then independence requires that a monetary system with CBDC be inefficient. In other words, the interest rate on CBDC must be sufficiently low relative to the interest rate on government debt.

5 Conclusion

We have constructed a model of banking, and means of payment so as to explore the role of central bank digital currency (CBDC), and how this matters for monetary policy. CBDC potentially increases welfare for three reasons. First, in substituting for physical currency, it serves to limit criminal activity. Second, CBDC can bear interest, which introduces an additional policy instrument for the central bank, and simplifies the problem of eliminating intertemporal inefficiency typically corrected by a Friedman rule. Third, in substituting for private bank deposits as means of payment, CBDC mitigates incentive problems in private banking, provided the central bank can be trusted.

If the advantages of CBDC over physical currency, and possibly private bank deposits, are important enough, then the demand for CBDC is potentially high. Under these circumstances, if the quantity of government debt is limited, then there could be insufficient debt to back CBDC, except when the nominal interest rate on CBDC is low. If the central bank cannot purchase private assets outright – or it decides that doing so is a bad idea – then CBDC issue could be expanded through central bank lending. This implies a tradeoff between increased efficiency from CBDC issue, and decreased efficiency because of the incentive problems associated with lending to private sector banks.

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