Monetary Policy under Demographic Transitions*

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Abstract

During the post-war period, the share of old (65 and above) individuals has doubled from 7.5% to about 15% in high-income economies. This paper analyzes how aging affects monetary transmission to aggregate consumption. Recent empirical evidence suggest that young households are more responsive to interest rate changes than old households, suggesting that population aging will weaken monetary transmission. But by how much? To answer this question we build a lifecycle New-Keynesian model consistent with the empirical evidence and present two main results. First, population aging weakens monetary transmission considerably. From 1970 to 2015, monetary transmission is [XX] percent weaker due to demographic changes. Second, while the baby-boom affects monetary transmission along the transition path, the increase in life expectancy is far more important in accounting for weaker monetary transmission, implying that it will only weaken going forward. Indeed, monetary transmission will be [75] percent weaker in 2100 than in 1970. We propose two fiscal reforms that may strengthen monetary transmission: a change in the pension system from a defined-contribution to a defined-benefit system and a tax reform that shifts the burden of taxation from labor income to capital.

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†Some of the quantitative values will change with pension recalibration.

1 Introduction

Is monetary transmission weaker in an economy with an older population? This is an important question not only because the population share of individuals older than 65 in high-income economies has doubled from about 7.5% in 1950 to 15% today, but it is also projected to double once more by 2100.[1] If population aging affects monetary transmission, the effects will only become stronger in the future. Although the potential effects of aging on monetary transmission may be substantial, this question has received limited attention in the economics literature.[2] This paper bridges this gap. We build a quantitative general equilibrium model with a fine-grained lifecycle structure consistent with recent empirical evidence on monetary transmission by age groups. Using the model, we argue that the demographic transition since the 1970s with gradually higher life expectancy and a post-war baby boom has weakened monetary transmission by [XX] percent, and is projected to weaken monetary transmission by another [XX] percent by 2100.

In a nutshell, this paper is about the relative size of the income and wealth effects of interest rate changes. For an individual household, an increase in the interest rate has three effects: a substitution, an income, and a wealth effect. The substitution effect is always negative, a higher interest rate increases the relative price of consumption today and the household therefore reduces current consumption. The income effect is always positive, a higher interest rate reduces the present value of future consumption and the household is therefore wealthier. But the income effect is dampened by a wealth effect: the higher interest rate reduces the present value of future income. The strength of monetary transmission depends on the relative size of these three effects. Since a young households hold relatively little wealth, the income and wealth effects cancels, and there is only a substitution effect. Old households, on the other hand, hold relatively more wealth, but less human wealth, meaning that they in addition to the substitution effect also face a strong income effect. Old households are therefore less sensitive to interest rate changes than young households. This theoretical prediction is also consistent with recent empirical evidence. Indeed, Holm, Paul, and Tischbirek (2019) estimate households' consumption responses to monetary policy shocks in Norway and find that the consumption responses to an interest rate increase are strong and negative for young households, but positive for old households.[3]

The prediction of the model above is that population aging will weaken monetary transmission. However, this conclusion ignores general equilibrium effects. In addition, our main concern lies not in understanding whether aging affects monetary transmission, but rather in understanding whether the effects are *quantitatively relevant*. Our main contribution lies here. We build a lifecycle New-Keynesian model with capital consistent with recent empirical evidence on monetary transmission to household-level consumption along the age dimension from Holm et al. (2019). The model allows us to evaluate monetary transmission along the transition path of demographic changes.

Using the model, we first show that population aging weakens monetary transmission substantially. We analyze monetary transmission in two steady states: the 1970s with a relatively low life expectancy and an economy with a high life expectancy consistent with the situation today. The high life expectancy economy has more wealth per individual, a lower real interest rate, and substantially weaker monetary transmission. Indeed, monetary transmission is [XX] percent weaker today than it was in the 1970s.

We next consider the strength of monetary transmission under demographic transitions. In our main experiment, we analyze monetary transmission during the demographic transition from 1970 to 2100. We

[1]See UN World Population Prospects (`https://population.un.org/wpp/`): percentage of total population older than 65 years in high-income countries (World Bank income groups).

[2]Notable exceptions are Fujiwara and Teranishi (2008), Kantur (2013), and Kara and von Thadden (2016), see the discussion below.

[3]Wong (2018) finds the same empirical relationship in US data.

highlight two results. First, while the post-war demographic transition included both a baby-boom and an increase in life expectancy, we show that changes in life expectancy are far more important for monetary transmission. The increase in life expectancy accounts for [XX] percent of the weakening of monetary transmission during the post-war period. In addition, life expectancy is projected to increase even further going forward, and our model suggests that monetary transmission will weaken by [XX] percent by 2100 compared with the 1970s. Second, we show that the equilibrium interest rate is a sufficient statistic for the strength of monetary transmission. Monetary transmission is weak when the interest rate is low, and monetary transmission is strong when the interest rate is high. Both low interest rates and weak monetary transmission have the same underlying cause: high wealth. With more wealth, the return on capital is lower, but monetary transmission is also weaker since the economy-wide ratio of financial to human wealth is higher. The secular stagnation in real interest rates during the past decades is therefore not only resulting in the zero lower bound to bind more often, but it is also an indication of weak monetary transmission. Both these effects point toward a bleak future for monetary policy.

With the model and the results above at hand, we ask: what economic policies may help strengthen monetary transmission? Intuitively, policies that reduce the financial to human wealth ratio in the economy will strengthen monetary transmission. We consider two such reforms: a pension and a tax reform.

Our first policy proposal is a pension reform. The intuitive idea behind pension reform is to move resources from financial to human wealth. The difference between financial and human wealth is whether future income depends on the interest rate. At one end is a defined-contribution pension system in which households save a fixed amount and pension income is determined by the return on capital. In this case, pension wealth is just like financial wealth. At the other end is a defined-benefit pension system in which households contribute to the pension system and are guaranteed a fixed pension income after retirement. In this case, pension income is independent of the interest rate and is therefore counted as part of human wealth. The point is this: the type of pension system directly affects the financial to human wealth ratio. By moving from a defined-contribution toward a defined-benefit pension system, one moves resources from financial to human wealth at the household-level and thus strengthen monetary transmission. We evaluate the potential effects a pension reform may have on monetary transmission by considering two corner cases for the pension system: defined-benefit and defined-contribution. We find [XX].

Our second policy proposal is a tax reform. Any tax reform that shifts the burden of taxation from labor income to capital will lower the level of wealth in the economy, but at the same time increase after-tax income and thus human wealth. Both these effects are desirable from the perspective of the monetary policy authority because they lower the ratio of financial to human wealth in the economy and thus strengthen monetary transmission. We evaluate the potential effects of such a reform by simulating the monetary transmission under demographic transition in two extreme cases of government financing: only labor income taxes and only capital income taxes. We find [XX].

Related literature. The literature most related to our paper is concerned with the interactions between monetary transmission and demographic changes.[4] Fujiwara and Teranishi (2008), Kantur (2013), and Kara and von Thadden (2016) use Gertler (1999)-type overlapping generation models.[5] Similar to us, they also

[4]A related literature investigates how monetary policy redistributes between age groups. Reiter and Hergovich (2016) and Doepke, Schneider, and Selezneva (2019) use lifecycle models with housing and find that monetary policy has long-lasting impact on the intergenerational wealth distribution. Sterk and Tenreyro (2018) use an OLG-model to study the transmission of expansionary open market operations through redistribution.

[5]Miles (2002) uses an OLG-model to show how aggregate steady-state consumption responses to a permanent change in the real interest rate depend on the demographic structure. Although he does not analyze monetary transmission in the conventional sense since he looks at permanent interest rate changes, he highlights the importance of the pension

find that monetary transmission is less effective as society ages. We extend their analysis in two dimensions. First, since demographic changes are slow moving, we analyze how monetary policy effectiveness varies over the transition path. Second, we build a lifecycle model with finer age dimensions that allows us to match the financial to human wealth ratio by age, which is the main determinant of monetary transmission in a lifecycle model, and to understand how different demographic changes (i.e. baby boom and aging) interact over time.

Our paper is also related to the literature studying monetary policy in heterogeneous agent frameworks. Kaplan, Moll, and Violante (2018) study monetary policy in a two-asset model with heterogeneous households and argue that the indirect channel of monetary policy is important.[6] Auclert (2019) studies monetary transmission and shows that an important channel of monetary policy is redistribution between low-MPC agents to high-MPC agents.[7] Our paper contributes to this literature by explaining how heterogeneity in the age dimension affects monetary policy.

Our paper also connects with the large literature on demographic changes, asset accumulation, and real interest rates. Prominent papers include Ríos-Rull (2001), Abel (2003), and Krueger and Ludwig (2007) who analyze the effects of demographic transitions on asset accumulation and interest rates.[8] More recent work by Eggertsson, Mehrotra, and Robbins (2019) and Auclert, Malmberg, Martenet, and Rognlie (2019) highlight the importance of aging for the recent increase in wealth to income ratios and decline in real interest rates. Cooley and Henriksen (2018) argue that there are important differences between various demographic transitions (life expectancy vs. birth rate) for the growth rate and Carvalho, Ferrero, and Nechio (2016) show that changes in life expectancy is most important for explaining the recent decline in the real interest rate. We complement this literature by explaining how higher life expectancy and enlarged birth rates interact to affect aggregate wealth accumulation and monetary transmission during a demographic transition.

There has been a recent revival of the empirical literature on the interactions between demographic transitions and monetary policy. Several papers argue that monetary policy has become less effective over time (see e.g. Boivin and Giannoni, 2002; Primiceri, 2005; Boivin, Kiley, and Mishkin, 2010). Imam (2015) investigates empirically the role of aging on monetary transmission in five major economies. He finds that the recent weakening of monetary transmission may be partly attributed to aging. Holm et al. (2019) document that monetary transmission to household-level consumption is decreasing in age using administrative tax data from Norway. Wong (2018) uses household survey data to investigate how monetary policy transmission varies by age in the US and finds that young households are more responsive to monetary policy than old. Berg, Curtis, Lugauer, and Mark (2019) also estimate consumption responses to monetary policy across age groups and find that the *aggregate* response of old households is greater than that of the young. Leahy and Thapar (2019) estimate state-level income and employment responses to monetary policy and find that state-level variables respond more to monetary policy if the share of middle-aged households is higher. We complement these empirical results by constructing a model consistent with the empirical facts on monetary transmission by age at the household-level and use this model to explain the recent decline in

system for how interest rate changes affect household behavior.

[6]Similarly, Ottonello and Winberry (2018) study monetary policy in a framework with heterogeneous firms and argue that firms with low default risk are most responsive to monetary policy.

[7]A related literature investigate the state- and time-dependence of monetary policy. Berger, Milbradt, Tourre, and Vavra (2018) and Eichenbaum, Rebelo, and Wong (2018) analyze how monetary transmission depends on recent monetary policy, highlighting the importance of past policy for monetary transmission today. Our paper also highlights the importance of time dependence in the sense that monetary transmission depends on the current demographic composition of the economy.

[8]Other examples include Castaneda, Diaz-Gimenez, and Rios-Rull (2003) and Benhabib, Bisin, and Zhu (2011) who investigate the interaction between lifecycle wealth accumulation and wealth inequality.

monetary policy effectiveness.

Our paper also relates to the recent empirical literature on household heterogeneity and monetary transmission. Cloyne, Ferreira, and Surico (2019) investigate the transmission of monetary policy to households in the US and the UK, and find that homeowners with debt make up a substantial part of the aggregate response to monetary policy. Similarly, Crawley and Kuchler (2018) use Danish data and argue that redistribution from debtors to creditors in response to a contractionary monetary policy shock is quantitatively important because debtors have higher MPCs than creditors. Fagereng, Holm, and Natvik (2018) use Norwegian data and find that consumption responses to lottery prizes are decreasing in age, indicating that age is an important dimension of household heterogeneity. Further, Flodén, Kilström, Sigurdsson, and Vestman (2017) and La Cava and Kaplan (2019) argue that the household cash flow channel of monetary transmission is quantitatively important. A common feature of this recent literature is that it emphasize other monetary transmission channels than intertemporal substitution. We complement this literature by highlighting the income and wealth effects as important monetary transmission channels and show that the variation in these effects due to demographic transitions may produce considerable time-variation in monetary transmission.

Structure. Section 2 first explains the intuition for how aging affects monetary transmission in a partial equilibrium framework. Section 3 then presents empirical evidence on wealth, earnings, and monetary transmission by age that make up the main calibration targets for our model. In Section 4, we describe our general equilibrium New-Keynesian lifecycle model. Section 5 contains our results on how demographic changes affect monetary transmission, while Section 6 presents our results on how fiscal reforms may affect monetary transmission. Section 7 concludes.

2 Why may demographics matter for monetary transmission?

In this section, simple models are used to show the intuitions for how the demographic structure of the economy may affect monetary transmission.

2.1 A consumption-saving model

First, we use a consumption-saving model to explain what matters for a household's consumption response to interest rate changes in the short-run. Households live for T periods and maximize discounted life-time consumption

$$\max_{\{c_{t+k}\}_{k=0}^{T}} \sum_{k=0}^{T} \beta^k \frac{c_{t+k}^{1-1/\gamma}}{1-1/\gamma}$$

subject to the lifetime budget constraint

$$c_t + \frac{1}{1+r}\left(c_{t+1} + \frac{c_{t+2}}{1+\bar{r}} + \frac{c_{t+3}}{(1+\bar{r})^2} + \ldots\right) = a + \frac{1}{1+r}\overbrace{\left(y_{t+1} + \frac{y_{t+2}}{1+\bar{r}} + \frac{y_{t+3}}{(1+\bar{r})^2} + \ldots\right)}^{y}$$

where c is consumption, r is the short-term interest rate (between period t and $t+1$), $\bar{r}$ is the interest rate that prevails after period $t+1$, a is wealth (cash-on-hand), y is the net present value of future income in period $t+1$, β is the discount factor, and γ is the elasticity of intertemporal substitution.

Optimal consumption in period t is

$$c_t \approx \frac{a + \frac{y}{1+r}}{\beta^{\gamma}(1+r)^{\gamma-1}T}$$

where the approximation holds for large T and $\beta(1+r) \approx 1$.[9] The derivative of period-t consumption with respect to the *short-term* interest rate r is

$$\frac{\partial c_t}{\partial r} \approx \overbrace{\frac{1}{\beta^{\gamma}(1+r)^{\gamma}T}}^{\zeta}\left[\underbrace{-\gamma\left(a + \frac{y}{1+r}\right)}_{\text{substitution effect}} + \overbrace{\left(a + \frac{y}{1+r}\right)}^{\text{income effect}} \underbrace{-\left(\frac{y}{1+r}\right)}_{\text{wealth effect}}\right].$$

There are three effects of an interest rate change on current consumption. First, a higher r reduces the price of future consumption relative to today (substitution effect). Second, a higher r increases real income of the household because the future consumption plan is cheaper (income effect). Third, a higher r reduces the net present value of future income (negative wealth effect). A household's consumption response to an interest rate change depends on the relative size of these three effects.

To illustrate how age matters, consider two cases: young ($a \approx 0$) and old ($y \approx 0$).

$$\frac{\partial c_t}{\partial r} \approx \begin{cases} -\zeta\gamma\left(\frac{y}{1+r}\right) < 0 & \text{if young } (a \approx 0) \\ \zeta(1-\gamma)a \gtreqless 0 & \text{if old } (y \approx 0) \end{cases}$$

When the household is young, it holds relatively little wealth. In that case, interest rate changes only affect consumption through the substitution effect since the income and wealth effect cancel out each other. The consumption response of young households is therefore unambiguously negative. Old households, on the other hand, hold wealth, but relatively little human wealth. The interest rate affects period t consumption through both the substitution and income effect. As a result, the consumption response of old households depends on the size of the elasticity of intertemporal substitution γ relative to 1. For low γ, the consumption response to an increase in the current interest rate may even be positive. However, even if the elasticity of intertemporal substitution is greater than 1, the presence of the income effect for old households implies that their consumption response to interest rate changes is weaker.

In general, there are two determinants of individual households' interest rate sensitivity: the elasticity of intertemporal substitution γ and the wealth-to-human wealth ratio a/y. One way to see this is to derive the sufficient condition for $\frac{\partial c_t}{\partial r} > 0$:

$$\frac{a}{y} > \frac{\gamma}{(1-\gamma)(1+r)}.$$

In words, a household responds to a higher interest rate by *increasing* current consumption if $\gamma < 1$ and the wealth-to-human-wealth ratio a/y is sufficiently high.

What matters for monetary transmission by age is therefore how the wealth-to-human-wealth ratio varies over the lifecycle. Figure 1 illustrates the empirical patterns of wealth, human wealth, and the implied consumption response to an interest rate change. There are two observations that are important for our context. First, the wealth-to-human-wealth ratio increases over the lifecycle. Young households hold relatively little wealth, but their human wealth is high because they expect high labor income in the future. Old households hold more wealth, but their human wealth is lower since they have already earned most

[9] The approximation around large T does not necessarily require the remaining lifetime to be long. Large T also holds if the household has a bequest motive. The value of T would in the case with bequests be the remaining lifetime plus the value of bequests in year-equivalent units.

of their labor income. Young households should therefore respond to a higher interest rate by reducing consumption (substitution effect), while old households will have weaker negative consumption response or even a positive consumption response (substitution and income effect).

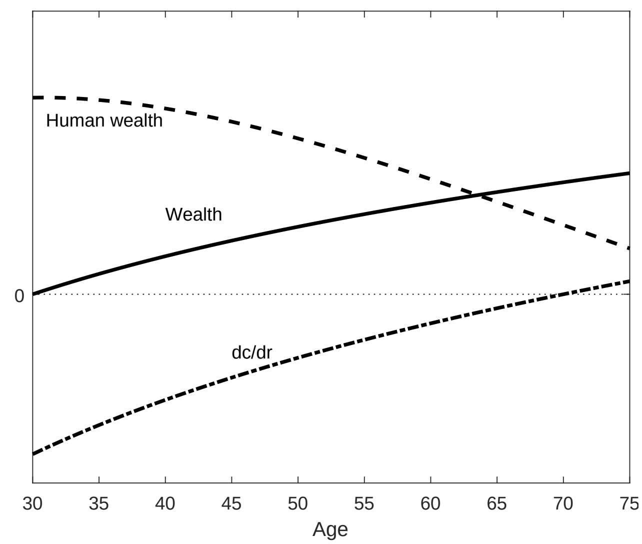


Figure 1: Wealth, human wealth, and consumption responsiveness to current interest rate changes over the lifecycle.

Second, what matters for monetary transmission is whether future income depends on the interest rate or not. Labor income does not depend on the interest rate and therefore only has a negative wealth effect. Capital income does depend on the interest rate and therefore has no negative wealth effect. This is important when we think about pension systems. With a defined-contribution system, future pension income depends on the interest rate so pension wealth is like wealth. With a defined-benefit system, future pension income does not depend on the interest rate so pension wealth is like human wealth. Monetary transmission, and in particular how aging affects monetary transmission, therefore depends on the type of pension system. Monetary transmission is weaker in the presence of a defined-contribution rather than a defined-benefit pension system because the wealth-to-human-wealth ratio is higher for all age groups in a defined-contribution system. We will return to the role of the pension system when we evaluate potential policies that may affect monetary transmission in Section 6.

The difference between wealth and human wealth is less clear in general equilibrium. When one includes market clearing in a model, future income (or any price in general) will depend on the current interest rate. In addition, even if the pension system is defined-benefit, expected pension income might depend on the current interest rate because the government might adjust pension payments in response to interest rate changes. However, as long as the effect of the interest rate on future income is only partial, meaning less than for capital income, the interest rate response will depend on the wealth-to-human-wealth ratio, albeit less starkly than in a partial equilibrium framework. Our model in Section 4 includes a general equilibrium market clearing so that we can explore the full *quantitative* implications of demographic transitions on monetary transmission.

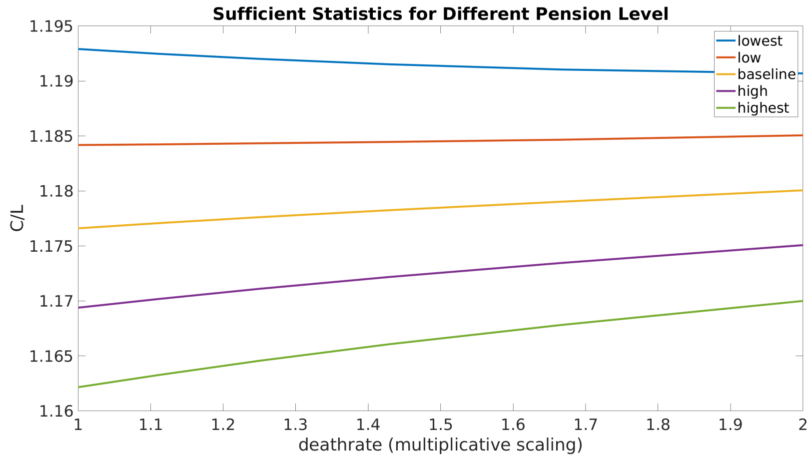


Figure 2: The dependency ratio values given different levels of deathrate and pension level.

2.2 Simple Economy with Productive Capital

The literature also argued that the natural rate of interest has drifted down due to the demographic change we have seen so far. To see the simple intuition, consider the economy with Cobb-Douglas production function

$$Y_t = K^\alpha L^{1-\alpha}$$

with the depreciation rate of capital at δ_k. From the market-clearing condition of the economy, we have

$$K^\alpha L^{1-\alpha} = C + \delta_k K$$

which results in

$$\frac{C}{L} = \tilde{K}^\alpha - \delta_k \tilde{K}$$

where $\tilde{K} := \frac{K}{L}$. With the market clearing price equation of the interest, one has

$$r + \delta_k = r_k = \alpha \tilde{K}^{\alpha-1}$$

This equation shows that the dependency ratio defined as $\dfrac{C}{L}$ is a sufficient statistic for the market-clearing interest rate. In English, the higher the dependency ratio, the lower the interest rate. This is different from the standard dependency ratio where people divide the number of working-aged people from the number of retirement-aged people, but it more relevant to the point for the interest rate determination. For example, given an increase in life-expectancy, households might endogenously work more during their prime-working years and or postpone their retirement age. This behavioral change would be captured with $\frac{C}{L}$, but not with the static population-proportion definition.

Hence, the impact of the demographic and policy changes will have on the market interest rate is given by how the dependency ratio would change from the given demographic change. For example, figure 2 shows the sufficient statistics for given different levels of deathrate and pension scheme in the economy based on the heterogenous household defined below. For example, by looking at the line with highest level of pension level, one can see that market clearing interest will *increase* when the life-expectancy increases. This is consistent with the baseline calibration of Fujiwara and Teranishi (2008) where they also find that the increase in life expectancy will lead to an increase in interest rate. In the worker-retiree model of Fujiwara and Teranishi (2008), this "paradox" can not be addressed further. However, in the full heterogenous-agent model, it can be seen that different pension arrangements will decide whether the natural rate of interest rate will increase or decrease. In fact, one can see that the strength of the pension leads to strong difference of the interest than the deathrate. Given this, one can infer that the historical change from the defined-benefit pension system into the defined-contribution pension system had a marked impact on the natural rate of interest.

3 Wealth, earnings, and monetary transmission by age in the data

The goal of this paper is to understand how demographic transitions affect monetary transmission. In order to do that, we build a general equilibrium lifecycle model in Section 4. Before we describe our model, we present three empirical moments by age that are relevant for calibrating our model: wealth, earnings, and monetary transmission.

3.1 Data sources

We combine several registry databases maintained by Statistics Norway. This allows us to construct a rich longitudinal data set containing the balance sheet, income, and demographic variables for every adult Norwegian from 1993 to 2015. We further supplement this dataset with information from the earnings records from 1972 to 2015. In this section, we describe each variable used.

Sample restrictions. We restrict our sample to the adult population (30-75 years) born in Norway and with business income less than the basic amount in the social security system and non-negative wealth.

Wealth. Norway has a wealth tax and the tax authorities collect information on the balance sheets of the universe of households. The balance sheet components are third-party reported, limiting the scope for tax evasion.[10] For most variables, the tax value is the market value and we can use the tax values to compute net worth, but there are two exceptions. First, privately held firms are typically valued close to the book value by the tax administration, which arguably is a lower bound on the market value. However, while privately held firms are important at the top of the wealth distribution, it does not significantly impact the life-cycle patterns we use to calibrate the model in this paper. Second, housing wealth is valued at an assessed value by the tax authorities. We instead use re-estimated housing wealth from Fagereng, Holm, and Torstensen (2019).[11] Our definition of wealth is the sum of all assets (deposits, stock funds, bonds, stocks, non-listed stocks, vehicles, outstanding claims, and housing wealth) minus all liabilities (mortgages, consumer debt, and student debt).

Earnings. Our measure of earnings is individual male pre-tax labor earnings from 1972 to 2015. Our measure of income is pre-tax earnings, which is the sum of wages, idiosyncratic productivity, and age effect and labor supply in the model. Under the assumptions that wages, idiosyncratic productivity, and labor supply are not affected by age, the age profile of pre-tax earnings will reflect the model-relevant age-gradient.

[10] The only exception is non-listed stocks which is partly self-reported.

[11] Fagereng et al. (2019) combine transaction data and housing unit characteristics to estimate housing wealth using an ensemble machine learning method between 1993 and 2015.

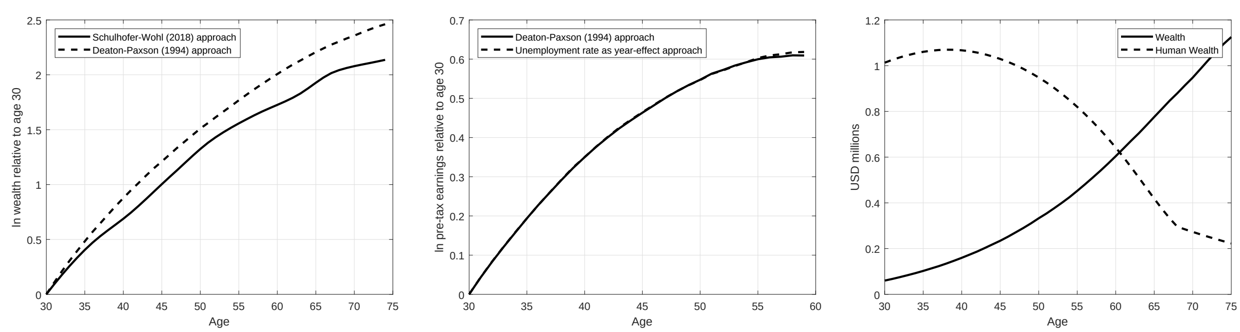
3.2 Wealth, earnings, and human wealth by age

We now consider the observed age patterns in Norwegian administrative data. We are interested in two age patterns: financial and human wealth. The main challenge is to compute age patterns that are relevant for the model calibration below. For that purpose, one has to compute the age pattern of individual households.

There are three reasons why computing age patterns is challenging. First, to identify age effects, the researcher needs to observe a panel of households. A cross-sectional dataset does not allow separation between age and cohort effects because these two are collinear. For example, the cross-sectional observation of average wealth by age in a given year is not the relevant age effect because it may contain cohort effects. The administrative Norwegian data follow the same households across multiple years, allowing us to circumvent this issue.

Second, even with a panel of households as in Norway, we need to make additional assumptions to separate time, age, and cohort effects. Time, age, and cohort are perfectly collinear so any linear trend can therefore be attributed to any linear combinations of the three effects. There are several standard approaches to deal with this. The most common approach is to assume that time-effects are orthogonal to a time-trend (Deaton and Paxson, 1994). However, Schulhofer-Wohl (2018) show that all approaches based on strong identifying assumptions produce inconsistent estimators of structural parameters and are unnecessary because one can obtain consistent estimates of structural parameters using weaker assumptions. Furthermore, as noted by Lagakos, Moll, Porzio, Qian, and Schoellman (2018), the identifying assumption in Deaton and Paxson (1994) is problematic if the underlying time-trend shifts during the sample period. We use at least two approaches for each variable to decompose the time, age, and cohort effects. For earnings, we use the approach by Deaton and Paxson (1994) to estimate age effects of earnings because the assumption of an orthogonal time-effect seems reasonable since the average growth rate of earnings is stable in our sample. For wealth, where the orthogonal time-effect assumption might be more problematic since there might be changing growth trends in asset prices, we follow Schulhofer-Wohl (2018) and identify the age effect using only higher order moments of the age profile.

Third, there may be selection on survival in the data. Wealth is correlated with lower mortality so old households (survivors) tend to be selected on high wealth. We follow Attanasio and Hoynes (2000) and correct for selection on mortality by (i) estimating a probit model of mortality on observed parameters (e.g. wealth, age, and cohort) and (ii) re-weight households by their inverse survival probability when we compute age patterns.



(a) Wealth (b) Pre-tax earnings (c) Financial and human wealth

Figure 3: Wealth, earnings, and human wealth by age.

Wealth. Figure 3(a) presents our estimated wealth pattern by age using two approaches: the Schulhofer-Wohl (2018) approach and the Deaton and Paxson (1994) approach. For wealth, the identifying assumption of Deaton and Paxson (1994) is problematic because one important component of wealth changes from year to year is capital gains, and these capital gains often experience long periods of high growth. We therefore do not want to impose the assumption that year-effects are orthogonal to a linear time trend. Instead we follow the approach in Schulhofer-Wohl (2018) where we remain agnostic about where to allocate the linear trend, but instead identify the model using only higher order moments of the age pattern.

Our approach proceeds in four steps. First, we estimate a probit model of survival on wealth percentile and a quadratic function of age. This model gives us the probability of survival to a specific age (cumulative survival probabilities up to that age) for an individual in our sample. Second, we compute an age trend using the orthogonal time trend assumption and weighting the regressions by the inverse survival probabilities. Third, we de-trend the age-effect by taking out the linear trend because we know that it is unidentified. Fourth, we use the structural model from Section 4 to identify the age pattern that best matches the higher than first order moments (the curvature) of the age trend.

Figure 3(a) compares the age effects using our preferred approach (Schulhofer-Wohl, 2018) and the Deaton and Paxson (1994) approach. The two methods produce similar age effects: wealth grows over the lifecycle and continues to grow even after retirement. The age pattern implies that a households can expect to have between 6 and 7 times as much wealth when they are 60 compared with when they are 30.

Earnings. Figure 3(b) presents the age patterns of pre-tax earnings from age 30 to 60.[12] Recall that our measure of pre-tax earnings is total labor market income of individual males. We identify the age effect of earnings in Figure 3(b) using the assumption that the year effect is orthogonal to a time trend (Deaton and Paxson, 1994). As discussed in Lagakos et al. (2018), this assumption is problematic if there are changes in growth trends during the sample period. Since Norway has a relatively stable growth rate during our sample period from 1972 to 2015, we argue that the identifying assumption where the year-fixed effects are orthogonal to a time-trend is reasonable. For robustness, we also show the age pattern of pre-tax earnings using the unemployment rate as the year-fixed effect in Figure 3(b). The estimated age patterns of earnings are almost identical across the two identifying assumptions, both implying that a household can expect to earn about 80 percent more when they are 60 compared with when they are 30.

Wealth and human wealth by age. As discussed in Section 2, interest rate sensitivity of households is determined by the ratio of financial to human wealth. Figure 3(c) presents financial and human wealth by age. We construct financial wealth by fixing wealth at 75 to the median wealth level for households between 67 and 79 years in 2015. We then use the estimated age pattern from Figure 3(a) to compute financial wealth at all other ages relative to age 75.

Human wealth is the net present value of future income and requires additional assumptions to compute. First, we fix income at age 60 equal to the median after-tax income of households between age 55 and 66 years. The age profile from Figure 3(b) then allows us to compute income at all age levels from 30 and 60 years. Second, we assume that income between age 60 and 67 (retirement) is equal to income at age 60. Third, since the main share of Norwegian households rely on the public defined-benefit pension system, we include pensions in human wealth. We assume that pensions are equal to the median after-tax pension of individuals between age 67 and 75. With these three assumptions, we can earnings by age from age 30

[12]We do not present age patterns for earnings above the age of 60 because we want our age gradient to represent full-time work and the incidence of part-time work and graded retirement increases after age 60. For the same reason, we start at age 30 to ensure that individuals in our sample are working full-time.

until death. Human wealth is the net present value of future earnings at a specific age, discounted by the discount rate (implied by the model in Section 4) and the age-dependent mortality rate.

Figure 3(c) shows that at age 30, the average household holds more than [20] times as much human wealth as financial wealth. As they age, they accumulate financial wealth while human wealth decreases. At around 65, financial wealth and human wealth are approximately the same, while after retirement, financial wealth is greater than human wealth. The ratio of financial to human wealth by age is one of our main calibration targets for the model in Section 4.

3.3 Consumption responses to monetary policy by age

Our next empirical evidence is the estimated household-level consumption responses to monetary policy shocks along the age dimension.

Norwegian evidence. Holm et al. (2019) use tax registry data from Norway and estimate the response of household-level imputed consumption to contractionary monetary policy by age. Figure 4 presents their results. Consistent with the model in Section 2, monetary transmission is strong and negative for young households, while monetary transmission weakens with age and eventually turns positive, although not significantly so, for the oldest age group.

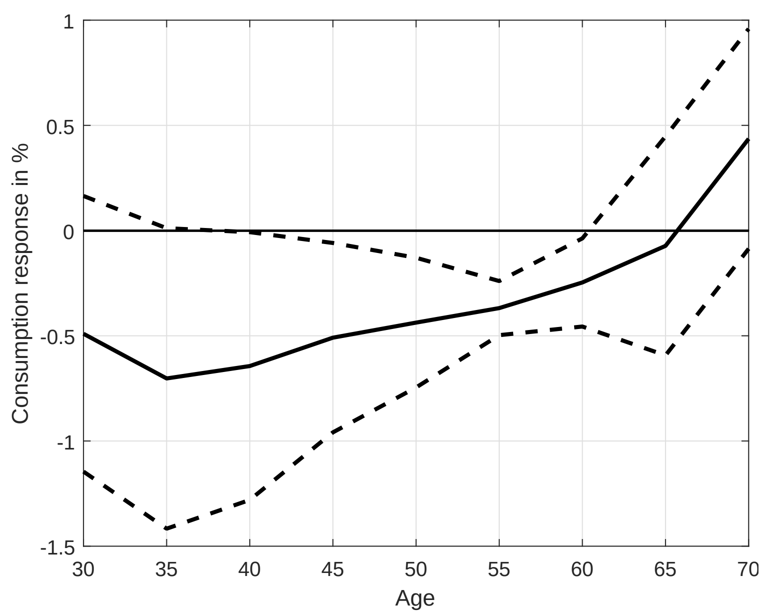


Figure 4: Consumption responses in year 1 to a monetary policy shock (1 pp. increase in interest rate) in year 0 by age (Holm et al., 2019). The dashed lines show 95 % confidence intervals.

US evidence. Wong (2018) uses survey evidence on household-level consumption from the US to estimate the annual consumption elasticities of households to a one standard deviation contractionary monetary policy shocks across the age distribution. Table 1 presents her main results. Across three different definitions of consumption data (CEX total, CEX non-durables, and food consumption from Nielsen home-scanner data), she finds monetary transmission to consumption to be decreasing with age. Young households are responsive to monetary policy, while old households are relatively insensitive to monetary policy.

Table 1: Annual consumption elasticities to contractionary monetary policy shocks by age from Wong (2018).

	Young 25-34		**Middle** 35-64		**Old** 65 +
CEX data					
Total	-4.59		-0.79		1.15
	[-7.17, -2.01]		[-3.02, 1.44]		[-2.5, 4.8]
Non-durables	-2.24		-0.47		-0.12
	[-3.82, -0.67]		[-1.65, 0.7]		[-2.07 ,1.83]
			Age groups		
	25-34	35-44	45-54	55-64	65+
Nielsen data					
Non-durables (food)	-0.79	-0.50	-0.60	-0.38	-0.03
	[-1.31, -0.28]	[-0.78, -0.21]	[-0.83, -0.36]	[-0.63, -0.14]	[-0.28, 0.23]

Notes: This table is a reprint of Table 2 in Wong (2018). It presents annual consumption elasticities by age in response to a one standard deviation contractionary monetary policy shock using CEX or Nielsen home-scanner data. The brackets depict 80 percent confidence intervals.

4 The Model

The model consists of four blocks: households, firms, government, and the monetary policy authority. Apart from the households, each sector is as close as possible to a standard New-Keynesian model with productive capital. Hence, the model represents the least deviation from a standard model that includes life-cycle considerations. This section presents a detailed description of each model block and our calibration.

Households. The economy is populated by a continuum of households indexed by their age a, wealth x, and idiosyncratic labor productivity z. At each point in their lifetime, households choose how much to consume and how much labor to supply. At age 67, they retire and receive pensions from the government. Mortality rate is age-dependent, but households die with certainty at age 105. At death, they receive utility from a warm-glow bequest motive on their remaining wealth.

Households receive a utility flow u from consumption c and a disutility flow from supplying labor l. Preferences are time-separable and the future is discounted at rate $\rho \geq 0$:

$$\max_{\{c_t, l_t\}_{t=0}^{T}} \quad \mathbb{E}_0 \left\{ \int_0^T e^{-\rho t} u(c_t, l_t) dt + e^{-\rho T} \mathbb{B}(x_T) \right\}. \tag{1}$$

The expectation is taken with respect to the stochastic time of death, T, and realizations of idiosyncratic labor productivity shocks. Households can save in an asset x with an exogenous borrowing limit $\underline{x}$ at the real interest rate r. Labor earnings is the product of the aggregate wage w, an age-specific component q, idiosyncratic labor productivity e^z, and labor supply l. The household also pays taxes on labor income τ^l, capital income τ^c, and wealth τ^w, and receives lump-sum transfers T and pension Λ after retirement.

$$\dot{x}_t = r_t x_t + w_t e^{z_t} q_a l_t + \Gamma_{at} - c_t, \tag{2}$$

$$\Gamma_{at} = \left[(1-\tau^l)T_t + (1-\tau^l)\Lambda_a - \tau^l w_t e^{z_t} q_a l_t - \tau^c r_t x_t - \tau^w x_t\right], \tag{3}$$

$$x_t \geq \underline{x}. \tag{4}$$

Pension is defined as a replacement rate multiplied by the median max earnings $T_t = \chi \max\{w_t l_t q_a\}$, similar to the pension rule in the Norwegian public pension system up until 2010. Idiosyncratic labor productivity moves according to an Ornstein-Uhlenbeck in logs

$$dz_t = \mu(z_t)dt + \sigma(z_t)dW_t \tag{5}$$

where μ is the drift and σ is the diffusion of the income process.

We specify the utility function as

$$u(c,l) = \frac{(c)^{1-1/\gamma}}{1-1/\gamma} - \phi_0 q_a e^{z_t} \frac{l^{1+1/\phi_1}}{1+1/\phi_1} \tag{6}$$

where γ is the elasticity of intertemporal substitution, ϕ_0 is a labor shifter, and ϕ_1 is the Frisch elasticity. We do not want to introduce any variation in labor income by age other than through the estimated earnings profiles from the data. We therefore assume a utility function of the GHH-form where we include q_a and z_t in the labor shifter. This utility specification implies that an individual's labor supply only depends on the aggregate wage.

The bequest function is

$$\mathbb{B}(x) = \psi_0 \frac{\left((1-\tau^b)x + \psi_1\right)^{1-1/\gamma}}{1-1/\gamma} \tag{7}$$

where ψ_0 and ψ_1 jointly determine the "luxuriousness" of bequests, and τ^b is the inheritance tax. We assume that there are no inter-vivos transfers so that all bequests happen at death. Bequests are distributed to all households according to the inheritance distribution by age in the data.

Final goods firm. We assume there exists a competitive representative final-good producer that aggregates a continuum of intermediate goods

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

where $\epsilon > 0$ is the elasticity of substitution between goods. The corresponding demand for good j and aggregate price index are

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\epsilon} Y_t, \qquad P_t = \left(\int_0^1 p_{j,t}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}.$$

Intermediate goods firms. Intermediate goods firm j produces according to the production function

$$y_{j,t} = \tilde{k}_{j,t}^{\alpha} n_{j,t}^{1-\alpha}$$

where $n_{j,t}$ is the amount of labor hired by firm j at the competitive real wage w_t and $\tilde{k}_{j,t}$ is the (efficiency unit) amount of capital hired by firm j at the competitive real rate r_t^k. Since all firms solve the same problem, they have the same real marginal cost

$$m_t = \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_t^k}{\alpha}\right)^{\alpha}$$

We follow Kaplan et al. (2018) and assume that each intermediate goods producer chooses its price to maximize profits subject to quadratic adjustment costs

$$\Theta_t\left(\frac{\dot{p}_t}{p_t}\right) = \frac{\theta}{2}\left(\frac{\dot{p}_t}{p_t}\right)^2 Y_t$$

where $\theta > 0$. Kaplan et al. (2018, Online Appendix B.2) show that the New Keynesian Phillips curve for this problem can be expressed as

$$\left(r_t - \frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\epsilon}{\theta}(m_t - m^*) + \dot{\pi}_t \tag{8}$$

where $m^* = \frac{\epsilon-1}{\epsilon}$ is the mark-up that would prevail if there were no adjustment costs.

The intermediate goods producers exit at exogenous rate of ν^{π}, and new firms will enter every period to replace the exiting firms keeping the total number of firms fixed. The equity of the intermediate goods firms are owned by a household through a mutual fund, and hence, the profit stream will go to the mutual fund as explained below.

Productive capital. There is productive capital in the economy. We follow the standard approach for New Keynesian models with productive capital where baseline capital is determined by investments with adjustment cost while the short-term efficiency-unit of capital is determined by capacity "utilization."[13] The joint capital investment and utilization decision is given by

$$\max_{\iota_t, u_t} \int_0^{\infty} e^{\int_t^{\infty} -r_\tau d\tau}(r_k \cdot k_t - \iota_t)\, dt$$

subject to a law of motion for capital of

$$\dot{k}_t = \iota_t(1 - \phi(\nu_t)) - \delta^k k_t$$
$$\dot{\iota}_t = \nu_t$$

where k_t is capital, r_t^k is return on capital, ι_t is investment, ν_t is the change in investment, δ^k is the depreciation rate on capital, and $\phi(\nu_t) = \frac{\theta_\iota}{2}\nu_t^2$ is the capital adjustment cost similar to Fujiwara and Teranishi (2008). r_t is the market rate of return explained further in the next section.

Investment, utilization, profit of capital producing firm, price of capital, and the law of motion of capital

[13]Capacity-utilization is the standard terminology in the literature, but this should be considered a joint short/long-term decision for capital. In English, one can buy more machines (investment) to increase production or (ab)use the current stock of machines at a cost of faster depreciations.

are given by

$$
\begin{aligned}
\nu^* &= \left(\phi'\right)^{-1}\left(\frac{\eta_t}{\lambda_t i_t}\right) \\
\dot{\eta} &= r_t\eta_t + 1 - \lambda_t(1-\phi(\nu)) \\
\dot{\lambda}_t &= (r_t+\delta^k)\lambda_t - r_t^k
\end{aligned}
$$

where λ_t is the price of capital and η_t the shadow-price of adjustment cost of investment. See Appendix A for details. We further assume that there is entry and exit of capital-producing firms in each period, equal to ν^k.

Mutual fund. In this economy, intermediate goods firms and capital producing firms earn profits. There are therefore three assets in the economy: capital, equity of the intermediate goods firms, and equity in the capital producing firm. All assets are held by the mutual fund where households invest. We assume no-arbitrage between assets, resulting in one market return, r_t, on all assets. Given r_t, the prices of equity of the intermediate goods firms, and equity of the capital producing firm are

$$
\begin{aligned}
\dot{q}_t^{\pi} &= (r_t+\nu^i)q_t^{\pi} - \Pi_t^{\pi}, \\
\dot{q}_t^{k} &= (r_t+\nu^k)q_t^{k} - \Pi_t^{k}
\end{aligned}
$$

with the per-period profits given by

$$
\begin{aligned}
\Pi_t^{\pi} &= (1-m_t)Y_t - \Theta_t\left(\frac{\dot{p}_t}{p_t}\right), \\
\Pi_t^{\iota} &= (\lambda_t - 1)\cdot\iota^* - \phi(\iota^*).
\end{aligned}
$$

To summarize, the value of the mutual fund is given by

$$
\lambda_t \cdot k_t + q_t^{\pi} + q_t^{k}
$$

with return of r_t.

Government. We assume the government runs a balanced budget equal to

$$
T_t + \int \Lambda_a d\mu_t = \tau^l w_t \int e^{z_t} q_a l_t d\mu_t + \tau^l \int \Lambda_a d\mu_t + \tau^l T_t + \tau^c r_t \int x_t d\mu_t + \tau^w \int x_t d\mu_t + \tau^b \int beq_t d\mu_t \tag{9}
$$

where the left hand side is total government expenditure on transfers and pensions while the right hand side is total income from the labor, capital income, wealth, and inheritance taxes. When the economy is not in a steady state, we assume that the government adjusts the general component of transfers in response to changes in tax revenue or interest expenses while pensions remain fixed.

Monetary policy. The monetary policy sets the nominal interest rate according to a Taylor rule

$$\begin{aligned} i_t &= \bar{r}_t + \phi^{\pi}\pi_t + \phi^{y}y_t + \nu_t \\ d\nu_t &= -\rho^{v}\nu_t dt + \sigma_{\nu}dW_t \end{aligned} \tag{10}$$

where $\bar{r}$ is the natural rate of interest,[14] ($\pi = 0$ in steady state), ϕ^{π} and ϕ^{y} are the coefficients on inflation and output in the Taylor rule, ν_t is a monetary policy shock, and ρ^{v} is the persistence of the monetary policy shock.

Equilibrium. An equilibrium in this economy is defined as paths for individual household and firm decisions $\{c_t, l_t, \iota_t, u_t, k_t\}_{t\geq 0}$, prices $\{w_t, \pi_t, \lambda_t, q_t^{\pi}, q_t^{k}\}_{t\geq 0}$, interest rates $\{i_t\}_{t\geq 0}$, transfers $\{T_t\}_{t\geq 0}$, measures $\{\mu_t\}_{t\geq 0}$, and aggregate quantities such that at every t

i. households maximize (1) subject to (2), (4), and (5),

ii. firms maximize profits, with the resulting Phillips curve (8),

iii. the government budget constraint holds, and

iv. all markets clear.

There are three markets in this economy: the asset market, the labor market, and the goods market. The asset market is cleared by a mutual fund (or no arbitrage), i.e., total asset in the economy equals to the asset demand of households

$$\int x_t d\mu_t = B_t + \lambda_t \cdot k_t + q_t^{k} + q_t^{\pi}.$$

(Productive) capital market clears when the efficiency unit demanded is equal to the efficiency unit of capital supplied.

$$\tilde{K}_t = u_t \cdot k_t.$$

The labor market clears when total labor demand equals total labor supply by households

$$\int e^{z_t} q_t l_t d\mu_t = L_t,$$

The goods market clears when total production in the economy equals the sum of consumption, investments, investment adjustment cost, firm entry costs, and price adjustment costs

$$\tilde{K}_t^{\alpha} \cdot L_t^{1-\alpha} = Y_t = C_t + I_t + \nu^{\pi} q_t^{\pi} + \nu^{k} q_t^{k} + \Theta_t\left(\frac{\dot{p}_t}{p_t}\right).$$

[14] The natural rate of interest rate can differ from the steady-state interest rate. For example, the natural rate of interest will vary along the transition path from a mortality rate change. Also, even without transition, the monetary policy authority does not set the interest rate away from the current market return, i.e.,

$$\bar{r}_t = r_{ss} + (r_t^{k} - r_{ss}^{k})$$

instead of

$$\bar{r}_t = r_{ss}.$$

4.1 Calibration

We calibrate the model in the following way: (i) we calibrate all observable parameters to their data counterpart, (ii) we set parameters that are common in the macro literature to its usual value in the literature, and (iii) we use the remaining parameters to match wealth and the wealth-to-human-wealth ratio by age.

Observable parameters in the data. We first use the Norwegian death tables from 2015 to compute the mortality rate for each age group.[15] We further take the initial distribution of income and wealth at age 25-29 from the Norwegian administrative tax data as our initial distribution. We distribute inheritance across the living population using the age distribution of bequest recipients in the data.[16]

Fixed parameters. We fix a number of parameters to their standard values in the literature. We first set γ, the elasticity of intertemporal substitution to 0.5. We set the labor supply parameters, ϕ_0 and ϕ_1 to 1.186 and 1, to match the average hours supplied during the day to 0.5 (ϕ_0) and the Frisch elasticity (Chetty, Guren, Manoli, and Weber, 2011). We further set the real interest rate to 0.03 in steady state and fix the borrowing constraint at 0.

We calibrate the income process with an annual autocorrelation of idiosyncratic income innovations equal to 0.937 and a standard deviation of 0.155, estimated in Fagereng et al. (2018). The production side of the economy is similar to Kaplan et al. (2018) with a curvature on the production function equal to 0.33, the elasticity of substitution between goods equal to 10 (profit share of approximately 11 %), and the cost of price adjustments, θ, equal to 100 to match a slope of the Phillips curve equal to 1/10. The monetary policy side of the economy is also the same as in Kaplan et al. (2018) with a Taylor coefficient of 1.25 on inflation and no response to output.

Calibrated parameters. We calibrate the remaining parameters (ρ and ψ_0) to match wealth by age, the ratio of wealth to human wealth by age, and an equilibrium interest rate equal to 0.03. Our main target is the wealth-to-human-wealth ratio because, as explained in Section 2, it determines the relative strength of the substitution, income, and wealth effects at the household level. We target wealth by age and equilibrium interest rate to ensure that the model is reasonable in its levels.

We follow a simulated method of moments (SMM) procedure where we minimize the distance between the data and the model-implied moments. Figure 5 presents the calibration results. We are able to match the equilibrium interest rate exactly (0.03) and we provide a reasonably good match of wealth and wealth-to-human-wealth ratio by age.

[15] We combine the number of deaths per age from Statistics Norway Table 10325 and with the population per age in Table 07459 to compute the mortality rate.

[16] We approximate the distribution of inheritances across age by matching a normal distribution to the data for the year 2000 $\approx \mathbb{N}(51, 95)$.

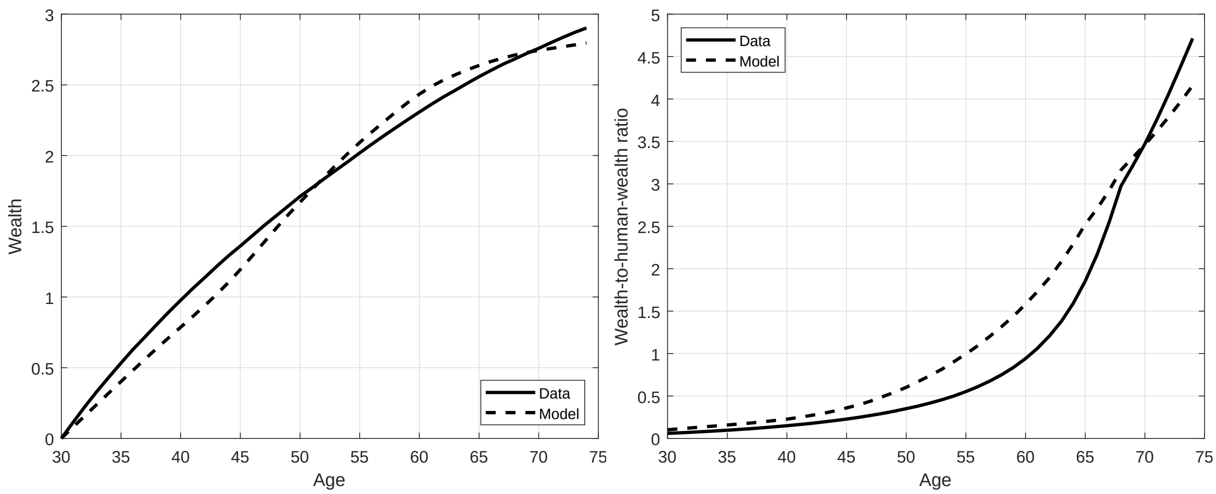
Table 2: Model calibration

Fixed parameters

	Value	Description	Source
Preferences			
γ	0.5	elasticity of intertemporal substitution	
ϕ_0	1.186	shifter on labor supply	time spent on labor during a day = 0.5
ϕ_1	1	Frisch elasticity	
r	0.03	real interest rate	
$\underline{x}$	0	borrowing constraint	
Income process			
μ	0.937	annual autocorrelation	Fagereng et al. (2018)
σ	0.155	standard deviation	Fagereng et al. (2018)
Production			
α	0.33	production curvature	Standard
ϵ	10	elasticity of substitution in $y_{j,t}$	Profit share of 11 %
θ	100	cost of price adjustment	Slope of Phillips curve, $\epsilon/\theta = 0.1$
Monetary policy			
ϕ^{π}	1.5	Taylor coefficient on inflation	Kaplan et al. (2018)
ϕ^{y}	0	Taylor coefficient on output	Kaplan et al. (2018)
Tax system			
τ^{l}	0.32	Labor income tax	Average tax on labor income and transfers
τ^{c}	0.22	Capital income tax	Average tax on capital income
τ^{w}	0.0085	Wealth tax	Wealth tax in 2015
τ^{b}	0.15	Inheritance tax	Inheritance tax in 2013
χ	0.7	Replacement rate of pension	

Parameters used to match data

	Value	Description	
ρ	0.0397	discount rate	calibrated
ψ_0	435	bequest shifter	calibrated



(a) Wealth (b) Wealth-to-human wealth ratio

Figure 5: Wealth and wealth-to-human-wealth ratio by age in data and model.

5 Results

5.1 Monetary transmission in steady state

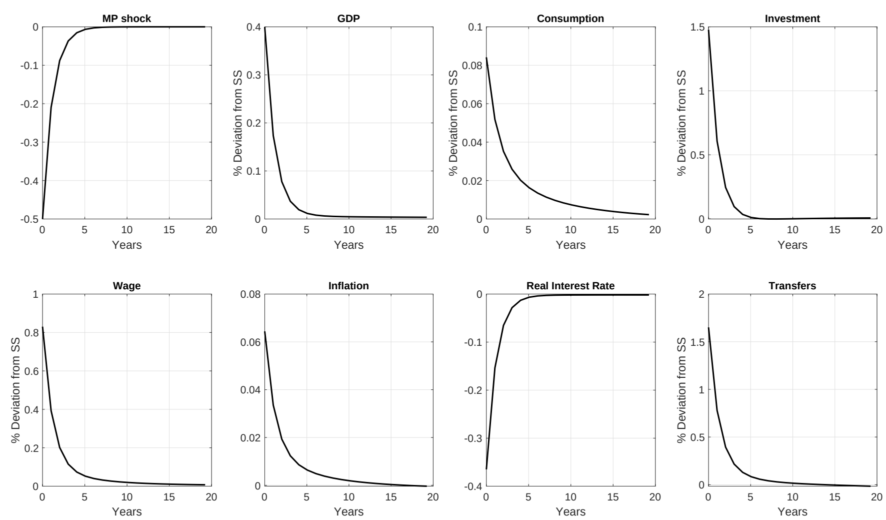


Figure 6: Aggregate responses to an expansionary 0.5 pp monetary policy shock. [Note: this figure is from old set up without capital and other labor specification.]

Aggregate responses to monetary policy.

Responses to monetary policy by age. Compare with Holm et al. (2019).

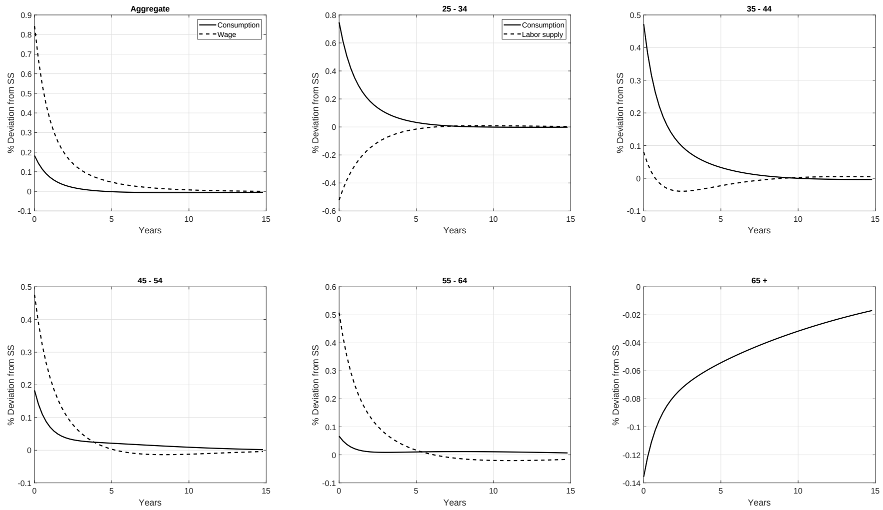


Figure 7: Consumption and labor supply responses to an expansionary 0.5 pp monetary policy shock by age groups. [Note: this figure is from old set up without capital and other labor specification.]

5.2 Effects of demographic transitions on monetary transmission

Higher life-expectancy.

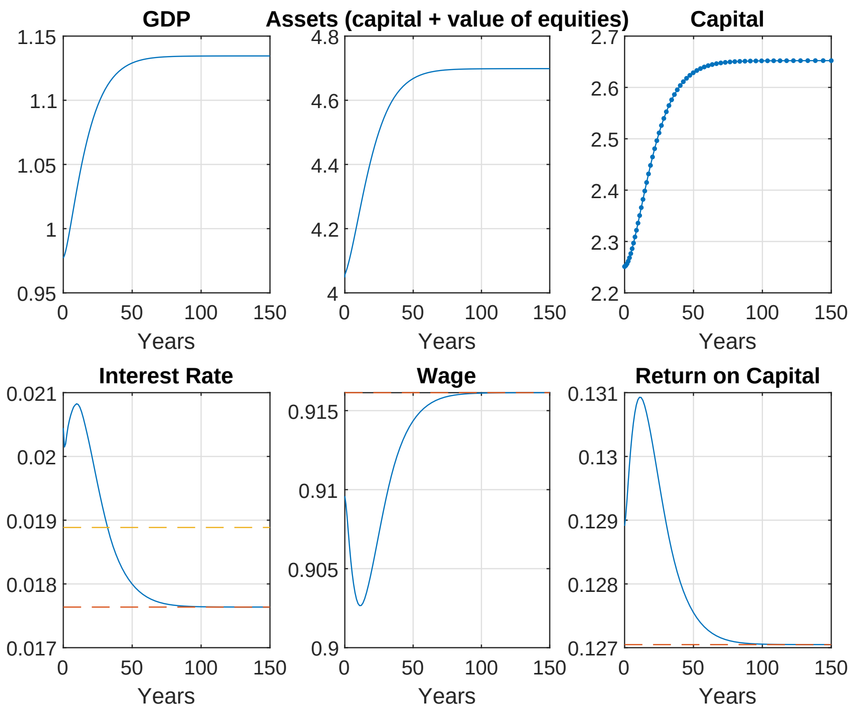


Figure 8: Transition paths from an increase in life expectancy.

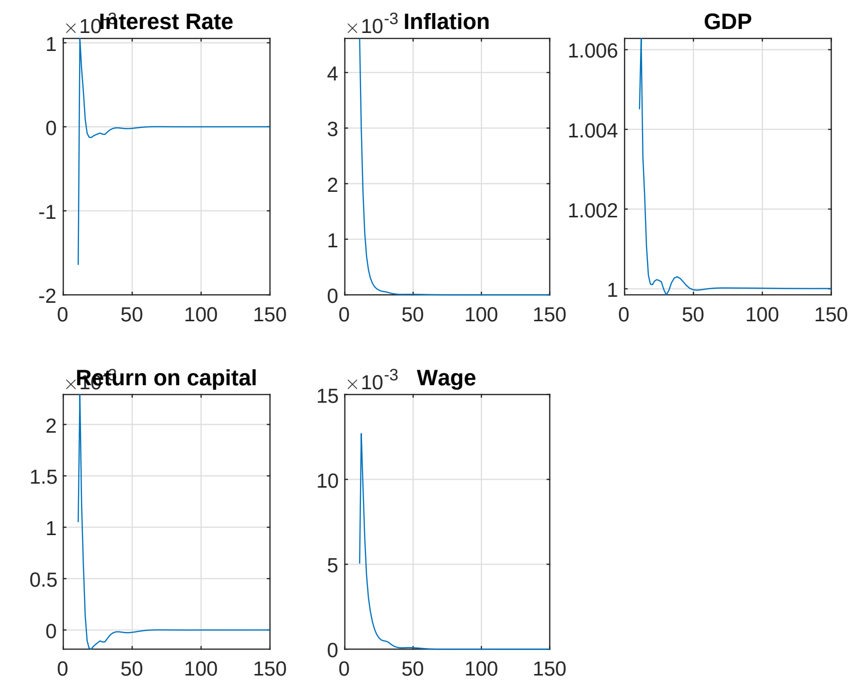


Figure 9: Monetary transmission after 10 years of transition path from increase in life expectancy.

Baby boom. [Early / Late, discuss in relation to the intuition section.]

5.3 Life expectancy + boomer shock

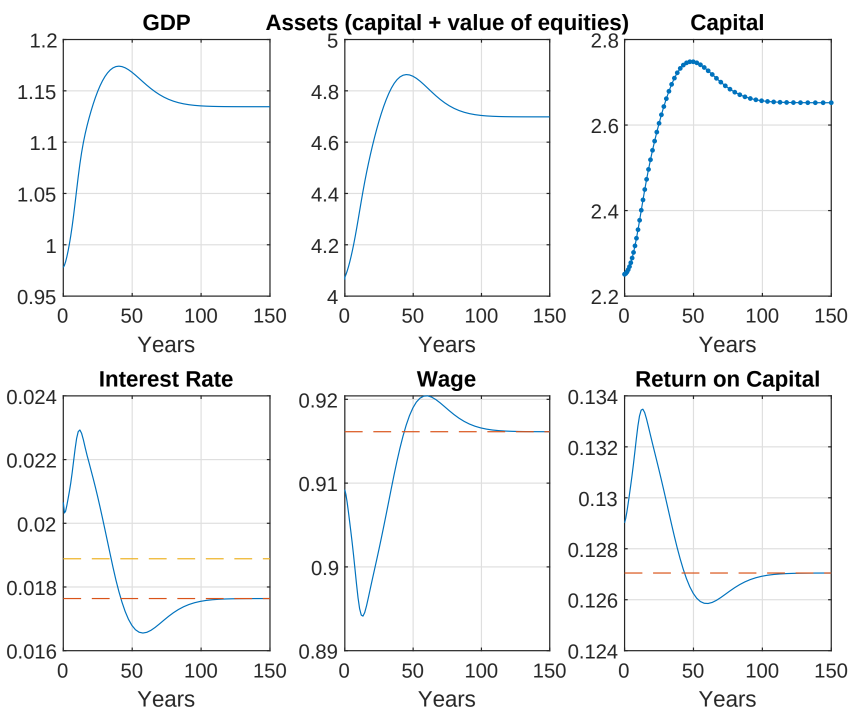


Figure 10: Transition paths from a joint increase in life expectancy and a baby boom.

6 Fiscal Reforms

7 Conclusion

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A Capital Production with Utilization

Given the optimization problem of

$$\max_{\iota,u} \int_t e^{\int -r_\tau d\tau} \int_t (r_t^k u_t k_t - \iota_t) dt \quad \text{s.t.}$$
$$\dot{k}_t = \iota_t(1-\phi(\nu_t)) - \delta^k k_t$$
$$\dot{\iota}_t = \nu_t$$

the corresponding Lagrangian is

$$\begin{aligned}\mathcal{L} &= \int_t e^{\int -r_\tau d\tau} \left[r_t^k k_t - \iota_t - \lambda_t \left(\dot{k}_t - \iota_t(1-\phi(\nu_t)) + \delta^k k_t \right) - \eta_t(\dot{\iota}_t - \nu_t) \right] dt \\ &= \int_t e^{\int -r_\tau d\tau} \left[r_t^k k - \iota_t - r_t \lambda_t k_t + \dot{\lambda}_t k_t + \lambda_t \iota_t (1-\phi(\nu_t)) - \lambda_t \delta^k k_t - r_t \eta_t \iota_t + \dot{\eta}_t \iota_t + \eta_t \nu_t \right] dt\end{aligned}$$

where λ_t and η_t are the Lagrange multipliers. The first-order conditions are

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial i} &= -1 + \lambda_t(1-\phi(\nu_t)) - r_t \eta_t + \dot{\eta}_t = 0 \\ \frac{\partial \mathcal{L}}{\partial k} &= r_k - r_t \lambda_t + \dot{\lambda}_t - \lambda_t \delta_k \\ \frac{\partial \mathcal{L}}{\partial \nu} &= -\lambda_t i_t \phi'(\nu_t) + \eta_t = 0\end{aligned}$$

which can be rearranged into

$$\begin{aligned}\nu^* &= \left(\phi'\right)^{-1} \left(\frac{\eta_t}{\lambda_t i_t} \right) \\ \dot{\eta} &= r_t \eta_t + 1 - \lambda_t (1-\phi(\nu)) \\ \dot{i} &= \nu^* \\ \dot{k} &= i(1-\phi(\nu)) - \delta_k k \\ \dot{\lambda} &= (r_t + \delta_k)\lambda_t - r_k \\ \text{profit} &= \lambda i_t (1-\phi(\nu)) - i_t\end{aligned}$$

Steady state. In the steady state, all the dynamics variables are zero, $\dot{x}_t = 0$

$$\begin{aligned}\nu_{ss} &= 0 \\ \eta_{ss} &= 0 \\ \lambda_{ss} &= 1 \\ r_{ss}^k &= r_{ss} + \delta_k \\ \iota_{ss} &= \delta^k k_{ss}\end{aligned}$$