

# Additivity simulations

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$$y = m(x_1, x_2) + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \sigma^2)$$

## 1 Scenario 1

$$m(x_1, x_2) = x_1 + 0.415 \cdot \exp(-20 \cdot (x_1^2)) + x_2^4$$

Setting	Method	B-splines	
		10%	5%
$L' = 6$	KS bootstrap	0.141	0.08
$N = 1000$	CvM bootstrap	0.096	0.048
$\sigma = 0.25$	AD bootstrap	0.123	0.067
$L' = 9$	KS bootstrap	0.127	0.076
$N = 1000$	CvM bootstrap	0.085	0.049
$\sigma = 0.25$	AD bootstrap	0.118	0.063
$L' = 12$	KS bootstrap	0.123	0.072
$N = 1000$	CvM bootstrap	0.0755	0.049
$\sigma = 0.25$	AD bootstrap	0.11	0.064

Table 1: Scenario 1

## 2 Scenario 2 (power 1)

$$m(x_1, x_2) = x_1 + 0.415x_2^4 \cdot \exp(-20 \cdot x_1^2)$$

Setting	Method	B-splines	
		10%	5%
$L' = 6$	KS bootstrap	0.204	0.108
$N = 1000$	CvM bootstrap	0.167	0.087
$\sigma = 0.25$	AD bootstrap	0.293	0.155
$L' = 9$	KS bootstrap	0.194	0.119
$N = 1000$	CvM bootstrap	0.179	0.087
$\sigma = 0.25$	AD bootstrap	0.332	0.173
$L' = 12$	KS bootstrap	0.201	0.12
$N = 1000$	CvM bootstrap	0.181	0.09
$\sigma = 0.25$	AD bootstrap	0.343	0.215
$L' = 6$	KS bootstrap	0.778	0.601
$N = 1000$	CvM bootstrap	0.798	0.55
$\sigma = 0.1$	AD bootstrap	0.986	0.924
$L' = 9$	KS bootstrap	0.826	0.677
$N = 1000$	CvM bootstrap	0.837	0.659
$\sigma = 0.1$	AD bootstrap	0.996	0.965
$L' = 12$	KS bootstrap	0.887	0.742
$N = 1000$	CvM bootstrap	0.887	0.721
$\sigma = 0.1$	AD bootstrap	0.997	0.981

Table 2: Scenario 2

### 3 Scenario 3 (power 2)

$$m(x_1, x_2) = (x_1 + 0.415 \exp(-20 \cdot x_1^2)) \cdot x_2^4$$

Setting	Method	B-splines	
		10%	5%
$L' = 6$	KS bootstrap	0.477	0.314
$N = 1000$	CvM bootstrap	0.492	0.316
$\sigma = 0.5$	AD bootstrap	0.585	0.405
$L' = 9$	KS bootstrap	0.49	0.362
$N = 1000$	CvM bootstrap	0.508	0.368
$\sigma = 0.5$	AD bootstrap	0.648	0.463
$L' = 12$	KS bootstrap	0.511	0.371
$N = 1000$	CvM bootstrap	0.543	0.0.383
$\sigma = 0.5$	AD bootstrap	0.666	0.525
$L' = 6$	KS bootstrap	0.982	0.933
$N = 1000$	CvM bootstrap	0.988	0.947
$\sigma = 0.25$	AD bootstrap	0.995	0.982
$L' = 9$	KS bootstrap	0.986	0.959
$N = 1000$	CvM bootstrap	0.991	0.961
$\sigma = 0.25$	AD bootstrap	0.997	0.988
$L' = 12$	KS bootstrap	0.988	0.967
$N = 1000$	CvM bootstrap	0.997	0.979
$\sigma = 0.25$	AD bootstrap	1	0.996

Table 3: Scenario 3