WINNING BY DEFAULT: WHY IS THERE SO LITTLE COMPETITION IN GOVERNMENT PROCUREMENT?

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Abstract. Government procurement contracts generally have a small number of participants, and it is not uncommon that only one seller is considered. We develop, identify, and estimate a principal-agent model in which the buyer chooses whether to solicit more than one bid and if so, how much effort to exert for a more competitive field; then she negotiates with prospective sellers, offers a menu of contracts, and chooses a winner and the payment schedule. Our structural estimates based on IT and telecommunications service contracts for the United States federal government show that the optimal contract yields a lower equilibrium number of bidders than a first-price sealed-bid auction. In counterfactual analyses, we show that if buyers are stripped of their discretion and their project-specific knowledge, the total cost of procurement would rise even if the average bidding costs per seller were halved.

1. Introduction

The market for the United States federal government procurement constitutes about 20% of the federal government spending. Despite its vast size, the extent of competition for a procurement contract is not very intense. Contracts generally have a small number of participants, and it is not uncommon that only one seller is considered. In this paper, we develop, identify, and estimate a procurement model to empirically quantify the factors determining the extent of competition observed in the data.

To conduct this analysis, we integrate three important institutional features of federal government procurement that have attracted attention from the literature, but have not

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been studied jointly. First, a procuring agency, or a buyer hereafter, chooses the extent and method by which a contract will be competed. The regulations permit contracting without providing for full and open competition under certain circumstances which are broadly defined, and the buyers have discretion in the extent of efforts and the scope of activities to attract more sellers and the solicitation procedure.

Second, a sealed-bid auction is not the dominant procedure to select a seller, depending on the nature of the products or services to be procured. An alternative solicitation procedure is by *negotiation*, through which the proposals submitted by sellers are evaluated, negotiated, and selected. After the request for proposals is posted, the sellers submit their proposals and discussions occur between the participating sellers and the buyer. During the discussions, the contract terms and prices are considered together.

Third, the final contract price may differ from the initial price due to two reasons: contingency plans specified in the initial contract and ex-post renegotiations. The federal regulations refer to the former price changes as *unilateral*, the latter as *bilateral*. The key difference between these two types of price changes is that the latter requires a new agreement between both parties while the former does not.

As documented in the literature review below, previous empirical work has analyzed each feature individually. However, to the best of our knowledge, this paper is the first paper that addresses all three features simultaneously. In our model, the extent of competition is endogenously determined by the buyer, who negotiates with prospective sellers to screen them by offering a menu of contracts (i.e.m payment schedules). We estimate the model using definitive contracts with a large size, from \$300,000 to \$5 million, for commercially unavailable IT and telecommunications service, awarded during FY 2004-2012. While the empirical results of this study may be specific to these contracts, the empirical framework in this paper is general, and can be readily applied to other similar types of contracts.

The estimates of the structural model show that negotiations substantially reduce the rent to low-cost sellers, and the optimal contract yields a lower equilibrium number of bidders than a first-price sealed-bid auction. We find that the expected price under the negotiated acquisition process with one seller is similar to that under the auction with two bidders. We estimate that using the auction format (in lieu of negotiation and optimal contracting) would increase the number of participants by 45 percent on average, but it would also increase the average contract price by 3 percent.

Although negotiations reduce the benefit of attracting an additional participant for a procurement project compared to sealed-bid auctions, the observed extent of competition could still be suboptimal from the public's point of view if the buyers' decisions on competition are affected by their private benefits. Our counterfactual analyses show that the effects of potential government capture on the contract prices are fairly limited. For example, mandating competition with at least two bids would decrease the average contract price by 4.5 percent, but such a cost reduction is offset by an increase in the bidding costs associated with a larger number of bids. Furthermore, if buyers are stripped of their discretion and their project-specific knowledge, the total cost of procurement would rise by 2.4 percent even if the average bid costs per seller were halved.

Our analysis is related to multiple strands of both theoretical and empirical literature on procurement. One strand of the literature studies endogenous determination of competition in procurement. Li and Zheng (2009), Krasnokutskaya and Seim (2011), and Athey, Coey and Levin (2013), amongst others, show the importance of allowing endogenous entry when assessing restrictive competition policies. Similar to Bandiera, Prat and Valletti (2009) and Coviello, Guglielmo and Spagnolo (2014), we focus on the choice of the buyer, not the prospective sellers, to determine the extent of competition.

Another strand of the literature studies nonstandard selection procedures, such as scoring auctions (Asker and Cantillon, 2010), multi-attribute auctions (Krasnokutskaya, Song and Tang, 2013), or negotiations (Bulow and Klemperer, 1996; Bajari, McMillan and Tadelis, 2009), where the price is not the only factor in selecting a seller. We consider an optimal direct revelation mechanism in a competitive environment, studied by Laffont and Tirole (1987), McAfee and McMillan (1987), and Riordan and Sappington (1987). We extend their models by allowing the buyer to choose the optimal extent of competition and to offer a menu of price schedules as a function of contract outcomes that are correlated with unobserved seller type. In this regard, our paper is related to the empirical literature on contracts, such as Wolak (1994), Chiappori and Salanie (2000), and Gagnepain, Ivaldi and Martimort (2013), to name a few.

The existing empirical literature, such as Bajari, Houghton and Tadelis (2014), has focused on bilateral price changes or renegotiation, but unilateral price changes have received little attention. We follow the approach in the literature that renegotiations may result from unanticipated modification in design or specification such as additional work, which is unrelated to private information of sellers. On the other hand, we allow that unilateral price changes may depend on contract outcomes related to sellers' private information. The empirical distinction of these two types of price changes is one of our contributions to the literature.

The rest of the paper is organized as follows. In Section 2, we delineate the institutional setting and the data, and present some empirical features that motivate our model, which is described in Section 3. The identification of the model follows in the next section. We present the estimation results in Section 5, and in the following section, we describe various counterfactual analyses and provide our empirical answers to the title question of the paper. Lastly, we conclude in Section 7.

2. Institutional Background and Data

The data is drawn from the Federal Procurement Data System–Next Generation. For each procurement project, we observe the solicitation procedure, the number of bids, the history of price and duration changes, product/service code, contracting agency (e.g., Department of Defense), and the location of contract performance. The purpose of this section is to describe the institutional background and the features of the data that are the most pertinent to our analysis.

2.1. **Definitive, Negotiated Contracts.** We study *definitive*, commercially unavailable contracts that were initiated during the fiscal years of 2004–2012. Definitive contracts have specified terms and conditions.[1] In this paper, we focus on the information technology (IT) and telecommunications service contracts to consider a relatively homogeneous set of contracts in terms of the nature of the provided service.[2] The IT and telecommunications services include, but are not limited to, IT strategy and architecture, programming, cyber security, data entry, backup, broadcasting, storage, and distribution, and telephone/Internet services.

We further restrict our attention to the negotiated contracts that were (i) paid for \$300,000–\$5 million, (ii) expected to take longer than 30 days to complete, (iii) completed before the end of FY 2014, (iv) not terminated prematurely, and (v) performed

[1] An alternative type of contract is a indefinite delivery, indefinite quantity contract. We focus on definitive contracts because a large fraction of the government procurement budget is allocated for these contracts. For example, in FY 2010, 94% (\$507 billion) of the total amount of money that the government was obliged to pay is for definitive contracts.

[2] Specifically, we consider the contracts with a product and service code of Category D3. The federal procurement data system requires that a product and service code be reported for each contract, and the codes are divided into three groups: research and development (R&D), service, and products. Among the service codes, there are 48 categories, and Category D3 is for IT and telecommunications' services.

within the the continental United States.[3] There are in total 2,203 such contracts in the data, costing the government $3.17 billion collectively, as shown in Table 1.

2.2. **Competition and Negotiation.** Table 1 presents summary statistics of the contracts in our sample by the extent of competition as specified in the data. The *full and open competition* is default in the acquisition process, but the Federal Acquisition Regulation (FAR) specifies the circumstances under which a procuring agency is allowed to limit competition.[4,5] More than two thirds of the contracts in the sample were not fully competed. The stated reasons in the data can be categorized into three cases: (i) unavailable for competition due to domestic statutes or international agreements, (ii) set-aside for small business concerns due to statutory requirements such as section 8(a) of the Small Business Act and the Historically Underutilized Business Zones Act of 1997, to name a few, and (iii) discretionary. As for the third category, the detailed reasons include the existence of limited rights in data, patent rights, copyrights, secret processes, or brand (54 percent in our data), follow-on contract (19 percent), urgency (8 percent), and other/unspecified (18 percent).

Even when a contract is competed, having only one bid is quite common (36 percent) and the median number of bids is 2. The number of bids can be affected by the efforts of the buyer to exchange information with potential bidders in advance, via pre-solicitation notices, draft requests for proposals, requests for information, industry conferences, public hearings, market research, and one-on-one meetings. Note that these efforts are costly to the buyer, because attracting and evaluating an additional bid or proposal incurs an extra administrative burden.[6]

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[3]The lower threshold of $300,000 is chosen because the contracts of an anticipated size less than the threshold are generally expected to be reserved for small business concerns. See FAR 13.003(b)(1) and FAR 2.101. Note that we use the actual payment, not the expected payment, for the threshold–this is because the anticipated payment amount does not appear in the data. We exclude the projects performed outside of the continental United States because the cost structure could be very different from those in our sample.

[4]See FAR 6.202–8 and FAR 6.302.

[5]According to FAR 1.6, authority and responsibility to contract for supplies and services are vested in the government agency head. The agency head may establish contracting activities and delegate broad authority to contracting officers. In our analysis, a *buyer* refers to a government agency head.

[6]Furthermore, there is anecdotal evidence that the risk of receiving a bid protest from losing bidders is not small. Federal Times reported in July 2013 on how bid protests are slowing down procurements. The article quoted Mary Davie, assistant commissioner of the Office of Integrated Technology Services at the General Services Administration: "We build time in our procurement now for protests. We know we are going to get protested."

Table 1. Competition for IT Service Contracts (FY 2004-2012)

Extent of competition	Obs.	Size ($M)		One Bid	Num. Bids	
		Mean	SD	Ratio	Mean	Median
No/limited competition	1,631	1.49	1.20	0.93	1.39	1
Unavailable for competition	796	1.67	1.19	0.98	1.06	1
Set-aside for small business	183	1.71	1.31	0.44	4.20	2
Not competed by discretion	652	1.20	1.12	1.00	1.00	1
Full and open competition	572	1.30	1.10	0.36	4.08	2
Negotiated acquisition	310	1.38	1.16	0.27	4.55	3
Sealed-bid auction	9	2.14	1.32	0.78	1.56	1
Other solicitation procedure	253	1.18	0.99	0.45	3.59	2
Total	2,203	1.44	1.17	0.78	2.09	1

Notes: This table provides summary statistics of all definitive, commercially unavailable IT and telecommunications service contracts of FY 2004-2012 with a large size ($0.3–5 million) and a long expected duration ($\geq$ 30 days). *Size* refers to the CPI-adjusted total amount of obligated money to the government per contract as of FY 2014, where CPI of December 2010 is 100.

Various procedures are used to solicit bids for competed contracts. We partition them into three: negotiated acquisition, sealed-bid auction, and others.[7] Table 1 shows that sealed-bid auctions are rarely used; only 9 contracts out of the 572 fully competed ones were auctioned. FAR 6.4 delineates the conditions under which sealed-bid auctions are required.[8] When these conditions are not met, the most prevalent solicitation method is negotiation. In a negotiated acquisition, a buyer issues a request for proposal, upon which interested sellers submit their proposals.[9] After receipt of proposals, the buyer negotiates price, project schedule, technical requirements, and contract type.

We study negotiated contracts, both noncompetitive and competitive. As for the former, we focus on those associated with discretion of a buyer. We exclude the non-competitive contracts related to the government statutes in order to study the role of discretion. This produces a final sample of 652 noncompetitive contracts and 310 competitive ones, shown in Table 1.

Based on the final sample, we find that the contracts awarded by the military-related agencies (the Departments of Defense, State, and Homeland Security) tend to be less

[7]Examples of the *other* solicitation procedures are architect-engineer, two-step, basic research, and simplified acquisition.

[8]These conditions are: (i) time permits the solicitation, submission, and evaluation of sealed bids, (ii) the award will be made on the basis of price and other price-related factors, (iii) it is not necessary to conduct discussions with the responding sellers about their bids, and (iv) there is a reasonable expectation of receiving more than one sealed bid.

[9]A typical request for proposal describes (i) the requirement, (ii) the anticipated terms and conditions that will apply to the contract, (iii) the information required to be in the proposal, and (iv) the proposal evaluation criteria.

Table 2. Competition and Military Contracts

	Noncompetitive	One Bid
Military agency	0.144***	0.120***
	(0.0417)	(0.0258)
Log(expected duration in days)	-0.0367	-0.0263**
	(0.0220)	(0.0126)
4-digit product/service code FE	Yes	Yes
State, year, month FE	Yes	Yes
N	962	962
R^2	0.182	0.193

Note: The dependent variables are (i) *Noncompetitive*, a dummy variable that indicates whether the contract was competitive; and (ii) *One Bid*, a dummy variable that indicates whether only one bid was considered (including noncompetitive and competitive). All contracts in the final sample are included; and the standard errors are clustered at the 4-digit product/service code level, and provided in parentheses; $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$. *Military agencies* include the Departments of Defense, State, and Homeland Security.

competitive than others, controlling for the product/service code, the location of the contract performance, and the period of the contract award, as can be seen in Table 2. This may be related to more pre-qualification requirements to work for these agencies, such as security clearance, than others. In our analysis, we control for these two types of government agencies, and study why military contracts are less competitive than other government contracts.

2.3. **Unilateral versus Bilateral Changes.** The contract price at the time of the award, *base price*, can be different from the actual price at the end of the contract, *final price*. We define the base price as the total amount of money that the government is obliged to pay at the beginning of the contract; the final price as the sum of all payments. The final price is typically higher than the base price, but not always.

Similarly, the duration of a contract may change ex-post. The *base* duration is the difference between the expected completion date and the starting date as in the initial contract record.[10] The *final* duration is the difference of the expected completion date of the last contract action record and the starting date of the base contract record. A *delay* is then the difference between the final duration and the base duration.

[10]In the data, there are three variables on the dates of each contract action: (i) effective date, which is the starting date for the contract action; (ii) current completion date, which is the scheduled completion date for the base contract and any options exercised at time of award; and (iii) ultimate completion date, which is the estimated or scheduled completion date including the base contract and all options. For the *expected completion date* in our analysis, we use the current completion date variable, and for the *starting date*, we use the effective date of the base contract.

Table 3. Price and Duration by Competition

	All	Noncompetitive	Competitive	
			1 Bid	2+ Bids
Number of Observations	962	652	83	227
Size ($M)	1.26 (1.13)	1.20 (1.12)	1.20 (1.11)	1.45 (1.17)
Contracts with price changes				
Unilateral	0.58	0.58	0.60	0.54
Bilateral	0.37	0.38	0.35	0.35
Amount of price changes ($M)				
Unilateral	0.36 (0.66)	0.35 (0.63)	0.35 (0.62)	0.39 (0.74)
Bilateral	0.18 (0.51)	0.19 (0.51)	0.18 (0.45)	0.16 (0.53)
Total duration (years)	2.16 (1.77)	2.08 (1.74)	2.55 (1.92)	2.24 (1.80)
Length of duration changes (years)				
Unilateral	0.69 (1.28)	0.63 (1.23)	0.96 (1.48)	0.77 (1.39)
Bilateral	0.30 (0.85)	0.32 (0.85)	0.27 (0.73)	0.27 (0.90)

Notes: All contracts in our final sample are included in this table, and standard deviations are provided in parentheses. *Size* refers to the CPI-adjusted total amount of obligated money to the government per contract as of FY 2014, where CPI of December 2010 is 100. *Bilateral* changes are associated with additional work, supplemental agreement for work within scope, change order, and definitization of letter contracts or change orders, and *unilateral* changes are associated with other reasons such as an administrative action, an exercise of an option, and close-out.

The federal regulations distinguish two types of ex-post price and duration changes: *bilateral* or *unilateral*. A bilateral change must be signed by both the seller and the buyer. They are used to make negotiated adjustments resulting from ex-post agreements modifying the terms of contracts. A unilateral change, on the other hand, requires the approval of the buyer alone, following the predetermined terms of a contract. Table 3 shows that both types of price changes are frequent and considerable in size. Among the contracts in the sample, 58 percent of them underwent unilateral ones, and 37 percent experienced bilateral ones. The average amount of ex-post price changes per contract is $544,454, which is the sum of $362,377 (unilateral) and $182,076 (bilateral).

The history of all changes in the price or duration of a contract, along with the reasons for each change, is reported in the data. The reasons for changes fall into one of twenty different categories, including additional work, supplemental agreement for work within scope, change order, definitization of letter contracts, and definitization of change orders. We consider price and duration changes due to these five reasons as *bilateral*, while the other remaining reasons, such as an administrative action, an exercise of an option, and close-out, as *unilateral*.[11]

[11]The full list of the remaining fifteen categories of reasons for modification are: C. Funding Only Action, E. Terminate for Default, F. Terminate for Convenience, G. Exercise an Option, J. Novation

Table 4. Relationship between Price Changes and Delays

	Unilateral price change		Bilateral price change	
	(1)	(2)	(3)	(4)
Unilateral delay[a]	0.792***	0.744***	-0.0226	-0.0558**
	(0.114)	(0.157)	(0.0383)	(0.0261)
Bilateral delay[b]	0.0560	0.177	0.981***	1.037***
	(0.107)	(0.117)	(0.174)	(0.145)
Unilateral delay[a] × Firm-fixed	-0.378***	-0.352**	0.0472	0.122***
	(0.138)	(0.170)	(0.0360)	(0.0394)
Bilateral delay[b] × Firm-fixed	-0.0263	-0.146	-0.278	-0.338
	(0.125)	(0.145)	(0.220)	(0.0359)
Base price and duration	No	Yes	No	Yes
4-digit product/service code FE	Yes	Yes	Yes	Yes
Agency, state, year, month FE	No	Yes	No	Yes
N	962	962	962	962
R^2	0.228	0.391	0.295	0.482

Note: The dependent variable in specifications (1) and (2) is the amount of the unilateral price change in thousand dollars; and the counterpart in specifications (3) and (4) is the amount of the bilateral price change in thousand dollars. a. *Unilateral delay* refers to the sum of all delays associated with reasons other than additional work, supplemental agreement for work within scope, change order, and definitization of letter contracts or changer orders. b. *Bilateral delay* refers to the sum of all delay associated with the aforementioned five reasons. All contracts in the final sample are included; standard errors are clustered at the 4-digit product/service code level, and provided in parentheses; $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

2.4. **Price and Duration.** Table 4 shows the statistical relationships between the ex-post price changes and delays. The results of the regressions of delays and other project attributes on the amount of the unilateral (bilateral) price changes are presented in Columns (1) and (2) (Columns (3) and (4)). There are a few notable trends manifest in the table. First, longer delays are associated larger ex-post price increases for both bilateral and unilateral changes. Second, bilateral price changes are more responsive to the length of the delays than unilateral ones. One-year delays due to unilateral actions are associated with an increase in the unilateral price change of about \$592,000-\$658,000, depending on the specifications; and one-year bilateral delays are associated with a much larger increase in the bilateral price change, about \$857,000-\$874,000. It seems as if contracts *reward* delays, which is at odds with a standard moral hazard model which would predict that tardiness is penalized.

Agreement, K. Close Out, M. Other Administrative Action, N. Legal Contract Cancellation, R. Re-representation of Non-Novated Merger/Acquisition, S. Change Procurement Instrument Identifier, T. Transfer Action, V. Vendor DUNS Change, W. Vendor Address Change, X. Terminate for Cause. Note that we drop contracts that were terminated prematurely or canceled.

Table 5. Competition and Price

	Log (Price)		Bilateral ΔPrice		Unilateral ΔPrice	
	(1)	(2)	(3)	(4)	(5)	(6)
Competitive	0.256***	0.0474	73.40*	40.01	75.40	22.54
	(0.0925)	(0.104)	(39.77)	(51.31)	(71.52)	(75.81)
Log (number of bids)		0.213***		34.06		53.91
		(0.056)		(52.54)		(74.42)
Base duration	Yes	Yes	Yes	Yes	Yes	Yes
4-digit product/service code FE	Yes	Yes	Yes	Yes	Yes	Yes
Agency, state, year, month FE	Yes	Yes	Yes	Yes	Yes	Yes
N	962	962	962	962	962	962
R^2	0.297	0.309	0.324	0.325	0.282	0.283

Note: The dependent variable for (1) and (2) is the log of the total contract price, the CPI-adjusted total amount of obligated money to the government per contract as of FY 2014. The dependent variable for (3) and (4) is the total ex-post unilateral price change in thousand dollars, and that for (5) and (6) is the total ex-post bilateral price change in the same unit. All contracts in the final sample are included; standard errors are clustered at the product/service code level, and provided in parentheses; $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Third, such price increases associated with delays tend to be slightly smaller for firm-fixed price contracts. It is notable, however, that even firm-fixed price contracts experience both types of price changes: 54 percent of firm-fixed contracts in our sample experienced a unilateral price change, and 34 percent of them experienced a bilateral price change.

2.5. **Competition and Price.** We find that greater competition is associated with higher contract prices. Table 3 shows that the average contract price (or size) does not decrease as competition intensifies from no competition to more than two bids. Furthermore, the frequency and the extent of the price and duration changes do not seem to vary with the extent of competition. These findings persist even after controlling for observed heterogeneity of each contract as shown in Table 5. This pattern is inconsistent with the equilibrium of standard auction models, which predicts that procurement price falls as the number of bidders increases.

Repeated interactions occur infrequently overall. Table 6 presents the summary statistics of the 962 contracts in our sample by the seller's history of winning contracts. To allow that the reputation of a seller may also be built from similar contracts to those in our sample, we look at all definitive contracts for IT and telecommunications services with a contract size greater than or equal to $300,000, initiated during the period of our study. There are in total 8,199 contracts, which were performed by 3,244 unique sellers

Table 6. Non-repeat vs. Repeat Sellers

	Num. Sellers	Num. Contracts	Competed	Num. Bids	Military Agencies	Size ($K)
Non-repeat sellers	284	284	0.33	2.38	0.35	1,160.6
	46.0%	29.5%	(0.03)	(0.54)	(0.03)	(65.0)
Repeat sellers (≤ 10)	282	405	0.28	1.69	0.35	1,257.7
	45.6%	42.1%	(0.02)	(0.10)	(0.02)	(54.4)
Repeat sellers (> 10)	52	273	0.37	2.57	0.32	1,358.0
	8.4%	28.4%	(0.03)	(0.40)	(0.03)	(73.9)
Total	618	962	0.32	2.14	0.34	1,256.7

Notes: We divide the contracts in our sample into three categories based on the seller's history of winning any of the definitive contracts for IT and telecommunications contracts with a contract size greater than or equal to $300,000 (8,199 contracts in total): *non-repeat sellers*, *repeat sellers* with 2–10 contracts, and those with more than 10. *Military Agencies* include the Departments of Defense, State, and Homeland Security. *Size* refers to the CPI-adjusted total amount of obligated money to the government per contract as of FY 2014, where CPI of December 2010 is 100. The numbers in parentheses are standard errors.

collectively.[12] The 962 contracts in our sample were performed by 618 sellers, and 52 (8 percent) of them won more than ten of the 8,199 definitive contracts. These sellers won about 28 percent of the contracts in the sample, but the contracts won by them are on average more likely to be competed and tend to have more bids. For example, Table 6 shows that 37 percent of the contracts in the sample that were performed by them resulted from a competitive solicitation while 33 percent of the contracts performed by the sellers who did not win any other IT and telecommunications service contracts were competed. These statistics do not support the hypothesis that the sellers who win multiple contracts face less competition. This implies that discretionary restrictions in competition may not be associated with dynamic incentive schemes.

Furthermore, we do not observe the identity of the losing participants, while we observe the number of them for each competitive contract. This limits our capacity to study the possibility of collusion and reputation. However, most sellers win only one contract during the period of study (Table 6), and the contracts in our sample tend to appear irregularly in terms of size and requirements. These features make it difficult for sellers to maintain a collusive relationship (Porter and Zona, 1993). Although the data is not suitable to study inter-temporal incentives, we partially reflect such possibility by allowing the buyer not to minimize the government cost only in the model.

[12]To identify a unique seller, we use its *parent company*'s DUNS Number. A DUNS number is a unique nine-digit identification number for each physical location of a business, and is required for all businesses to register with the federal government for contracts or grants. For example, there are 45 unique DUNS numbers that are associated with AT&T as a parent company, and using the parent company DUNS number, we treat them as one seller in Table 6.

3. Model

The institutional features and stylized facts highlighted in the previous section are guideposts for developing a model that explains why there is so little competition in government procurement. This section exposits our model.

3.1. **Total Cost to a Buyer.** The rules described in Section 2 surrounding solicitation delegate responsibility to the buyer for deciding whether she will permit competition or not. This is a choice variable in our model. We denote by η a nonpecuniary cost to the buyer of choosing a competitive process rather than simply designating a seller for the project. As described in Section 2.2, the implicit cost of a competitive process may increase when the most suitable seller is apparent based on patent, copyrights, or follow-on contracts or when the project is urgent. We do not assume that the buyer's nonpecuniary costs reflect social welfare, but do allow for the possibility that they might.

Should she permit competition, the second choice confronting the buyer in our model is the extent of soliciting extra bids. The various activities mentioned in Section 2.2 testify to the range of instruments available to the buyers for publicizing the request for proposals. In our model we denote the level of effort by $\lambda \in \mathcal{R}^{+}$, which is the arrival rate of a Poisson probability distribution for the number of bids exceeding one. The greater the number of bids, denoted by n , the higher the administrative costs of soliciting and processing bids, denoted by $\kappa(n)$. We assume that $\kappa(n)$ is positive, increasing and convex in n.

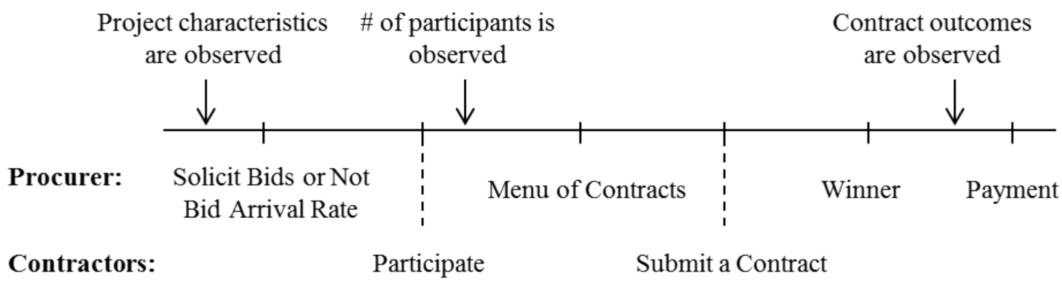
Section 2.3 distinguishes between the base price, which we now denote by p, and the final price, which we express as $p + \Delta$. Thus the total cost of the project to the buyer is:

$$\begin{array}{ll} p + \Delta + \kappa(n) + \eta & \text{if the project is competed with } n \text{ bids,} \\ p + \Delta + \kappa(1) & \text{if the project is not competed.} \end{array}$$

3.2. **Payoff to a Winning Seller.** In the model there are two types of sellers, low and high cost. The total cost of completing a given procurement project is the sum of the deterministic cost, $\alpha \in \mathcal{R}^{+}$ for a low-cost seller, and $\alpha + \beta > \alpha$ for a hight-cost seller, and ex-post stochastic cost change, denoted by $\epsilon \in \mathcal{R}$.

Liquidity concerns, or the cost of working capital, lead the winning seller to discount Δ, the ex-post price adjustment, and enlarge ϵ, unanticipated cost adjustments. Thus

Figure 1. Timeline of the Procurement Process in the Model



the payoff from contract (p, Δ) and realized value of ϵ is:

$$\begin{array}{ll} p + \psi(\Delta - \epsilon) - \alpha & \text{if the seller is low cost,} \\ p + \psi(\Delta - \epsilon) - \alpha - \beta & \text{if the seller is high cost,} \end{array}$$

where $\psi(\cdot)$ is a continuous real-valued function defined on $\mathcal{R}$, with $\psi(0) = 0$, $\psi'(0) = 1$, $\psi' > 0$, and $\psi'' < 0$.

3.3. **Timeline and Information.** Figure 1 represents the timeline of the model. When a project is realized, the buyer decides whether to hold a competitive solicitation procedure. If she chooses to hold a competitive solicitation, she also determines the level of effort to attract bids, λ, which stochastically determines n, the realized number of participating bidders.

The following procedure models the negotiation process. The buyer offers menu of contracts, each defined by its base price p plus a probability distribution over its ex-post price change Δ. She announces her preference ordering over the items on the menu. Then sellers simultaneously select a contract from the menu. Last, the buyer chooses a winner following her preference ordering.

After the project is initiated, the base price is paid. After the project is completed, contractible, stochastic outcomes are revealed to both parties, and the final payment is made. The outcomes consist of ex-post cost changes (ϵ) and delays unrelated to the cost changes, denoted by s. The key difference between the two outcomes is that ϵ is distributed independently of seller's type, while s is not. Let $\underline{F}(s)$ denote the cumulative distribution function of s for the low-cost sellers; the counterpart for the high-cost ones is $\overline{F}(s)$. We assume both functions are differentiable with densities $\underline{f}(s)$ and $\overline{f}(s)$, respectively.

The seller cost type is hidden information, known to the seller only. The procure knows the type distribution: a project specific parameter $\pi \in [0,1]$ denotes the proportion of the low-cost sellers in the population. We assume that s is informative but imperfect: $\underline{F}(s)$ and $\overline{F}(s)$ are defined on common support denoted by S, but $\underline{F}(s) \neq \overline{F}(s)$ for some $s \in S$.

3.4. **Ex-post Price Changes and Liquidity Constraints.** The description of the agreement between buyer and winning seller given in Section 2.3 distinguishes between unilateral ex-post price changes versus bilateral ones. Bilateral price changes may result from unanticipated modification in design or specification such as additional work, which is unrelated to private information of sellers. In our model the buyer is risk neutral, but the winning seller has liquidity concerns characterized by $\psi(\cdot)$. It is straightforward to show that since ϵ is revealed to both the buyer and the winning seller, it is optimal to fully insure him against ϵ on a cost-plus basis, so that bilateral changes in the price track this insurance against unanticipated cost changes observed by both parties.

Unilateral price changes may depend on contract outcomes related to sellers' private information on cost, such as the length of unilateral delays.[13] Netting out the insurance from bilateral costs changes, the resulting contract is a schedule determining the base price and unilateral price changes. Accordingly we define $q \equiv \Delta - \epsilon$ and express a contract on the menu in terms of a base price p and a probability distribution for q, recognizing that the total ex-post price adjustment is simply $\Delta = q + \epsilon$. This is consistent with the institutional feature, described in Section 2.3 that unilateral price changes arise following the initial contract, while bilateral changes occur via renegotiation.

We also assume there exists a maximal penalty the buyer can impose on sellers, denoted by $M \in \mathcal{R}^{-}$, such that $q \geq M$. In the theory the maximal penalty finesses situations where it might otherwise be optimal to impose an extremely steep penalty on a low-cost winner in the event of a very unlikely outcome for a high-cost seller to achieve an outcome very close to first best. In practice M reflects limited liability and bankruptcy constraints of sellers.

[13]The buyer often does not observe the cost to the sellers, but she observes the duration of a contract and keeps track of the changes of the expected duration over the period of a contract. The buyer may require the sellers to disclose in writing their cost accounting practices and to comply with the Cost Accounting Standards. However, only 10% of the contracts in our data have such requirements in place.

4. Equilibrium

It is optimal to offer a menu of two contracts: a preferred fixed price contract that depends on the number of bids; and a variable contract that does not depend on the number of bids but does depend on the informative contract outcome, s. Presented with an optimally designed contract menu, bidders truthfully reveal their cost type through their contract selection: low-cost sellers choose the fixed contract and high-cost ones choose the variable contract. The buyer selects a seller choosing the fixed contract if she can. In the case of a tie, the buyer randomly selects a winner. The extent of competition is chosen to minimize the expected total cost from using an optimal menu. This section characterizes and illustrates the optimal menu of contracts for given a number of bids, and then shows how the optimal extent of competition is derived.

4.1. **Contract Menu.**

Denote the number of sellers who bid by $n \in \{1, 2, \ldots\}$, the price of the fixed contract in the menu by $\underline{p}_n$, the base price of the variable contract by $\overline{p}$, and the variable component by $q(\cdot)$. Since bidders reveal their type through their choice of a contract in equilibrium, and the probability that a low-cost seller bids is $1-(1-\pi)^n$, the expected transfer from the buyer to the winning seller is:

$$[1-(1-\pi)^n]\,\underline{p}_n + (1-\pi)^n \left[\overline{p} + \int q(s)\overline{f}(s)ds\right]. \tag{1}$$

Appealing to the revelation principle, the buyer is limited to choosing $\underline{p}_n$ and $(\overline{p}, q(\cdot))$ subject to three constraints: (i) individual rationality (IR) conditions inducing both seller types to bid if presented with an opportunity to do so; (ii) incentive compatibility (IC) conditions inducing them to reveal their true type; and (iii) a limited liability condition restricting the range of $q(s)$. The IR constraints for the two types are:

$$\underline{p}_n \geq \alpha \quad \text{and} \quad \overline{p} + \int \psi[q(s)]\overline{f}(s)ds \geq \alpha + \beta.$$

To derive the IC constraint for the low-cost type, we first compute the winning probability if he chooses the fixed contract when the other bidders follow their equilibrium strategy, which is:

$$\underline{\phi}_n \equiv \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{\pi^k(1-\pi)^{n-1-k}}{k+1} = \frac{1}{n\pi} \sum_{j=1}^{n} \binom{n}{j} \pi^j (1-\pi)^{n-j} = \frac{1-(1-\pi)^n}{n\pi}.$$

If the bidder chooses the variable contract instead, the probability of winning is:

$$\overline{\phi}_n \equiv \frac{(1-\pi)^{n-1}}{n}.$$

Thus a low-cost seller prefers $\underline{p}_n$ to $(\overline{p}, q(s))$ if and only if:

$$\underline{\phi}_n\{\underline{p}_n - \alpha\} \geq \overline{\phi}_n\{\overline{p} + \int \psi[q(s)]\underline{f}(s)ds - \alpha\}.$$

The IC condition for the high-cost type can be similarly defined.

To characterize the optimal menu, four additional pieces of notation are helpful. Let $h : \mathcal{R}^+ \to \mathcal{R}$ denote the inverse of the first derivative of $\psi(q)$; that is $h[\psi'(q)] \equiv q$. Let $l(s) \equiv \underline{f}(s)/\overline{f}(s)$ denote the likelihood ratio of the probability density function of the low-cost type's s to the high-cost density. Define a threshold likelihood ratio associated with the limited liability condition by:

$$\tilde{l}(\pi, M) \equiv \frac{1}{\pi} - \frac{1-\pi}{\pi\psi'(M)}.$$

A lemma in the appendix proves the existence of a unique value of $\pi \in (0,1)$, which we denote by $\tilde{\pi}$, solving the equation:

$$\beta = \int_{l(s)<\tilde{l}(\pi,M)} \psi\left(h\left[\frac{1-\pi}{1-\pi l(s)}\right]\right)[\overline{f}(s) - \underline{f}(s)]ds - \psi(M)\int_{l(s)\geq\tilde{l}(\pi,M)}[\overline{f}(s) - \underline{f}(s)]ds.$$

Theorem 4.1. *The optimal menu of contracts consists of two contracts, a fixed contract and a variable contract. The price of the fixed contract in the menu, $\underline{p}_n$, is:*

$$\underline{p}_n = \alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left(\beta - \int \psi[q(s)]\left[1 - l(s)\right]\overline{f}(s)ds\right). \tag{2}$$

The base price of the variable contract, $\overline{p}$, is defined by:

$$\overline{p} = \alpha + \beta - \int \psi[q(s)]\overline{f}(s)ds, \tag{3}$$

and the price adjustment schedule, $q(\cdot)$, is characterized by:

$$q(s) = \begin{cases} h\left(\frac{1-\min\{\pi,\tilde{\pi}\}}{1-\min\{\pi,\tilde{\pi}\}l(s)}\right) & \text{if } l(s) \leq \tilde{l}(\min\{\pi,\tilde{\pi}\}, M), \\ M & \text{if } l(s) > \tilde{l}(\min\{\pi,\tilde{\pi}\}, M). \end{cases} \tag{4}$$

Intuitively, two contracts suffice to minimize costs because there are only two types of sellers. In the interior solution to this optimization problem $\underline{p}_n$ and $\overline{p}$ are found by solving two linear equations characterizing the IR constraint for the high-cost type and the IC constraint for the low-cost seller, both of which hold with equality, and minimizing the resulting expression for the buyer's cost with respect to q for each s. When $\pi > \tilde{\pi}$ it is optimal for the buyer to extract all the rent from the low cost seller by offering a fixed contract at α; i.e. the IR constraint for the low-cost type binds at optimum. When $\tilde{l}(\pi, M) > l(s)$ the buyer charges the maximal penalty to sellers selecting the variable contract to deter low-cost sellers.

Figure 2 illustrates the optimal menu of contracts for an example in which the distribution of s for the low-cost type is $Gamma(1, 1.5)$, the counterpart for the high-cost type is $Gamma(1, 2)$. We set the cost parameters to $\alpha = 1000$ and $\beta = 500$, the penalty M is high enough to be non-binding and the costs of liquidity are modeled as:

$$\psi(q) = -\psi_0 e^{-q/\psi_0} + \psi_0, \tag{5}$$

where $\psi_0 = 2500$.

The solid line in Panel (A) in represents the likelihood ratio $l(s)$, while the two dotted lines in Panel (A) in Figure 2 show $q(s)$, the price adjustment schedule of the variable contract for two values of the ratio of the low-cost type in the population, $\pi = 1/3$ and $\pi = 1/2$. Since $h(1) = 0$ and its derivative is negative, it follows from (4) that $q(s) \gtreqless 0$ as $l\,(s) \lesseqgtr 1$ with $q(s) = 0$ if and only if $l\,(s) = 1$. In words, if s is more likely to be generated by a high-cost seller than a low-cost one, then the variable component for s is positive, and vice-versa.

From (3) and (4), neither $\overline{p}$ and $q(\cdot)$ depend on n, and hence the variable price contract depends on π and $l(s)$ but not on the number of bidders and consequently the expected cost does not either. This is because the IR condition for the high-cost type is satisfied with equality. Consequently the expected cost of a variable contract does not depend on n. It is straightforward to prove that a higher π is associated with a steeper price adjustment schedule as illustrated in Panel (A).[14] Since a steeper price schedule is associated with a more volatile variable contract, in order to meet their certainty equivalent payment of $\alpha + \beta$ determined by the IR constraint, high-cost sellers must be paid a higher risk premium, defined as:

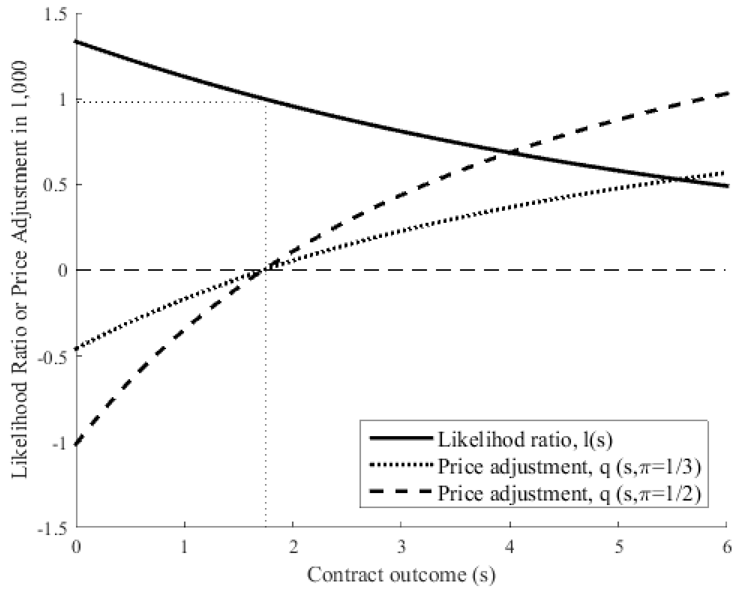
$$r \equiv \int \{q(s) - \psi[q(s)]\}\,\overline{f}(s)ds \tag{6}$$

As Panel (B) of Figure 2 shows, in our example the risk premium increases from 15 to 68 when π increases from 1/3 to 1/2 due to the steeper variable component.

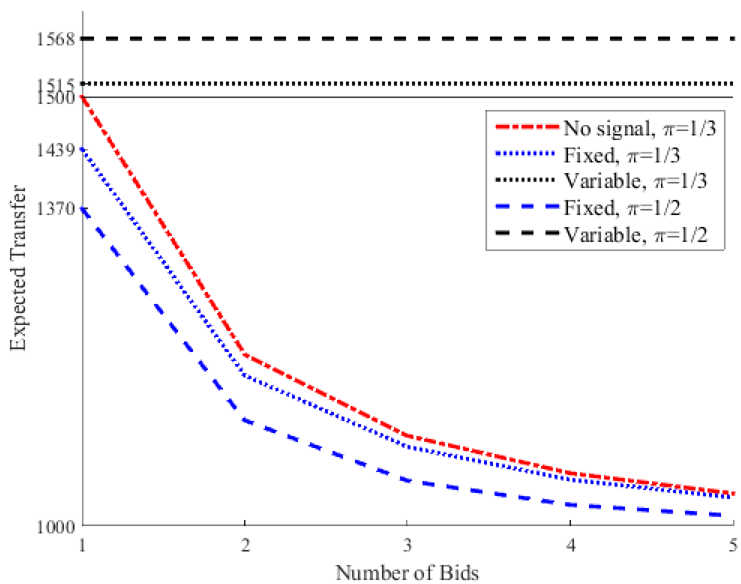
Panel (B) also depicts the expected transfer by contract type as a function of the number of bids for different combinations of the parameters. Differentiating (2), the fixed cost contract declines with the number of bidders, converging to α, almost achieved with only handful of bidders in the example depicted in Panel (B).

[14]In the interior solution $q(s)$ also depends on π so we can write $q(s;\pi)$ for $q(s)$. Partially differentiating $q(s;\pi)$ with respect to π establishes that if $l\,(s) < 1$ then $q(s;\pi)$ is increasing in π and if $l\,(s) > 1$ then $q(s;\pi)$ is decreasing in π. See Lemma 5.1 in the next section.

FIGURE 2. Optimal Contracts and Expected Transfer



(A) Optimal Price Adjustments as a Function of the Informative Contract Outcome



(B) Expected Transfer by Contract Type

Notes: The above graphs show the optimal variable contracts and the expected transfer by contract type for an example case with $\alpha = 1000$ and $\beta = 500$. The distribution of the informative contract outcome, s, for the low-cost sellers is $Gamma(1, 1.5)$ and the counterpart for the high-cost sellers is $Gamma(1, 2)$.

If s is uninformative, meaning $\underline{f}(s) = \overline{f}(s)$ for all $s \in S$, the optimal menu reduces to a menu of two fixed contracts. This menu consists of one preferred contract, $\alpha + \pi\left(1-\pi\right)^{n-1}\beta/[1-\left(1-\pi\right)^{n}]$ and a default contract of $\alpha+\beta$, which is selected only if no bidder chooses the preferred lower price contract. For the example when $\pi = 1/3$, these prices are 1500 and the highest downward sloping piecewise-linear line with the 1500 intercept (the red dotted line in the graph).

Using s and replacing fixed price of $\alpha+\beta$ with a variable price contract enables the buyer to reduce the fixed price contract offered to low-cost producers without violating their IC constraint. In the example when $\pi = 1/3$ the fixed price contract designed for the low-cost seller shifts down to 1439 when there is only one bidder. From (2), the expected amount extracted is:

$$\frac{\pi\left(1-\pi\right)^{n-1}}{1-\left(1-\pi\right)^{n}}\left(\int \psi[q(s)]\left[l\left(s\right)-1\right]\overline{f}(s)ds\right) \equiv \frac{\pi\left(1-\pi\right)^{n-1}}{1-\left(1-\pi\right)^{n}}\gamma \tag{7}$$

where γ is the expected amount of rent extracted from a low-cost seller when there is only one bid. As π increases the probability of settling with a high-cost seller falls, inducing the buyer to raise r and γ.

The buyer balances the gains of extracting rent from the low-cost type with the losses of the risk premium she pays to the high-cost type. Substituting (7) into (2), and (6) into (3) and the resulting expressions into (1) yields the expected transfer to a winning seller given n bids:

$$T(n) = \alpha + \left(1-\pi\right)^{n-1}\left[\beta - \pi\gamma + (1-\pi)r\right] \equiv \alpha + \left(1-\pi\right)^{n-1}\left[\beta+\Gamma\right] \tag{8}$$

where Γ is the expected net benefit of using the informative contract outcome where there is only one bidder. Equation (8) captures a basic intuition permeating through our analysis: in her quest to extract rent from the low-cost type when faced with the constraint of having to accept a high-cost type as a last resort, the buyer uses contract outcomes to discriminate between different cost types as a partial substitute for attracting more bidders.

4.2. **Extent of Competition.** Having solved the optimal menu of contracts and the expected transfer to a winning seller given a number of bids, we can now derive the expected total cost of competed procurement with effort λ, denoted by $U(\lambda)$. Recall that λ is the arrival rate of extra bids, denoted by j in the equation below, which follows a Poisson process.

$$\begin{aligned} U(\lambda) &\equiv \sum_{j=0}^{\infty} \frac{\lambda^j e^{-\lambda}}{j!} \left[T(j+1) + \kappa(j+1) \right] + \eta \\ &= \alpha + e^{-\lambda\pi}(\beta + \Gamma) + \mathbb{E}[\kappa(j+1)|\lambda] + \eta. \end{aligned}$$

The expected total cost of non-competed procurement, denoted by U_0, is:

$$U_0 = \alpha + \beta + \Gamma + \kappa(1).$$

The buyer chooses to hold a competitive solicitation if and only if $U_0 \geq \min_\lambda U(\lambda)$. Because $U(\cdot)$ is convex, it attains a global minimum at its unique stationary point, denoted by λ^*. If $\lambda^* \leq 0$, then the choice reduces to the sign of η. Alternatively if $\lambda^* > 0$, then a competitive solicitation is chosen if and only if:

$$\eta \leq (1 - e^{-\lambda^*\pi})(\beta + \Gamma) + \kappa(1) - \mathbb{E}[\kappa(j+1)|\lambda^*].$$

5. Identification

Our data comprises: whether the contract was competed, which we denote by setting $c = 1$, or not (setting $c = 0$); how many bids were tendered, n; whether the winning bid is a variable contract, denoted by setting $v = 1$, or not (setting $v = 0$); the informative contract outcome s for both contract types; the fixed price $\underline{p}_n$ if the winning contract is a fixed contract; and the base price $\overline{p}$ and the ex-post unilateral price change q if the winning contract is variable. We also observe price changes arising from bilateral price changes, ϵ. Thus an observation in the data set, which we denote by I, is defined as:

$$i \equiv \{c, n, v, s, v\overline{p}, vq, (1-v)\,\underline{p}_n, \epsilon\}.$$

We assume each $i \in I$ is generated by an independent draw of (π, s, ϵ).

Under the null hypothesis that π is constant conditional on observed project characteristics, this parameter is identified off the proportion of variable contracts $(1-\pi)^n$ for any given $n \in \{1, 2, \ldots\}$. Thus π is over-identified from variation in n. However, this null hypothesis is rejected. Accordingly we treat π as an unobserved project specific continuous variable filtering through the equilibrium and complicating identification.

After conditioning on the number of bidders, the contract type, and other observed characteristics, we allow costs to vary with π as well. Thus α, β, $\kappa(n)$ are now expressed as $\alpha\,(\pi)$, $\beta\,(\pi)$ and $\kappa\,(n, \pi)$. Similarly we make explicit the dependence of $\underline{p}_n$, $\overline{p}$, and $q\,(s)$ on π by writing $\underline{p}_n(\pi)$, $\overline{p}\,(\pi)$, and $q\,(s, \pi)$ respectively. The primitives of the econometric structure therefore comprise: the distribution of the proportion of the low-cost type,

$F_\pi(\cdot) \in \mathcal{F}_\pi$; the liquidity cost function, $\psi(\cdot) \in \Psi$; project costs, $\alpha(\cdot) \in \mathcal{A}$ and $\beta(\cdot) \in \mathcal{B}$; the distribution function of s, $\underline{F}(\cdot) \in \underline{\mathcal{F}}$ and $\overline{F}(\cdot) \in \overline{\mathcal{F}}$; the bid processing cost function $\kappa(n, \pi) \in \mathcal{K}$; and the distribution function of η, denoted by $F_\eta(\cdot) \in \mathcal{F}_\eta$. It is convenient to partition both the primitives and the identification analysis into those determining sellers' costs, namely:

$$\theta_1 \in \Theta_1 \equiv \mathcal{F}_\pi \times \Psi \times \mathcal{A} \times \mathcal{B} \times \underline{\mathcal{F}} \times \overline{\mathcal{F}}$$

and those determining the buyer's preferences, that is: $\theta_2 \in \Theta_2 \equiv \mathcal{F}_\eta \times \mathcal{K}$.

To preserve tractability, our empirical analysis makes the following two assumptions about the unobserved variables:

A1: $s \perp (\pi, \eta)$ and $\eta \perp \pi$.
A2: $F_\pi(\cdot)$ is strictly increasing for all $\pi \in \Pi$.

We also simplify the analysis by restricting the parameter space so that an interior solution invariably attains, meaning neither the IR constraint for the low-cost type nor the maximal penalty constraint bind. Therefore, instead of (4), the following first order condition characterizes $q(s, \pi)$ for any $(s, \pi) \in S \times \Pi$:

$$\psi'[q(s)]\,[1 - \pi l(s)] = 1 - \pi. \tag{9}$$

In addition we assume that as the proportion of the low-cost type increases, the expected cost of the project to either type declines.

A3: $\Pi \subset (0, \min\{\tilde{\pi}, 1\})$, and $l(s) \leq \widetilde{l}(\pi, M)$ for all $(s, \pi) \in S \times \Pi$.
A4: $\alpha(\cdot)$ and $\beta(\cdot)$ are nonincreasing in π.
A5: $\alpha(\cdot)$ and $\beta(\cdot)$ satisfy the following inequality for all π:

$$\frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} + \frac{h'[\psi'(M)]\,[\psi'(M)]^2[1 - \psi'(M)]}{\pi(1-\pi)} < 0.$$

The distributions of the informative contract outcome, $\underline{F}(\cdot)$ and $\overline{F}(\cdot)$, are directly identified off the data on the observed s of fixed and variable contracts. Identifying the remaining components of the model proceeds as follows. We begin with a preliminary lemma showing that the absolute value of the variable component is increasing in the proportion of low-cost sellers π, the base price is declining in π, and the fixed contract price declines in π for each number of bids, n.

Lemma 5.1. *(i) If A3 holds then* $\partial |q(s; \pi)| / \partial \pi > 0$. *(ii) If A3 and A5 hold then* $\partial \overline{p}(\pi) / \partial \pi < 0$. *(iii) If A3 and A4 hold then* $\partial \underline{p}_n(\pi) / \partial \pi < 0$ *for all* $n \in \{1, 2, \ldots\}$.

In the following, we assume that A1 through A5 are satisfied.

5.1. **Sellers' Costs.** Identification of $\psi(\cdot)$ is based on the rate at which the ex-post price change in the variable contract, q, decreases as the likelihood ratio l increases for any given π, i.e., an equation derived from totally differentiating (9) with respect to q and l. Since s only enters the optimal contract only through the likelihood ratio we can summarize outcomes of variable contracts in terms of $(\overline{p}, q, l)$ rather than $(\overline{p}, q, s)$, where $l = l(s)$. Since π is unobserved, (9) is redefined in terms of $\overline{p}$, which under our assumptions is monotone in π; we write $\pi = \pi(\overline{p})$ and rearrange the first order condition to define:

$$l^*(\overline{p}, q) \equiv \frac{1}{\pi(\overline{p})} - \frac{1 - \pi(\overline{p})}{\pi(\overline{p})\,\psi'(q)}. \tag{10}$$

Lemma 5.2. *For all variable contract outcomes $(\overline{p}, q, s)$ such that $l(s) = l$, $(\overline{p}, q, l) = (\overline{p}, q, l^*(q, \overline{p}))$ holds. For all $(\overline{p}, q)$ such that $l^*(\overline{p}, q) \neq 1$, the following equation holds:*

$$\psi''(q) = \left[\frac{1 - \psi'(q)}{1 - l^*(\overline{p}, q)}\right] \psi'(q) \frac{\partial l^*(\overline{p}, q)}{\partial q}. \tag{11}$$

We assume conditions on $\psi(\cdot)$ that guarantee $l^*(\cdot, \cdot)$ defined in (10) is uniformly Lipschitz continuous in q. Then the Picard–Lindelöf theorem applies, proving the differential equation (11) has a unique solution given the normalizing constant $\psi'(0) = 1$. It now follows that $\psi(\cdot)$ is solved from the other normalizing constant for the liquidity cost function, that $\psi(0) = 0$. Since $l^*(\cdot, \cdot)$ is identified off variable contract outcomes $(\overline{p}, q, l)$, so is $\psi(\cdot)$.

Since $\psi'(\cdot)$ is identified, π corresponding to each variable contract $(\overline{p}, q, s)$, denoted by $\pi_{q,s}$, is identified from the first order condition (9) by:

$$\pi_{q,s} \equiv \frac{1 - \psi'[q(s)]}{1 - l(s)\,\psi'[q(s)]}.$$

Noting that $\pi_{q,s}$ can be interpreted as a random draw from the $f_{\pi|c,n,v}(\cdot\,|c, n, 1)$ probability density, it now follows that $f_{\cdot|c,n,v}(\pi\,|c, n, 0)$ is identified. Let us define the odds ratio related to contract types conditional on (c, n) as:

$$\varphi_{c,n} \equiv \Pr(v = 1|c, n) / \Pr(v = 0|c, n).$$

We show in the appendix that $f_{\pi|c,n,v}(\pi\,|c, n, 0)$ is linked to $f_{\pi|c,n,v}(\pi\,|c, n, 1)$ by the conditional probability of a high-cost seller winning given π and n and $\varphi_{c,n}$ as follows:

$$f_{\pi|c,n,v}(\pi\,|c, n, 0) = \varphi_{c,n} \frac{[1 - (1 - \pi)^n]}{(1 - \pi)^n} f_{\pi|c,n,v}(\pi\,|c, n, 1),$$

for any $(\pi, c, n) \in \Pi \times \{0, 1\} \times \mathcal{N}$.

Lemma 5.3. $f_{\pi|c,n,v}(\cdot\,|c, n, v)$ *is identified for any* (c, n, v).

We identify project costs, $\alpha(\cdot)$ and $\beta(\cdot)$, by exploiting two identified mappings: (i) the mapping from (l,q) to $\overline{p}$, denoted by $\overline{p}^*(\cdot,\cdot)$; and (ii) the mapping from (π,c) to $\underline{p}_n$ for any given n, denoted by $\underline{p}_n^*(\cdot,\cdot)$. The existence and the derivation of the former mapping is guaranteed from (10), and the counterparts of the latter mapping from Lemma 5.1 (iii), showing that $\underline{p}_n$ is monotone (decreasing) in π. Let $G_{\underline{p}_n|c}(\cdot\,|c)$ denote the cumulative distribution function for $\underline{p}_n$ conditional on $c\in\{0,1\}$. Then the monotonicity guarantees that for any $(\pi,c)\in\Pi\times\{0,1\}$,

$$\underline{p}_n^*(\pi,c)\equiv G^{-1}_{\underline{p}_n|c}\left(\int_{\pi}^{\pi_{\max}} f_{\pi|c,n,v}\left(x\,|c,n,0\right)dx\middle|c\right).$$

To identify $\alpha(\pi)$ and $\beta(\pi)$ we substitute $\underline{p}_n^*(\pi,c)$ for $\underline{p}_n$ and $\overline{p}^*(q,s)$ for $\overline{p}$ in (2) and (3) for any s and any $n\in\{2,3,\ldots\}$, rearrange the resulting expressions and substitute out q using (9) to obtain:

$$\alpha(\pi) = \frac{1-(1-\pi)^n}{1-(1-\pi)^{n-1}}\underline{p}_n^*(\pi,c)-\frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^{n-1}}\underline{p}_1^*(\pi,c), \tag{12}$$

$$\beta(\pi) = \overline{p}^*\left(h\left[\frac{1-\pi}{1-\pi l(s)}\right],s\right)+\int\psi\left(h\left[\frac{1-\pi}{1-\pi l(x)}\right]\right)\overline{f}(x)\,dx-\alpha(\pi). \tag{13}$$

Taken together, the three lemmas in this section provide the critical arguments for establishing the identification of the seller side of the model.

Theorem 5.1. *$\theta_1\in\Theta_1$ is identified from $i\in I$.*

In fact, the seller sellerside of the model, θ_1, is over-identified. First, the optimal setting of $\underline{p}_1$ does not depend on whether competition is restricted or not, $\underline{p}_1^*(\pi,0)=\underline{p}_1^*(\pi,1)$. Second, varying $n\in\{2,3,\ldots\}$ in (12) yields testable restrictions. Third, setting $n=1$ in (2) and (3) and substituting $\overline{p}^*(q,s)$ for $\overline{p}$ and $\underline{p}_1^*(\pi,c)$ for $\underline{p}_1$ yields:

$$\int\psi\left(h\left[\frac{1-\pi}{1-\pi l(t)}\right]\right)\underline{f}(t)\,dt=\underline{p}_1^*(\pi,c)-\overline{p}^*\left[\frac{1-\pi}{1-\pi l(s)},s\right].$$

Varying π in the above equation provides further over-identifying information for $\psi(q)$.

5.2. **Buyer's Costs.** Note that Lemma 5.3 provides that the distribution of the number of bids conditional on π is identified. Given this, the bid cost function $\kappa(\cdot,\cdot)$ in a competitive solicitation is partially identified, and $F_\eta(\cdot)$ is partially identified from buyer choices, through variation in π transmitted through the identified costs to both parties.

To illustrate this point, consider a simple case where $\kappa(\cdot,\pi)$ is linear in n; i.e., $\kappa(n,\pi)=\kappa_0(\pi)n$. Under this specification, there is a closed-form solution for the the optimal arrival rate of extra bids for a project with π, denoted by $\lambda^*(\pi)$:

$$\lambda^*(\pi) = \max\left\{0, \frac{1}{\pi}\ln\left[\frac{\pi\left(\beta(\pi)+\Gamma(\pi)\right)}{\kappa_0(\pi)}\right]\right\},$$

where $\Gamma(\pi)$ is to express the dependence of Γ, defined in (8), on π. Then a competitive solicitation for a project with (π, η) is preferred if and only if

$$\eta \leq \Omega(\pi) \equiv \left(1 - e^{-\lambda^*(\pi)\pi}\right)\left[\beta(\pi)+\Gamma(\pi)\right] - \kappa_0(\pi)\lambda^*(\pi).$$

Note that $\lambda^*(\pi)$ is identified by:

$$\lambda^*(\pi) = \sum_{n=1}^{\infty} \frac{n f_{\pi,n|c}(\pi, n-1\,|1)}{f_{\pi|c}\,(\pi\,|1)}.$$

If $\lambda^*(\pi) = 0$, then only a lower bound of $\kappa_0(\pi)$ is identified; otherwise,

$$\kappa_0\,(\pi) = \pi[\beta(\pi)+\Gamma(\pi)]\exp\left[-\pi\lambda^*(\pi)\right].$$

By exploiting the variation in π, which is guaranteed by A1, we identify $F_\eta(\eta)$ for $\eta \in \{\Omega\,(\pi) : \lambda^*(\pi) > 0\}$ and $\eta = 0$. Specifically, if $\lambda^*(\pi) > 0$, then $\Pr(c = 0|\pi) = F_\eta[\Omega(\pi)]$; otherwise, $Pr(c = 0|\pi) = F_\eta(0)$.

In the estimated specification of the model, the bid processing cost is nonlinear in the number of bids:

$$\kappa(n, \pi) = (\kappa_1 + \kappa_2\pi)\,(n-1) + (\kappa_3 + \kappa_4\pi)\,(n-1)^2. \tag{14}$$

The first order condition for interior optimality simplifies to:

$$\kappa_1 + \pi\kappa_2 + \left[1 + 2\lambda^*\,(\pi)\right]\kappa_3 + \pi\left[1 + 2\lambda^*\,(\pi)\right]\kappa_4 = \pi\exp\left[-\pi\lambda^*(\pi)\right]\left[\beta\,(\pi)+\Gamma(\pi)\right].$$

Given the mappings $\lambda(\cdot)$, $\beta\,(\cdot)$ and $\Gamma(\cdot)$ from π, identified by our previous arguments, the vector of coefficients $(\kappa_1, \kappa_2, \kappa_3, \kappa_4)$ solves a system of linear equations for different values of π. The rank condition sufficient to identify $(\kappa_0, \kappa_1, \kappa_2, \kappa_3)$ can be checked directly in any given application.

6. Estimation Results

6.1. **Definition of the Variables.** Given the observed variables in the data, we define the variables of the model as follows. We define contract i is restricted ($c_i = 0$) if there was no or limited solicitation procedure and only one bidder was considered. The definition of the contract type, fixed ($v_i = 0$) or variable ($v_i = 1$), relies on whether or not there was a unilateral modification.[15] For fixed contracts, the difference in the final and the base prices is denoted by ϵ_i. For variable contracts, the base price is $\overline{p}_i$, the sum of all changes in the price related to unilateral modifications is q_i, and the difference between the final price and $\overline{p}_i + q_i$ is ϵ_i. We consider the length of delay related to unilateral modifications, divided by the base duration of the contract, as an observed informative contract outcome, s_i. If the final duration is shorter than the expected duration, $s_i = 0$.

6.2. **Parameterization and Estimation.** Following the identification argument, we can in principle nonparametrically estimate the model. However, given our sample size, non-parametric estimation is not feasible. Instead, we impose a parametric functional form to the primitives of the model as follows.

We assume that the distribution of π is $Beta(\alpha_\pi, \beta_\pi)$ on $(0, 1)$, $\psi(\cdot)$ takes the parametric form of (5) with $\psi_0 > 0$, and the maximal penalty is a constant, denoted by δ_0. The extra net cost of bypassing the formal solicitation procedure, η, is assumed to follow $N(\mu_\eta, \sigma_\eta)$ for non-military contracts and $N(\mu_\eta^m, \sigma_\eta^m)$ for military contracts. The parametric assumption for the bid costs $\kappa(\cdot, \cdot)$ is represented in (14).

The distribution of s for the low-cost sellers is:

$$\underline{F}_s(s) = \begin{cases} \underline{\rho} & \text{if } s = 0, \\ (1 - \underline{\rho})\underline{G}(s) & \text{if } s > 0, \end{cases}$$

where $\underline{G}(s)$ is the CDF of a Gamma distribution with shape parameter $\underline{\alpha}_s > 0$ and scale parameter $\underline{\beta}_s > 0$. The counterpart for the high-cost sellers, $\overline{F}_s(\cdot)$ is similarly assumed with $\overline{\rho}$ and $\overline{G}(s)$, where $\overline{G}(s)$ is the CDF of a gamma distribution with shape parameter $\overline{\alpha}_s$ and scale parameter $\overline{\beta}_s$.

We assume that the seller cost for the low-cost sellers, α, is linear in π, and the cost differential β is a fraction of α. We allow the costs to depend on whether the project is for military agencies, denoted by a binary variable, m.

$$\alpha(\pi, m) = \alpha_1 + \alpha_2 m + \alpha_3 \pi,$$

$$\beta(\pi, m) = [\beta_1(1 - m) + \beta_2 m]\, \alpha(\pi, m).$$

To employ the monotonicity result of Lemmas 4.2 and 4.3, we assume that $\alpha_2 \leq 0$.

[15]Note that our categorization of contract types does not coincide with the nomenclature of fixed or cost plus contracts. See Section 2.3 for our discussion on the firm-fixed price contracts frequently experiencing both types of ex-post price changes.

Table 7. Model Fit

	All		Non-military	
	Data	Model	Data	Model
Probability of				
No competition	0.6778	0.7374	0.6341	0.6876
One bid conditioning on competition	0.2677	0.3062	0.2500	0.2913
Up to two bids conditioning on competition	0.4258	0.5209	0.3664	0.5058
Up to five bids conditioning on competition	0.8516	0.9162	0.8534	0.9173
Fixed contracts conditioning on no competition	0.4156	0.4307	0.4254	0.4443
Fixed contracts conditioning on one bid	0.3976	0.4254	0.4655	0.5933
Fixed contracts conditioning on up to two bids	0.4091	0.4561	0.4310	0.4224
Fixed contracts conditioning on up to five bids	0.4621	0.5594	0.4000	0.4620
Average transfer ($M) of fixed contracts				
Conditioning on entry restriction	0.8256	0.7578	1.0459	1.0055
Conditioning on competition	1.1869	1.0863	2.4779	1.5879
Average transfer ($M) of variable contracts				
Conditioning on entry restriction	1.1397	1.0951	1.1835	1.0808
Conditioning on competition	1.2322	1.0153	1.0496	1.0640

We estimate the parameters of the model by an efficient simulated GMM estimator. The moment conditions are motivated by the identification argument: the joint probabilities regarding entry restrictions, number of bids, and contract type; some moments of the joint distribution of s and contract type; and the quantiles of contract prices conditional on contract type and number of bids. The Appendix defines the estimator and provides details on the estimation procedure.

6.3. **Parameter Estimates.** Using the estimated parameters, we simulate the data and calculate some key moments displayed in Table 7. The table shows the actual and predicted moments regarding the extent of competition, the contract types, and the contract prices. The predicted moments are based on a simulation of 5,000 observations using the estimated parameters. The overall fit of the simulated data to the actual data is good in both the level and the trend.

The parameter estimates are presented in Table 8. The standard errors are based on the asymptotic variance. Our estimates of the average direct cost of employing the formal solicitation procedure, or alternatively, the average direct benefits from no competition (μ_η for non-military contracts and μ_η^m for military ones) are [$11,150, $29,850] for non-military contracts and [$18,778, $48,422] for military contracts, where the numbers in brackets indicate 95 percent confidence intervals. The direct benefit from no competition for military contracts is greater than the counterpart for nonmilitary ones

Table 8. Parameter Estimates

Description	Parameter	Estimate	Parameter	Estimate
Low project cost (in \$M)	α_1	1.7178	α_2	0.0267
		(0.0698)		(0.0251)
	α_3	-2.1699		
		(0.1536)		
Project cost differential	β_1	0.3069	β_2	0.2583
		(0.0325)		(0.0333)
Bid cost (in \$M)	κ_1	-0.0105	κ_2	0.1530
		(0.0023)		(0.0261)
	κ_3	-0.0009	κ_4	0.0124
		(0.0004)		(0.0030)
Maximum penalty (in \$M)	δ_0	-0.0100		
		(0.1695)		
Liquidity cost (in \$M)	ψ_0	10.4020		
		(7.5244)		
Distribution of π	α_π	6.7445	β_π	10.7797
		(0.4396)		(0.8946)
Distribution of η	μ_η	-0.0205	σ_η	0.0150
		(0.0048)		(0.0042)
	μ_η^m	-0.0336	σ_η^m	0.0237
		(0.0076)		(0.0033)
Distribution of s	$\underline{\rho}$	0.9167	$\overline{\rho}$	0.3520
		(0.0155)		(0.0221)
	$\underline{\alpha}_s$	1.7828	$\underline{\beta}_s$	0.2881
		(0.4853)		(0.0715)
	$\overline{\alpha}_s$	0.8240	$\overline{\beta}_s$	2.8523
		(0.1739)		(0.7617)

Notes: Numbers in parentheses are asymptotic standard errors.

by \$13,100 with the asymptotic standard error being \$6,892. This difference is statistically significant at the 6 percent level, and it can partially explain why there is less competition for military contracts than non-military ones.

The average direct benefit from no formal solicitation conditional on no competition, \$30,545 (with the standard error being \$6,079) per noncompetitive contract, is larger than the unconditional average. However, such benefit seems to be dwarfed by the bid costs conditional on competition, \$61,662 (with the standard error being \$22,005) per competitive contract. The direct benefit associated with no competition may result from saving administrative costs, selecting favorites on the basis of unverifiable quality, and potentially receiving bribery. Assuming that bid costs are mostly administrative such as potential costs from bid protests by losing bidders and the opportunity cost of having to hire a potentially low-quality seller by broadening the pool of qualified

Table 9. Decomposition of Procurement Costs

	Non-military		Military	
(in $K)	Estimate	SE	Estimate	SE
Low project cost, $\alpha(\pi, m)$	884.1	40.4	910.8	45.8
Project cost difference, $\beta(\pi, m)$	271.3	32.1	235.3	32.1
Bid cost with two bidders, $\kappa(\pi, 2)$	52.1	8.9	52.1	8.9
Average formal solicitation cost, $\mathbb{E}(\eta\|m)$	20.5	4.8	33.6	7.6

Notes: Given that the project costs and bid costs vary with π, the numbers in this table are evaluated at the median value of π, 0.384. Furthermore, since the bid solicitation and processing cost depends on the total number of bids, we provide the cost when there are two bids.

sellers, we conclude that the role of corruption at the buyer level, if it at all exists, seems to be very limited in the federal government contracting.[16]

To compare the estimates of these buyers' costs with the sellers' project costs, Table 9 provides each cost, evaluated at the median value of the ratio of the low-cost sellers (π), 0.384.[17] The estimates indicate that absent cost shocks (ϵ), it takes \$0.88 (\$0.91) million for low-cost sellers to complete a procurement project for non-military (military) agencies, while it takes \$0.27 (\$0.24) million more for high-cost sellers. Compared to the cost differential, which is the maximum cost savings for a buyer in terms of transfer to sellers, the bid cost for one more bid is 19 percent (22 percent) and the direct competition cost is 8 percent (14 percent) for non-military (military) contracts.

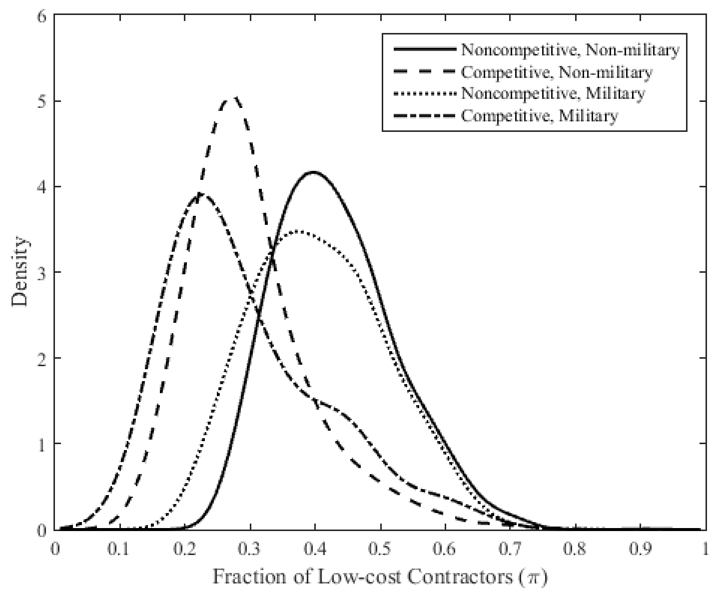
Note that the cost differential between low-cost and high-cost sellers is slightly higher for non-military contracts than for military ones. In terms of the parameter estimates of β_0 (for non-military contracts) and β_1 (for military ones), both of which measure the ratio of the cost differential to the cost of low-cost sellers for the respective contract category, the difference is 4.8 percent with the asymptotic standard error being 2.3 percent, statistically significant at 4 percent level. This provides another source that explains why there is less competition for military contracts than non-military ones.

Figure 3 shows the estimated distribution of the ratio of the low-cost sellers (π) conditional on the solicitation process (competitive or not). Based on the simulation, the median value of π is 0.384, the 5^{th} percentile π is 0.210, and the 95^{th} percentile is 0.587. A notable trend in the graph is that contracts with a smaller π tend to competitive than others. This trend implies that the bid costs are relatively small for projects with small π while they are high enough for projects with large π to be

[16]Note that we focus on the discretion of buyers, so we do not study noncompetitive contracts associated with small business concerns or appropriations or other existing law.

[17]Note that we allow the project costs and bid costs vary with the ratio of the low-cost sellers (π), which we treat as the contract-level unobserved heterogeneity.

Figure 3. Estimated π Distribution Conditional on Competition



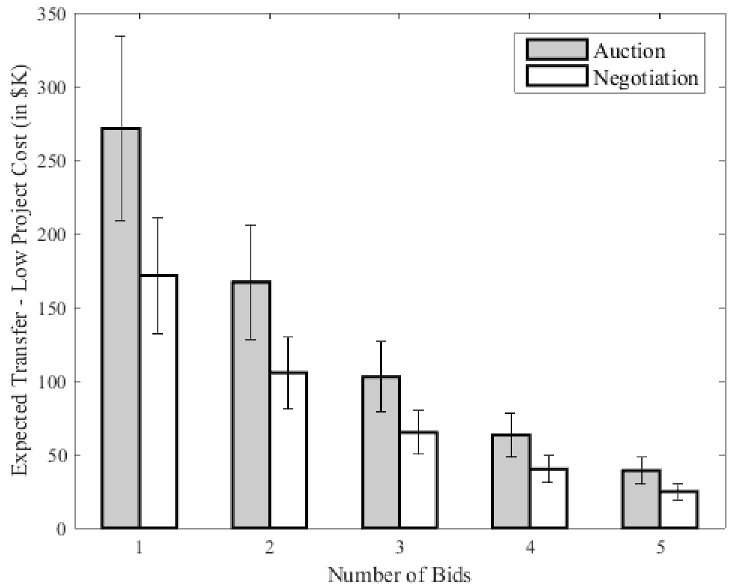
assigned to no competition. It also illustrates the importance of accounting for the endogeneity of the extent of competition.

7. Why So Little Competition?

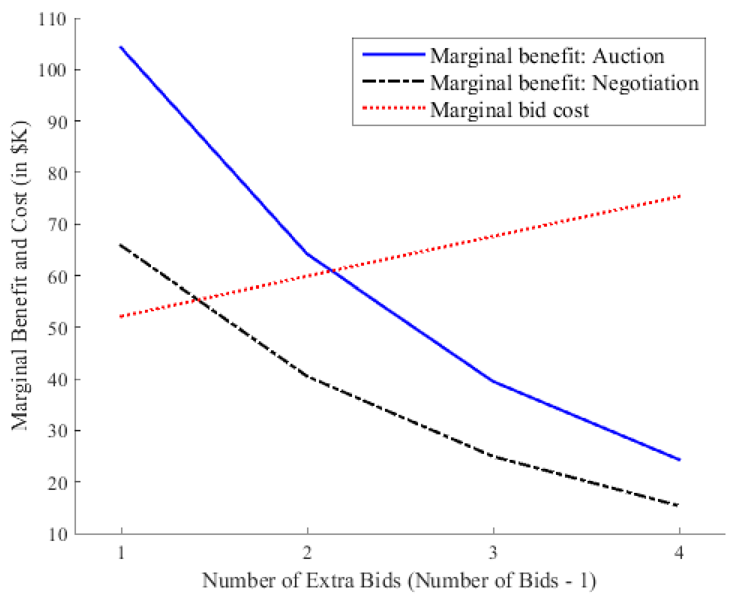
7.1. **Contract Negotiations and Informational Rent.** Our model shows that negotiations on contract terms help the buyer extract the informational rent from low-cost sellers. In Figure 4, we illustrate the cost savings from contract negotiations as opposed to a first-price sealed-bid auction, based on our estimated parameters. We consider a non-military contract with the median ratio of low-cost sellers ($\pi = 0.382$). In Panel (A), we show the expected transfer conditional on a given number of bids, under contract negotiations and first-price sealed bid auctions, respectively. Notable that the expected transfer under contract negotiations with one bidder is similar to the counterpart under a first-price auction with two bidders, and this pattern persists in the range of the number of bidders in the graph. This explains why little competition does not necessarily mean inefficiency in the allocation of procurement contracts. As can be seen in Panel (B), the optimal number of bids conditional on competition, is less than 2.5 two under negotiations and over 3 under auctions.

To quantify the extent to which contract negotiations reduce the procurement cost on average, we consider a policy where negotiations are not allowed and first-price

FIGURE 4. The Value of Contract Negotiations



(A) Expected Transfer



(B) Marginal Benefit and Cost of an Extra Bid

Notes: In Panel (A), we show the expected transfer for a non-military contract with the median ratio of low-cost sellers ($\pi = 0.382$) conditional on a given number of bids, under contract negotiations and first-price sealed bid auctions, respectively. The error bars represent 95 percent confidence intervals. In Panel (B), the decreases in the expected transfer in response to an additional bid under contract negotiations and first-price sealed bid auctions are represented as *marginal benefit* of an extra bid. More bids are associated with more bid costs, and the additional bid solicitation and processing cost is represented as *marginal bid cost*.

Table 10. Effects of Policies to Reduce Buyers' Discretion

	Base	(1)	(2)	(3)	(4)	(5)
Average number of bids	1.48	+0.67	+0.31	+0.81	+0.60	+2.03
Average costs ($K)						
Transfer (A)	1027.38	+32.08	-16.76	-45.73	+27.64	-38.64
Bid costs (B)	16.19	+31.44	+14.31	+50.12	+45.98	+63.23
Average *total* costs ($K)						
$A + B$	1043.57	+63.51	-2.45	+4.38	+73.62	+24.24
$A + B+$ Solicitation cost	1021.04	+70.21	+20.08	+26.91	+96.14	+47.11

Note: Under Policy 1, negotiations are not allowed and first-price auctions are the only available competitive solicitation procedure. All remaining alternative policies considered in this table mandate competitive solicitation. In Policy 2, there are no restrictions on the number of bids, while in Policy 3, at least two bids are required. In both policies, the buyer provides the optimal menu of contracts conditional on each project type, π. On the other hand, the last two policies require a first-price sealed-bid auction. Under Policy 4, the constrained optimal level of effort, $\lambda^* = 1.06$ (or on average 2.06 bids), is made for all contracts, regardless of π. The last policy is identical to the fourth policy except that we assume that the bid processing costs are halved. Given the new bid processing costs, we derive the constrained optimal level of effort for bids ($\lambda^* = 2.48$, or on average 3.52 bids) and simulate the expected contract outcomes.

auctions are the only available competitive solicitation procedure. The effects of this policy are presented in column (1) of Table 10. We find that the optimal number of bids under first-price auctions are larger than the counterpart under negotiations by 0.67 bids on average. Despite more bids, the expected transfer under first-price auctions is higher than its counterpart by $\$32,084$ per project on average, which amounts to 3.1 percent of the per-project expected transfer under the existing policy, \$1.03 million. Because more bids are costly to the buyers, the total cost amounts to an increase of 6.2-6.8 percent of the expected transfer, or \$63,520-70,210 per project, depending on the interpretation of the direct cost of using a competitive solicitation procedure as opposed to no competition. In other words, contract negotiations reduce the government cost by at least 6 percent of the contract prices paid to the sellers.

7.2. **Manipulating Competition.** Although negotiations reduce the informational rent and accordingly the benefit of more bids, the current extent of competition could be suboptimal from the public point of view. The buyers choose whether to solicit bids or contract with a default seller and determine the amount of effort to attract and process more bids. When making such decisions, they may receive private benefits from restricting the entry of competing sellers. To quantify the effects of their discretion on the extent of competition for contracts and procurement costs, we consider various policies that limit their discretion.

In Policy (2) in Table 10, competitive solicitation is mandatory for all projects, but the buyers have discretion in choosing the amount of effort for more bids and the menu of contracts. This mandatory competition certainly decreases the expected transfer to the sellers, on average by $\$16,757$ per project. This decrease amounts to 1.6 percent of the per-project expected transfer without the policy change. This amount provides the upper bound of the potential inefficiency due to the agency problem associated with buyers as agents of the federal government, assuming that (i) the choice of the winner given bidders is efficient, and (ii) the bid processing costs are mostly administrative and reducing such costs given the number of bidders are beyond the scope of a single buyer's responsibility.

It is notable that most of the cost savings associated with more competition are offset by an increase of bid processing costs. Hence the eventual cost savings are $\$2,448$ per project, or 0.2 percent of the expected transfer without the policy change. If the cost of implementing a competitive solicitation procedure is born by the government, which is estimated to be $\$21,955$ per project, then this mandatory competition policy is suboptimal and increase the total procurement cost by 2 percent of the expected transfer without the policy change.

In Policy (3) in Table 10, we limit the discretion of the buyers on the extent of competition further, in the sense that at least two bids are required. Under this requirement, the buyer's problem is to decide how much *extra* effort to attract and process more bids; if they exert no extra efforts, they will end up having only two bids, and any extra efforts will lead to more than two bids on average. Let $\tilde{\lambda}$ denote the average *extra* number of bids and $\tilde{U}(\pi,\tilde{\lambda})$ denote the expected total cost for the project type π:

$$\tilde{U}(\pi,\tilde{\lambda}) = \sum_{n=0}^{\infty} \frac{\tilde{\lambda} e^{-\tilde{\lambda}}}{n!} \left\{ T(\pi, n+2) + \kappa(\pi, n+2) \right\},$$

where $T(\pi, n)$ denotes the expected transfer under optimal contracts with type π and n bids and $\kappa(\pi, n)$ denotes the bid processing cost. Buyers are allowed to choose the optimal $\tilde{\lambda}$ for each project type π. As expected, this policy will increase the total procurement cost, and such an increase, 0.4 to 2.6 percent of the expected transfer depending on the interpretation of the competitive solicitation cost, is larger than that of Policy (2) because it limits the discretion further. Although more bids lead to cheaper contract prices, the associated cost savings are eclipsed by the bid processing costs.

7.3. **The Value of Discretion.** We find that tying the buyers' hands and mandating more competition can assure lower contract prices by 1.6-4.4 percent, but these cost savings are mostly offset by an increase in bid processing costs and the competitive procedure costs. These costs could be reduced by implementing a centralized, first-price auction where a certain, identical amount of efforts to attract and process bids is mandated for all projects. However, the trade-off is that the officers cannot use their information on the project type, π in our model. In this section, we quantify this trade-off based on our estimates.

In Table 10, we consider two scenarios regarding this one-size-fits-all policy and the simulation results are provided in the columns (4) and (5). In scenario (4), we assume that the bid processing costs are unchanged, while in scenario (5), we assume that these costs are halved to allow the possibility that using auctions, as opposed to negotiations, could lead to a decrease in the bid searching and processing costs.

Because the amount of bid searching and processing efforts is identical across projects, we consider the optimal effort under this constraint. Let $\check{U}(\pi, \lambda)$ denote the expected total cost for the project type π when a first-price auction is implemented and the average number of bids is λ. The constrained optimal λ^* is:

$$\lambda^* = \arg\min_{\lambda \geq 0} \int \check{U}(v, \lambda) f_\pi(v) dv.$$

For each of the two policies, we calculate the constrained optimal λ^*: 1.06 or on average 2.06 bids when the bid costs are unchanged and 2.48 or on average 3.48 bids when the bid costs are halved.

The decrease in the bid searching and processing costs leads to a large reduction in the average contract price, as can be seen in column (5) in Table 10. However, the total procurement cost is still larger than the counterpart in the current policy by \$24,582-\$47,106 per project on average, or 2.4-4.6 percent of the expected contract price under the current policy. This is because the reduction in the per-bidder bid processing costs leads to a large increase in the number of bids. This result highlights that allowing discretion to the buyers to set the level of competition and negotiate with the sellers using their knowledge of the project type reduces the government costs, even if there could be rent-seeking behaviors.

8. Conclusion

This paper is an empirical analysis of government procurement auctions that seeks to explain the low number of bidders and the features of their winning contracts by developing, identifying, and estimating a principal-agent model. In our model the buyer chooses whether to solicit bids, if so how much effort to exert to attract bidders, and propose a menu of contracts in order to select and reach an agreement with the winner.

In our model the primary reason why buyers use their discretion to restrict competition or to expend little effort to attract greater numbers of bidders is that the buyer can extract much of the rent from low cost sellers by offering a menu of contracts that induces bidders to reveal private information about their own costs. In equilibrium, high-cost sellers win a procurement project only if no low-cost sellers bid, and low-cost sellers surrender a substantial portion of the informational rent they would have garnered in an alternative first price sealed-bid auction.

Our empirical results show that the cost of soliciting, identifying, and processing an additional bid is relatively large compared to the cost savings from competition. Roughly speaking, the costs of soliciting one extra bidder can be saved by designing an optimal contract that uses contract outcomes to extract rent from low-cost producers. Our results are robust to the objectives of the buyer: in the extreme case when all the non-pecuniary benefits associated with choosing to restrict competition are wasted, or equivalently accrue to a buyer who has no social weight, we estimate that negotiated agreements would still be more efficient than a first price sealed-bid auction. We conclude that giving discretion to buyers to extract rent from sellers with private information can explain why there are so few bidders in government auctions.

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Appendix A. Proofs

A.1. Proof of Theorem 4.1. The following five lemmas collectively prove Theorem 4.1. The first lemma shows that variable contracts are only offered in conjunction with fixed contracts, not by themselves.

Lemma 8.1. *The equilibrium contract menu includes a fixed contract.*

Proof. The proof is by a contradiction argument. Suppose to the contrary that every contract on the menu is a variable contract. Denote by $\{p, q(s)\}$ one of the contracts on the menu. There are three cases to consider.

First, suppose $\overline{\mathbb{E}}\{\psi[q(s)]\} \equiv \int \psi[q(s)]\overline{f}(s)ds > \underline{\mathbb{E}}\{\psi[q(s)]\} \equiv \int \psi[q(s)]\underline{f}(s)ds$. Then, the buyer can offer an additional, fixed contract of $p' = p + \overline{\mathbb{E}}\{\psi[q(s)]\}$. The high-cost type would accept the contract, but the low-cost type will not. By strict concavity of $\psi(\cdot)$, we have $\overline{\mathbb{E}}\{\psi[q(s)]\} < \overline{\mathbb{E}}\{q(s)\}$. Therefore, the expected payoff of the buyer increases when the high-cost type accepts the fixed contract with any positive probability.

Second, suppose $\overline{\mathbb{E}}\{\psi[q(s)]\} < \underline{\mathbb{E}}\{\psi[q(s)]\}$. The buyer can offer an additional, fixed contract of $p' = p + \underline{\mathbb{E}}\{\psi[q(s)]\}$. The low-cost type would accept the contract, but the high-cost type will not. Since $\underline{\mathbb{E}}\{\psi[q(s)]\} < \underline{\mathbb{E}}\{q(s)\}$, the expected payoff of the buyer increases when the low-cost type to accept the new contract with any positive probability.

Lastly, suppose $\overline{\mathbb{E}}\{\psi[q(s)]\} = \underline{\mathbb{E}}\{\psi[q(s)]\}$. The buyer can offer instead an fixed contract of $p' = p + \underline{\mathbb{E}}\{\psi[q(s)]\}$. Both cost types would accept the contract. Since $\underline{\mathbb{E}}\{\psi[q(s)]\} < \underline{\mathbb{E}}\{q(s)\}$ the expected payoff of the buyer increases when either or both cost types to accept the new contract with any positive probability. □

Given Lemma 8.1, an optimal menu of contracts includes at least one fixed contract. We show that when there is only one fixed contract in the optimal menu, high-cost type never selects the fixed contract.

Lemma 8.2. *If a menu of contracts including a single fixed contract is offered, it is optimal to induce the high-cost type to select a fixed contract with probability zero.*

Proof. Suppose not; i.e., the high-cost type selects a fixed contract with positive probability. Then the fixed-price must be $\alpha + \beta$ so that the IR constraint for the high-cost type is satisfied. Notice that the IR constraint for the low-cost type is satisfied with strict inequality; otherwise, the low-cost type will select the fixed contract instead. Given this, the buyer's problem boils down to choosing the terms of the variable contract, p and $q(\cdot)$, to minimize expected total transfer:

$$\phi_n \{p + \underline{\mathbb{E}}[q(s)]\} + (1 - \phi_n)(\alpha + \beta), \tag{15}$$

where ϕ_n is the probability that a seller that chooses a variable contract becomes a winner given n participants, subject to the IC constraint for the low-cost type:

$$\underline{\phi}_n (p - \alpha + \underline{\mathbb{E}}\{\psi[q(s)]\}) \geq \overline{\phi}_n \beta, \tag{16}$$

where $\underline{\phi}_n$ and $\overline{\phi}_n$ denote the subjective probability that a seller that chooses the variable contract (or the fixed contract) wins, respectively. Since the IR constraint is satisfied with strict inequality, the IC constraint must bind. Solving for p when (16) holds with equality,

$$p = \frac{\overline{\phi}_n}{\underline{\phi}_n}\beta + \alpha - \mathbb{E}\left\{\psi\left[q\left(s\right)\right]\right\}.$$

Substituting for p in (15) and simplifying we obtain:

$$\alpha + \phi_n \mathbb{E}\left\{q\left(s\right) - \psi\left[q\left(s\right)\right]\right\} + \beta\left(1 - \phi_n + \phi_n \frac{\overline{\phi}_n}{\underline{\phi}_n}\right),$$

which is minimized with respect to $q\left(s\right)$ for each $s \in S$. Since $q\left(s\right) \gtrless \psi\left[q\left(s\right)\right]$ when $q\left(s\right) \lesseqgtr 0$ and $q\left(0\right) = \psi\left[q\left(0\right)\right]$, $q\left(s\right) = 0$ for all $s \in S$. This leads to a contradiction. $\square$

Lemma 8.3. *If two fixed contracts are offered, then it is optimal to offer $\alpha + \beta$ and $\alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\beta$, where the first priority going to sellers submitting the latter and the second priority to those who submit the former.*

Proof. To ensure the project is undertaken, the buyer must meet the IR constraint of the high-cost type, and the cheapest fixed price contract meeting this constraint is $\alpha + \beta$. To meet the IC constraint of the low-cost type, the buyer must offer terms that are at least as profitable as $\overline{\phi}_n\beta$, which are the expected profits to the low-cost type from selecting $\alpha + \beta$. Letting p denote any price that solves the IC constraint:

$$\underline{\phi}_n \left(p - \alpha\right) \geq \overline{\phi}_n\beta.$$

Appealing to the definition of $\underline{\phi}_n$ and $\overline{\phi}_n$, this inequality can be expressed as:

$$p - \alpha \geq \frac{\pi\left(1-\pi\right)^{n-1}}{1-\left(1-\pi\right)^n}\beta,$$

which is minimized by setting $p = \alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\beta$. $\square$

This leaves us two generic possibilities on the optimal menu of contracts. Either two fixed contracts comprise the optimal menu, or it consists of a fixed contract designed for the low-cost type and one or more variable contracts designed for the high-cost type. If the contract outcome s was of very high quality and the fraction of the low-cost type is high, we might expect the buyer to extract all the rent from the low-cost type, and limit his losses to the risk premium paid to the high-cost type. As proved in Theorem 4.1, this is indeed the case.

In preparation for that theorem we now define the expression:

$$H(\pi) \equiv \int_{l(s)<\tilde{l}(\pi,M)} \psi\left(h\left[\frac{1-\pi}{1-\pi l(s)}\right]\right)[\overline{f}(s)-\underline{f}(s)]ds+\psi(M)\int_{l(s)\geq\tilde{l}(\pi,M)}[\overline{f}(s)-\underline{f}(s)]ds,$$

where the cutoff $\tilde{l}(\pi, M)$ is defined in the text. Lemma 8.4 shows that if s is informative, then the expression $\beta - H(\pi)$ has a unique root, denoted by $\tilde{\pi}$.

Lemma 8.4. *A unique probability denoted by $\tilde{\pi} > 0$ solves $\beta = H(\pi)$.*

Proof. Note from equation (8) that $H(0) = 0$. We show that $H(\cdot)$ is strictly increasing in π. To see this, we rewrite $H(\pi)$ by

$$H(\pi) = \int \tilde{H}(\pi, s)\overline{f}(s)ds,$$

where $\tilde{H}(\pi, s)$ is defined by

$$\tilde{H}(\pi, s) = \begin{cases} \psi\left(h\left[\frac{1-\pi}{1-\pi l(s)}\right]\right)\{1-l(s)\} & \text{if } l(s) < \tilde{l}(\pi, M), \\ \psi(M)\{1-l(s)\} & \text{otherwise.} \end{cases}$$

When $l(s) \geq \tilde{l}(\pi, M)$, $\partial\tilde{H}(\pi, s)/\partial\pi = 0$. Otherwise, we can see that

$$\frac{\partial}{\partial\pi}\tilde{H}(\pi, s) = -\psi'\left(h\left[\frac{1-\pi}{1-\pi l(s)}\right]\right)h'\left(\frac{1-\pi}{1-\pi l(s)}\right)\frac{[l(s)-1]^2}{[1-\pi l(s)]^2} > 0.$$

Therefore, $H(\pi)$ is strictly increasing in π. □

Now we characterize the optimal menu of contracts when it consists of one fixed contract and one variable contract.

Lemma 8.5. *Suppose the optimal menu of contracts consists of one fixed contract, denoted by $\underline{p}_n$, and one variable contract, denoted by $\{\overline{p}, q(\cdot)\}$. The ex-post price adjustment schedule, $q(\cdot)$, is:*

$$q(s) = \begin{cases} h\left(\frac{1-\min\{\pi,\tilde{\pi}\}}{1-\min\{\pi,\tilde{\pi}\}l(s)}\right) & \text{if } l(s) \leq \tilde{l}(\min\{\pi, \tilde{\pi}\}, M), \\ M & \text{if } l(s) > \tilde{l}(\min\{\pi, \tilde{\pi}\}, M). \end{cases} \tag{17}$$

The base price of the variable contract is:

$$\overline{p} = \alpha + \beta - \int \psi[q(s)]\overline{f}(s)ds. \tag{18}$$

The price of the fixed contract is:

$$\underline{p}_n = \alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left[\beta - \int \psi[q(s)]\left\{\overline{f}(\mathbf{s}) - \underline{f}(s)\right\}ds\right]. \tag{19}$$

Proof. The buyer designs a menu of two contracts that minimizes the expected transfer:

$$[1-(1-\pi)^n]\,\underline{p}_n + (1-\pi)^n\left[\overline{p} + \overline{\mathbb{E}}(q(s))\right]. \tag{20}$$

subject to the constraints that the low-cost type selects the fixed contract, the high-cost type selects the variable contract, and the limited liability constraint is satisfied. A necessary condition of the optimal menu is that the IR constraint the high-cost type holds with equality (otherwise the base price $\overline{p}$ could be further reduced, reducing the price and strengthening the IC constraint for the low-cost type). Solving for $\overline{p}$ yields (18). The IC constraint for the low-cost type is:

$$\underline{\phi}_n(\underline{p}_n - \alpha) \geq \overline{\phi}_n\{\overline{p} + \underline{\mathbb{E}}[\psi(q)] - \alpha\},$$

Substituting for $\overline{p}$ using equation (18) and using the definitions of $\overline{\phi}_n$ and $\underline{\phi}_n$:

$$\underline{p}_n \geq \alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left(\beta - \int \psi[q(s)]\{1-l(s)\}\overline{f}(s)ds\right). \tag{21}$$

Note that the IC for the high-cost type will be satisfied with strict inequality at the optimum by Lemma **??**. Therefore, at least one of the two remaining constraints, IR and IC for the low-cost type, must bind. Otherwise, the price of the fixed contract could be reduced, earning the buyer higher revenue. This leads us to consider the following three cases separately.

Case 1: IC binds but IR does not Solving for $\underline{p}_n$ from the IC constraint, and substituting the resulting expressions for $\underline{p}_n$ and $\overline{p}$, obtained from equations (18) and (19), into the expected total cost for the buyer or (20), we obtain:

$$\begin{aligned}
&[1-(1-\pi)^n]\left\{\alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left(\beta - \int \psi[\overline{q}(s)]\{1-l(s)\}\overline{f}(s)ds\right)\right\} \\
&+(1-\pi)^n\left\{\alpha + \beta + \int \{q(s) - \psi[q(s)]\}\overline{f}(s)ds\right\} \\
= \;& \alpha + (1-\pi)^{n-1}\left(\beta + \int \left[(1-\pi)(q(s) - \psi[q(s)]) - \pi\psi[\overline{q}(s)]\{1-l(s)\}\right]\overline{f}(s)ds\right).
\end{aligned}$$

The (scaled) Lagrangian for the cost minimization problem can now be expressed as:

$$L = \int \left[(1-\pi)\{q(s) - \psi[q(s)]\} - \pi\psi[\overline{q}(s)]\{1-l(s)\} - \varkappa_1(s)\,(q(s) - M)\right]\overline{f}(s)ds,$$

where $\varkappa_1(s) \geq 0$ denotes the Kuhn Tucker multiplier for the linear constraint $q(s) \geq M$. The first order condition for $q(s)$ is:

$$(1-\pi)\,(1-\psi'[q(s)]) - \pi\psi'[q(s)]\{1-l(s)\} - \varkappa_1(s) = 0.$$

Rearranging terms we obtain:

$$\psi'\left[q(s)\right]=\frac{1-\pi-\varkappa_1\left(s\right)}{1-\pi l(s)}. \tag{22}$$

If $l(s)<\tilde{l}(\pi,M)$, then $q(s)=h\left[\frac{1-\pi}{1-\pi l(s)}\right]>M$ and $\varkappa_1\left(s\right)=0$ solve equation (22). If $l(s)\geq\tilde{l}(\pi,M)$, then $\varkappa_1\left(s\right)>0$ and $q(s)=M$ solve the equation.

Case 2: IR binds but IC does not When IR binds, $\underline{p}_n=\alpha$. Substituting for $\underline{p}_n$ and $\overline{p}$, using equation (18), the expected total transfer (20) simplifies to:

$$\alpha+(1-\pi)^n\left\{\beta+\int\left\{q(s)-\psi[q(s)]\right\}\overline{f}(s)ds\right\}.$$

Substituting for $\underline{p}_n$ in inequality (21) yields:

$$\beta\leq\int\psi[q(s)]\{1-l(s)\}\overline{f}(s)ds. \tag{23}$$

Notice the solution to this problem depends on neither π nor n. If IR binds but IC does not, then the first order condition for the Kuhn Tucker formulation is:

$$1-\psi'[q(s)]=\varkappa_1\left(s\right).$$

If $q(s)>M$, then the complementary slackness condition requires $\varkappa_1\left(s\right)=0$, and hence $1=\psi'[q(s)]$ or $q(s)=0$. Therefore, either $q(s)=M$, and the marginal benefit of imposing a harsher penalty would exceed its cost were it not for the limited liability constraint, or $q(s)=0$. Let us define S_M as the set of contract outcomes such that $q(s)=M$ and let μ denote $\Pr(s\in S_M)$. Note that for any $\mu\in[0,1]$, both IR constraints and the IC constraint for the high-cost type are satisfied. The total expected transfer can now be written as

$$\alpha+(1-\pi)^n\left\{\beta+[M-\psi(M)]\mu\right\}.$$

Notice that the above transfer is increasing in μ, while $\mu=0$ does not satisfy the IC condition for the low-cost type, or inequality (23). This implies that when both IR constraints bind, the IC for the low-cost type must bind.

Case 3: Both IR and IC bind If (23) holds with equality, the (scaled) Lagrangian for the minimization problem can be written as:

$$\begin{aligned} L &= \int \left(q(s) - \psi[q(s)]\right) \overline{f}(s)ds - \int \varkappa_1(s)\left[q(s) - M\right] \overline{f}(s)ds \\ &\quad + \varkappa_2 \left\{ \beta - \int \psi[q(s)][1 - l(s)]\overline{f}(s)ds \right\}. \end{aligned}$$

The first order condition with respect to $q(s)$ is:

$$1 - \psi'[q(s)] - \varkappa_1(s) - \varkappa_2 \psi'[q(s)][1 - l(s)] = 0.$$

This can be written as:

$$\psi'[q(s)] = \frac{1 - \varkappa_1(s)}{1 + \varkappa_2[1 - l(s)]}. \tag{24}$$

Substituting for $\varkappa_2 = \tilde{\pi}/(1 - \tilde{\pi})$ in equation (24) follows that the solution for $q(s)$ in this case can be obtained as in (17).

We have ruled out the second case, implying that the IC for the low-cost type always binds at the optimum. The IR constraint for the low-cost type does not always bind, i.e.

$$\beta - \int \psi[q(s)][1 - l(s)]\overline{f}(s)ds = \beta - H(\pi) \leq 0,$$

where $H(\pi)$ is defined in equation (8). As shown in Lemma 8.4, $\beta - H(0) = \beta > 0$, $H(\cdot)$ is increasing in π, and there always exists a unique root of $\beta - H(\pi)$, $\tilde{\pi}$. Therefore, if $\pi < \tilde{\pi}$, then the IR does not bind; otherwise, it binds. This completes the proof. □

We now show that the menu of contracts characterized in Lemma 8.3 is always dominated by that of Lemma 8.5 if the contract outcome s is informative. In other words, the buyer is better off exploiting the contract outcomes that are informative of the seller type.

Lemma 8.6. *Suppose $\overline{F}(s) \neq \underline{F}(s)$ for some contract outcome s in the support. Then the menu of contracts characterized in Lemma 8.5 minimizes the total expected transfer.*

Proof. Given that there are two types, the optimal menu includes two contracts. By Lemma 8.1, we have shown that at least one of them must be a fixed contract, and it is optimal to induce the low-cost type to choose a fixed contract in the menu, as shown in Lemma 8.2. There are two possibilities: one is to offer two fixed contracts, as characterized in Lemma 8.3, and the other is to offer one fixed contract for the low-cost type and one variable contract for the high-cost type, as characterized in Lemma 8.5. We show that the latter is cheaper than the former.

The expected total cost of offering the two fixed contracts to n bidders, as characterized in Lemma 8.3, denoted by T_n^F, is:

$$T_n^F = (1-\pi)^n(\alpha+\beta) + [1-(1-\pi)^n]\,\alpha + \pi(1-\pi)^{n-1}\beta = \alpha + (1-\pi)^{n-1}\beta.$$

Denoting by T_n^V, the total cost of offering the menu of contracts of Lemma 8.5 is:

$$\begin{aligned} T_n^V &= [1-(1-\pi)^n]\left[\alpha + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\{\beta-\gamma\}\right] + (1-\pi)^n[\alpha+\beta+r] \\ &= \alpha + (1-\pi)^{n-1}\{\beta - \pi\gamma + (1-\pi)r\}. \end{aligned}$$

Thus $T_n^V < T_n^F$ if and only if:

$$\Gamma \equiv -\pi\gamma + (1-\pi)r < 0.$$

This condition is satisfied if and only if $T_1^V < T_1^F = \alpha + \beta$. We complete the proof by showing that this inequality holds. The proof is done by construction that it is less profitable to offer one fixed contract than a menu of two contracts.

For some $\epsilon > 0$, we define $S \equiv \{s : \overline{f}(s) - \underline{f}(s) > \epsilon\}$. Let the probability that an outcome s is in S conditional on that the seller is low-cost as γ_1 and that conditional on that the seller is high-cost as γ_2. If $\overline{F}(s) \neq \underline{F}(s)$ for some outcome s in the support, there exists $\epsilon > 0$ such that $\gamma_1 \neq 0$ and $\gamma_2 \neq 0$. Note that $\gamma_2 > \gamma_1$. For any $\delta > 0$ choose $\mu(\delta)$ for a two-part variable contract in which $\overline{p} = c + \beta$ and:

$$q(s) = \begin{cases} \delta \text{ if } s \in S, \\ \mu(\delta) \text{ if } s \notin S, \end{cases}$$

where

$$\gamma_2\psi(\delta) + (1-\gamma_2)\,\psi(\mu(\delta)) = 0.$$

Note that the above equation implies that $\mu(\delta) < 0$. Because $\psi(\cdot)$ is strictly increasing, $\mu(\delta)$ is uniquely defined by the equation:

$$\mu(\delta) = \psi^{-1}\left[\frac{-\gamma_2}{1-\gamma_2}\psi(\delta)\right],$$

and is twice differentiable with:

$$\mu'(\delta) = \frac{-\gamma_2}{1-\gamma_2}\frac{\psi'(\delta)}{\psi'(\mu(\delta))},$$

where $\mu(0) = 0$. The fixed contract takes the form:

$$\begin{aligned} \underline{p} &= \alpha + \beta + \gamma_1 \psi(\delta) + (1-\gamma_1)\,\psi(\mu(\delta)), \\ &= \alpha + \beta + \gamma_1 \psi(\delta) - (1-\gamma_1)\left(\frac{\gamma_2}{1-\gamma_2}\right)\psi(\delta). \end{aligned}$$

Note that the IC constraint is satisfied with equality by the low-cost type and strict inequality by the high-cost type because $\gamma_1 < \gamma_2$. Similarly, the participation constraint is satisfied with equality by the high-cost type and strict inequality by the low-cost type as long as $\delta > 0$ is small enough. The expected price to the buyer is:

$$\begin{aligned} \mathbb{E}(T|\delta) &= \alpha + \beta + \pi\left[\gamma_1\psi(\delta) + (1-\gamma_1)\,\psi(\mu(\delta))\right] + (1-\pi)\left[\gamma_2\delta + (1-\gamma_2)\,\mu(\delta)\right], \\ &= \alpha + \beta + \pi\left[\gamma_1\psi(\delta) - \frac{(1-\gamma_1)\,\gamma_2}{1-\gamma_2}\psi(\delta)\right] + (1-\pi)\left[\gamma_2\delta + (1-\gamma_2)\,\mu(\delta)\right]. \end{aligned}$$

We now show this expression is decreasing in the neighborhood of $\delta = 0$. Differentiating with respect to δ yields:

$$\frac{\partial\mathbb{E}(T|\delta)}{\partial\delta} = \pi\left[\gamma_1\psi'(\delta) - \frac{(1-\gamma_1)\,\gamma_2}{1-\gamma_2}\psi'(\delta)\right] + (1-\pi)\left[\gamma_2 - \gamma_2\frac{\psi'(\delta)}{\psi'(\mu(\delta))}\right].$$

Evaluating $\frac{\partial\mathbb{E}(T|\delta)}{\partial\delta}$ at $\delta = 0$ gives us:

$$\frac{\partial\mathbb{E}(T|\delta = 0)}{\partial\delta} = \pi\frac{\gamma_1 - \gamma_2}{1-\gamma_2} < 0,$$

which shows that a fixed contract fails to meet a first order necessary condition. □

A.2. Proof of Lemma 5.1. If A3 holds, then $q(s)$ satisfies the first order condition, (4). Rewriting (4) while replacing $q(s)$ by $q(s,\pi)$ to emphasize the dependence of q on π,

$$\psi'\left[q(s,\pi)\right]\left[1 - \pi l(s)\right] = 1 - \pi.$$

Note that $q(s,\pi) = 0$ if $l(s) = 1$ and $q(s,\pi) > 0$ if $l(s) < 1$. Similarly $q(s,\pi) < 0$ if $l(s) > 1$. Totally differentiating the first order condition with respect to π yields:

$$\psi''\left[q(s,\pi)\right]\frac{\partial q(s,\pi)}{\partial\pi}\left[1 - \pi l(s)\right] - \psi'\left[q(s,\pi)\right] l(s) = -1.$$

Rearranging to make $\partial q(s,\pi)/\partial\pi$ the subject of the equation gives:

$$\frac{\partial q(s,\pi)}{\partial\pi} = \frac{l(s) - 1}{\psi''\left[q(s,\pi)\right]\left[1 - \pi l(s)\right]^2}.$$

Noting $\psi''(\cdot) < 0$ it follows that $\partial q(s,\pi)/\partial\pi > 0$ when $l(s) < 1$ and $\partial q(s,\pi)/\partial\pi < 0$ when $l(s) > 1$. Therefore,

$$\frac{\partial q(s,\pi)}{\partial\pi} \begin{array}{l} > 0 \text{ if } q(s,\pi) > 0, \\ = 0 \text{ if } q(s,\pi) = 0, \\ < 0 \text{ if } q(s,\pi) < 0. \end{array}$$

as was to be proved.

Rewriting (2) while making the dependence of $\underline{p}_n$, $q(s)$, α, and β on π:

$$\underline{p}_n(\pi) = \alpha(\pi) + \frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\left[\beta(\pi) - \int \psi(q(s,\pi))[\overline{f}(s) - \underline{f}(s)]ds\right].$$

To show that $\underline{p}_n'(\pi) < 0$ we consider the two expressions involving π separately. First:

$$\frac{\partial}{\partial\pi}\ln\left[\frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\right] = \frac{1-n\pi-(1-\pi)^n}{\pi(1-\pi)\left[1-(1-\pi)^n\right]}$$

Note that the derivative is zero at $n = 1$ and that at $n = 2$ is $-\pi^2$, which is negative. Now suppose it is negative for all $n \in \{2, \ldots, n_0\}$. Then for $n_0 + 1$ the denominator is clearly positive and the numerator is:

$$1-(n_0+1)\pi-(1-\pi)(1-\pi)^{n_0} < \pi(1-\pi)^{n_0} - \pi < 0.$$

The first inequality follows from an induction hypothesis, and the second one from the inequalities $0 < \pi < 1$. Therefore $\pi(1-\pi)^{n-1}/\pi(1-\pi)^{n-1}$ is decreasing in π for all $n > 1$.

Second, we note that:

$$\frac{\partial}{\partial\pi}\int \psi[q(s,\pi)][1-l(s)]\overline{f}(s)\,ds = \int \psi'[q(s,\pi)]\frac{\partial q(s,\pi)}{\partial\pi}[1-l(s)]\overline{f}(s)\,ds$$

$$= \int (1-\pi)\frac{\partial q(s,\pi)}{\partial\pi}\left[\frac{1-l(s)}{1-\pi l(s)}\right]\overline{f}(s)\,ds = \int \frac{(\pi-1)[1-l(s)]^2}{\psi''[q(s,\pi)][1-\pi l(s)]^3}\overline{f}(s)\,ds > 0.$$

The second equality follows from using the first order condition to substitute out $\psi'[q(s,\pi)]$. Note that we can use the first order condition because A3 holds. The third equality results from the expression we derived for $\partial q(s,\pi)/\partial\pi$. The last inequality appeals to the concavity of $\psi(\cdot)$. Finally note that since the participation constraint is satisfied with an inequality for the low-cost seller under A3.

$$\beta(\pi) - \int \psi[q(s,\pi)][1-l(s)]\overline{f}(s)\,ds > 0,$$

for all $\pi \in \Pi$. Hence, if $\alpha(\pi)$ and $\beta(\pi)$ are nonincreasing in π, as assumed in **A4**, the following inequality holds as claimed.

$$\begin{aligned}\frac{\partial}{\partial \pi}\underline{p}_n(\pi) &= \alpha'(\pi) + \frac{\partial}{\partial \pi}\left[\frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\right]\left\{\beta(\pi) - \int \psi[q(s,\pi)][1-l(s)]\overline{f}(s)\,ds\right\} \\ &+ \frac{\pi(1-\pi)^n}{1-(1-\pi)^n}\left\{\beta'(\pi) - \int \frac{[1-l(s)]^2}{\psi''[q(s,\pi)][1-\pi l(s)]^3}\overline{f}(s)\,ds\right\} < 0.\end{aligned}$$

Recall that

$$\overline{p} = \alpha + \beta - \int \psi[q(s)]\overline{f}(s)\,ds.$$

Differentiating with respect to π yields:

$$\begin{aligned}\frac{\partial \overline{p}}{\partial \pi} &= \frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} - \int \psi'[q(s)]\frac{\partial q(s;\pi)}{\partial \pi}\overline{f}(s)\,ds \\ &= \frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} - \int h'\left[\frac{1-\pi}{1-\pi l(s)}\right]\frac{1-\pi}{1-\pi l(s)}\frac{(1-\pi)l(s)-[1-\pi l(s)]}{[1-\pi l(s)]^2}\overline{f}(s)\,ds \\ &= \frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} + \int h'\left[\frac{1-\pi}{1-\pi l(s)}\right]\frac{1}{[1-\pi l(s)]^2}\frac{(1-\pi)}{1-\pi l(s)}[1-l(s)]\overline{f}(s)\,ds.\end{aligned}$$

First note that the integral is negative for $l(s) < 1$ and positive for $l(s) > 1$ because $h'(\cdot) < 0$ and $\pi l(s) < 1$ by A3. Therefore,

$$\frac{\partial \overline{p}}{\partial \pi} < \frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} + \int_{l(s)>1} h'\left[\frac{1-\pi}{1-\pi l(s)}\right]\frac{(1-\pi)[1-l(s)]}{[1-\pi l(s)]^3}\overline{f}(s)\,ds.$$

We define $m(\pi, l)$ by

$$m(\pi, l) \equiv h'\left[\frac{1-\pi}{1-\pi l}\right]\frac{(1-\pi)[1-l]}{[1-\pi l]^3}.$$

It can be seen that the derivative of $m(\pi, l)$ with respect to the second argument is positive if $h'' > 0$ and $l > 1$.

$$\begin{aligned}&\frac{\partial m(\pi, l)}{\partial l} \\ &= h''\left[\frac{1-\pi}{1-\pi l}\right]\frac{(1-\pi)^2[l-1]\pi}{[1-\pi l]^5} - h'\left[\frac{1-\pi}{1-\pi l}\right]\frac{(1-\pi)}{[1-\pi l]^3} + 3h'\left[\frac{1-\pi}{1-\pi l}\right]\frac{\pi(1-\pi)[1-l]}{[1-\pi l]^4} \\ &= h''\left[\frac{1-\pi}{1-\pi l}\right]\frac{(1-\pi)^2[l-1]\pi}{[1-\pi l]^5} + h'\left[\frac{1-\pi}{1-\pi l}\right](1-\pi)[2\pi(1-l)+\pi-1].\end{aligned}$$

Therefore,

$$\frac{\partial\overline{p}}{\partial\pi} < \frac{\partial\alpha}{\partial\pi} + \frac{\partial\beta}{\partial\pi} + h'\left[\frac{1-\pi}{1-\pi\tilde{l}\left(\pi,M\right)}\right]\frac{\left(1-\pi\right)\left[1-\tilde{l}\left(\pi,M\right)\right]}{\left[1-\pi\tilde{l}\left(\pi,M\right)\right]^{3}}. \tag{25}$$

Using the definition of $\tilde{l}\left(\pi,M\right) \equiv \frac{1}{\pi} - \frac{1-\pi}{\pi\psi'(M)}$, we have

$$\begin{aligned} 1-\widetilde{l}\left(\pi,M\right) &= 1-\frac{1}{\pi}+\frac{1-\pi}{\pi\psi'\left(M\right)} = \frac{\pi\psi'\left(M\right)-\psi'\left(M\right)+\left(1-\pi\right)}{\pi\psi'\left(M\right)} \\ &= \frac{\left(1-\pi\right)\left(1-\psi'\left(M\right)\right)}{\pi\psi'\left(M\right)}, \\ 1-\pi\widetilde{l}\left(\pi,M\right) &= 1-1+\frac{1-\pi}{\psi'\left(M\right)} = \frac{1-\pi}{\psi'\left(M\right)}. \end{aligned}$$

Using these, we simplify the RHS of (25) as:

$$\frac{\partial\overline{p}}{\partial\pi} < \frac{\partial\alpha}{\partial\pi} + \frac{\partial\beta}{\partial\pi} + \frac{h'\left[\psi'(M)\right]\left[\psi'(M)\right]^{2}\left[1-\psi'(M)\right]}{\pi(1-\pi)}.$$

Therefore, $\frac{\partial\overline{p}}{\partial\pi} < 0$.

A.3. Proof of Lemma 5.2. Appealing to **A5**, $\overline{p}$ is monotone decreasing in π, implying the existence of a mapping $\pi\left(\overline{p}\right)$ such that $l^{*}\left(q,\overline{p}\right)$ defined in (10) satisfies:

$$\psi'\left(q\right) = \frac{1-\pi}{1-\pi l^{*}\left(q,\overline{p}\right)}.$$

Making π the subject we obtain:

$$\pi = \frac{1-\psi'\left(q\right)}{1-l^{*}\left(q,\overline{p}\right)\psi'\left(q\right)}.$$

Differentiating with respect to q holding π and $\overline{p}$ constant yields:

$$\psi''\left(q\right) = \frac{\pi}{1-\pi l^{*}\left(q,\overline{p}\right)}\psi'\left(q\right)\frac{\partial l^{*}\left(q,\overline{p}\right)}{\partial q}.$$

Using these two equations we substitute π out to obtain (11).

A.4. Proof of Lemma 5.3. The joint probability that the contract type is fixed and $\pi \leq \pi^{*}$ can be expressed as:

$$\begin{aligned} \Pr\left\{\pi \leq \pi^{*}, v=0\left|c,n\right.\right\} &= F_{\pi|c,n,v}\left(\pi^{*}\left|c,n,0\right.\right)\Pr\left(v=0\left|c,n\right.\right) \\ &= \int_{\pi=\underline{\pi}}^{\pi^{*}} f_{\pi|c,n}\left(\pi\left|c,n\right.\right)\left[1-\left(1-\pi\right)^{n}\right]d\pi. \end{aligned} \tag{26}$$

Taking the derivative with respect to π^* yields:

$$f_{\pi|c,n,v}\left(\pi^* \left|c,n,0\right.\right)\Pr\left(v=0\left|c,n\right.\right)=f_{\pi|c,n}\left(\pi^*\left|c,n\right.\right)\left[1-\left(1-\pi^*\right)^n\right]. \tag{27}$$

Similarly:

$$\begin{aligned}\Pr\left\{\pi\le\pi^*,v=1\left|c,n\right.\right\} &= F_{\pi|c,n,v}\left(\pi^*\left|n,v=1\right.\right)\Pr\left(v=1\left|c,n\right.\right)\\ &= \int_{\pi=\underline{\pi}}^{\pi^*} f_{\pi|c,n}\left(\pi\left|c,n\right.\right)\left(1-\pi\right)^n d\pi,\end{aligned}$$

and taking the derivative with respect to π^* yields:

$$f_{\pi|c,n,v}\left(\pi^*\left|c,n,1\right.\right)\Pr\left(v=1\left|c,n\right.\right)=f_{\pi|c,n}\left(\pi^*\left|c,n\right.\right)\left(1-\pi^*\right)^n. \tag{28}$$

Rearranging the quotient of (27) and (28) to make $f_{\pi|c,n,v}\left(\pi^*\left|c,n,v=0\right.\right)$ the subject of the resulting equation, and relabeling π^* as x, we obtain:

$$f_{\pi|c,n,v}\left(x\left|c,n,0\right.\right)=\varphi_{c,n}\frac{1-\left(1-x\right)^n}{\left(1-x\right)^n}f_{\pi|c,n,v}\left(x\left|c,n,1\right.\right), \tag{29}$$

where $\varphi_{c,n}\equiv\Pr\left\{v=1\left|c,n\right.\right\}/\Pr\left\{v=0\left|c,n\right.\right\}$, as defined in the text.

Appendix B. Estimation and Counterfactual Analyses

B.1. Optimal Competition under the Parametric Specification. Under the specification on $\kappa(\pi,n)$ in (14), the expected total cost of competed procurement with effort λ, denoted by $U(\pi,\lambda)$, is:

$$\begin{aligned}U(\pi,\lambda) &= \alpha(\pi)+\exp^{-\lambda\pi}[\beta(\pi)+\Gamma(\pi)]+\mathbb{E}[\kappa(\pi,j+1)|\lambda]+\eta\\ &= \alpha(\pi)+\exp^{-\lambda\pi}[\beta(\pi)+\Gamma(\pi)]+\tilde{\kappa}_1(\pi)\lambda+\tilde{\kappa}_2(\pi)\lambda(1+\lambda)+\eta,\end{aligned}$$

where $\tilde{\kappa}_1(\pi)\equiv\kappa_1+\kappa_2\pi$ and $\tilde{\kappa}_2(\pi)\equiv\kappa_3+\kappa_4\pi$.

Taking the first order condition:

$$\pi\exp^{-\lambda\pi}[\beta(\pi)+\Gamma(\pi)]=\tilde{\kappa}_1(\pi)+\tilde{\kappa}_2(\pi)(1+2\lambda). \tag{30}$$

Because the LHS is decreasing in λ while the RHS is increasing in λ, there exists a unique solution to the above equation for any given π, denoted by $\tilde{\lambda}(\pi)$. Because $\lambda\ge 0$, $\lambda^*(\pi)=\max\{\tilde{\lambda}(\pi),0\}$. In our estimation, we numerically solve for $\lambda^*(\pi)$ for each π.

Given $\lambda^*(\pi)$, it is optimal for the buyer to hold a competitive solicitation if and only if

$$U[\pi,\lambda^*(\pi)]\le U_0(\pi).$$

The above inequality can be rewritten as:

$$\eta \leq (1 - e^{-\lambda^*(\pi)\pi}[\beta(\pi) + \Gamma(\pi)] - \tilde{\kappa}_1 \lambda^*(\pi) - \tilde{\kappa}_2 \lambda^*(\pi)[1 + \lambda^*(\pi)]. \tag{31}$$

Equations (2), (3), (4), (30), and (31) characterize the equilibrium contracts and competition. Given the parameters of the model and these equations, we simulate the model.

B.2. Simulated GMM Estimator. Let us denote the vector of the parameters of the model by θ. Our estimator minimizes a weighted sum of squared distances:

$$g_n(\theta)' W g_n(\theta), \text{ with } g_n(\theta) = \frac{1}{n} \sum_{t=1}^{n} g(w_t; \theta),$$

where W is a symmetric positive-definite weighting matrix. The $g(w_t; \theta)$ vector is associated with 40 moment conditions: (i) 17 moment conditions on competition, contract type, and contract price for all projects, (ii) the same 17 moment conditions for non-military projects, and (iii) 6 moment conditions on the distribution of s, or the standardized delay. The 17 moment conditions consist of $\Pr(c_i = 0)$, $\Pr(n_i = 1|c_i = 1)$, $\Pr(n_i \leq 2|c_i = 1)$, $\Pr(n_i \leq 5|c_i = 1)$, $\Pr(v_i = 0|c_i = 0)$, $\Pr(v_i = 0|c_i = 1)$, $\Pr(v_i = 0|c_i = 1, n_i = 1)$, $\Pr(v_i = 0|c_i = 1, n_i \leq 2)$, $\Pr(v_i = 0|c_i = 1, n_i \leq 5)$, $\mathbb{E}[n_i]$, $\mathbb{E}[p_i|c_i = c, v_i = v]$ for $(c, v) \in \{0, 1\} \times \{0, 1\}$ where $p_i \equiv \underline{p}_i(1 - v_i) + (\overline{p}_i + q_i)v_i$, $\mathbb{E}[p_i|c_i = 1, n_i = 1]$, $\mathbb{E}[p_i|c_i = 1, n_i \leq 2]$, and $\mathbb{E}[p_i|c_i = 1, n_i \leq 5]$. The 6 moment conditions on the distribution of the standardized delay are $\Pr(s_i = 0, v_i = 0)$, $\Pr(s_i = 0, v_i = 1)$, $\Pr[s_i(1 - v_i)]$, $\Pr[s_i^2(1 - v_i)]$, $\Pr[s_i v_i]$, and $\Pr[s_i^2 v_i]$. Note that the moments as a function of θ are calculated using simulation. In our estimation, the simulation size is 5,000.

We use the two-step procedure to obtain the efficient simulated GMM estimator. We start with a positive definite weighting matrix and obtain a first-step estimator, denoted by $\tilde{\theta}_n$. The asymptotic variance of $\sqrt{n} g_n(\theta_0)$, S, is estimated by:

$$\hat{S} \equiv \frac{1}{n} \sum_t g(w_t, , \tilde{\theta}_n) g(w_t; \tilde{\theta}_n)'.$$

Then we re-estimate the parameters using the optimal weighting matrix $\hat{S}^{-1}$. Let us denote this efficient simulated GMM estimator by $\hat{\theta}_n$.

Under certain regularity conditions, the efficient simulated GMM estimator is asymptotically normally distributed. A consistent estimator of the asymptotic variance of

$\sqrt{n}(\hat{\theta}_n - \theta_0)$ is:

$$\left(\frac{\partial g_n(\hat{\theta}_n)}{\partial \theta'}' \hat{S}^{-1} \frac{\partial g_n(\hat{\theta}_n)}{\partial \theta'} \right)^{-1}.$$

Since the moments are calculated by simulation, we use a numerical derivative of $g_n(\cdot)$ to estimate the asymptotic variance of the estimator.

C. Counterfactual Analyses. We consider five counterfactual policies and report the results in Table 10. Under Policies 1, 4, and 5, negotiations are not allowed and first-price auctions sealed-bid are the only available competitive solicitation procedure. This scenario is identical to a case where only two fixed contracts are allowed to be offered. In Lemma 8.3, we show that the two fixed contracts are $\alpha+\beta$ and $\alpha+\frac{\pi(1-\pi)^{n-1}}{1-(1-\pi)^n}\beta$, where the priority is given buyers submitting the latter. Given this, we solve backwards to solve for the optimal decision rule on competition for each π.

Policies 4 and 5, on the other hand, do not allow π-specific competition policy and all procurement is mandated to be competed. Under Policy 4, the constrained optimal level of effort, $\lambda^* = 1.06$ (or on average 2.06 bids), is made for all contracts, regardless of π. The last policy is identical to the fourth policy except that we assume that the bid processing costs are halved. Given the new bid processing costs, we derive the constrained optimal level of effort for bids ($\lambda^* = 2.48$, or on average 3.52 bids) and simulate the expected contract outcomes. The derivation of the constrained optimal level of effort is described in Section 6.3.

Lastly, Policies 2 and 3 allow negotiations. However, they both mandate competitive solicitation, and the difference between the two policies lies in that there are no restrictions on the number of bids, while in Policy 3, at least two bids are required. To implement Policy 2, we simply calculate the optimal bid arrival rate for each π, regardless of whether competition is preferred or not. Under Policy 3, the buyer is assumed to be provided with two random bidders for each procurement project. This implies that her problem is to find the rate of bid arrival in addition to the two bids, denoted by $\tilde{\lambda}$, in order to minimize the total expected cost of procurement, defined by:

$$\begin{aligned}\tilde{U}(\pi, \tilde{\lambda}) &= \sum_{n=0}^{\infty} \frac{\tilde{\lambda} e^{-\tilde{\lambda}}}{n!} \{T(\pi, n+2) + \kappa(\pi, n+2)\} \\ &= \alpha(\pi) + [\beta(\pi) + \Gamma(\pi)](1-\pi)e^{-\pi\tilde{\lambda}} + \tilde{\kappa}_1(\pi)(\tilde{\lambda}+1) + \tilde{\kappa}_2(\pi)(\tilde{\lambda}^2 + 3\tilde{\lambda} + 1).\end{aligned}$$

Given this, we solve backwards to solve for the optimal $\tilde{\lambda}(\pi)$ for each π.